

LUKAS GIANINAZZI, TAL BEN-NUN, MACIEJ BESTA, SALEH ASHKBOOS, YVES BAUMANN, PIOTR LUCZYNSKI, TORSTEN HOEFLER

Energy-Optimal and Low-Depth Algorithmic Primitives for Spatial Dataflow Architectures



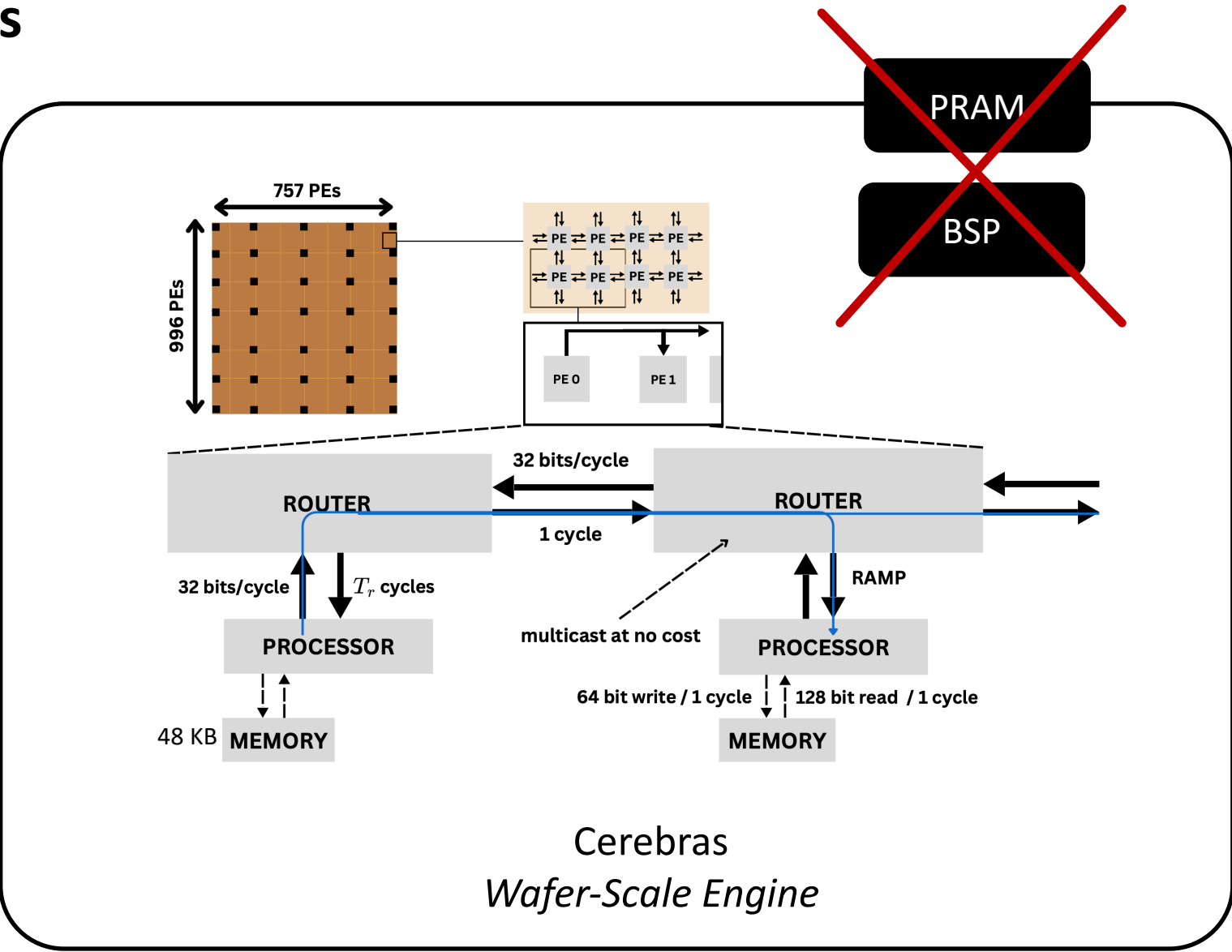
Spatial Dataflow Architectures

AMD
Versal Adaptive SoCs

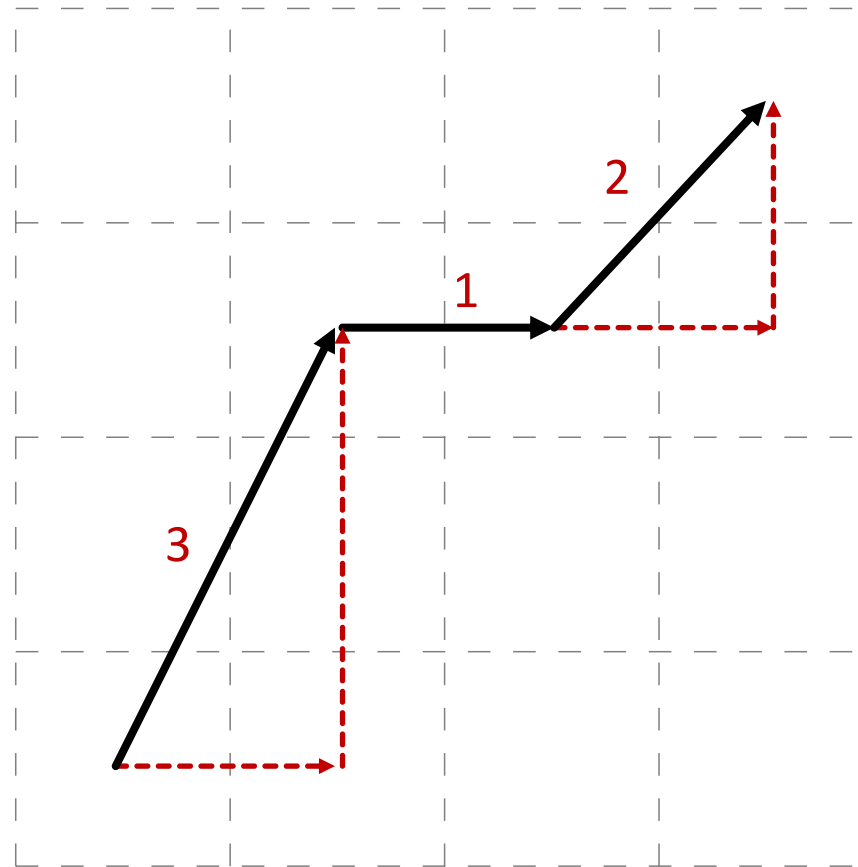
Samba Nova
Reconfigurable Dataflow Architecture

...

Graphcore
IPUs

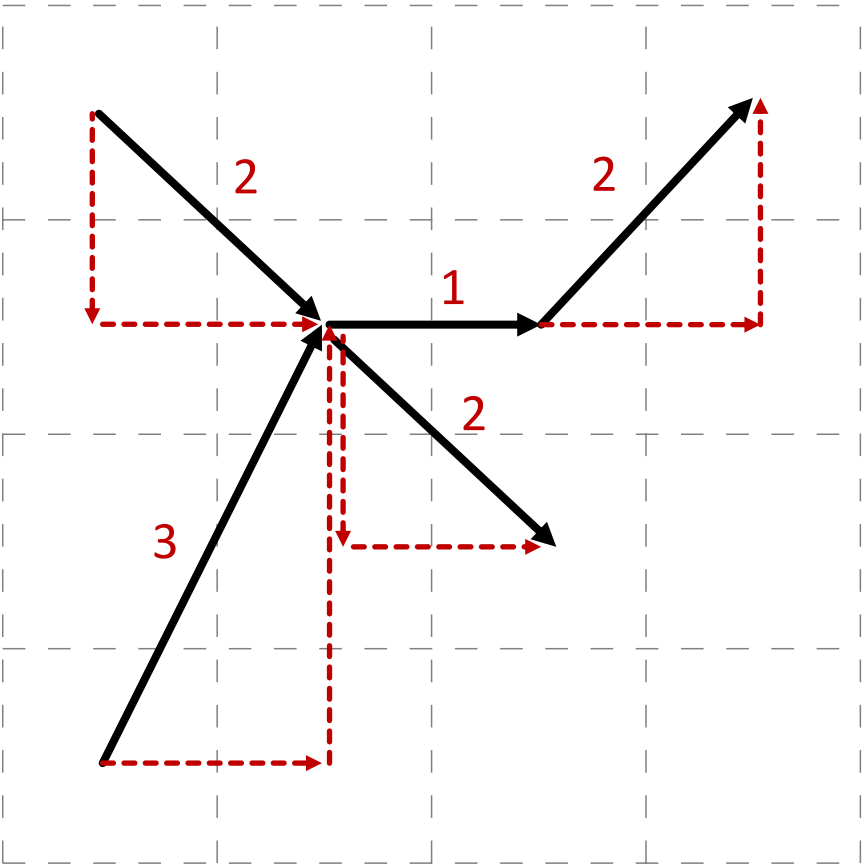


Model of Computation



Distance

Model of Computation

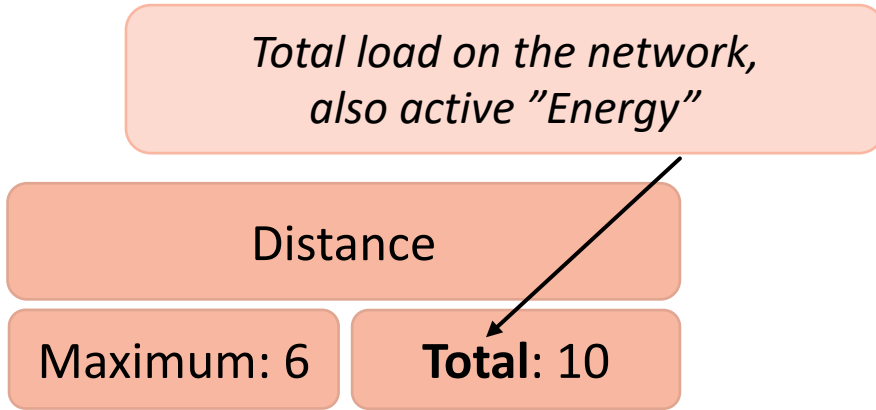
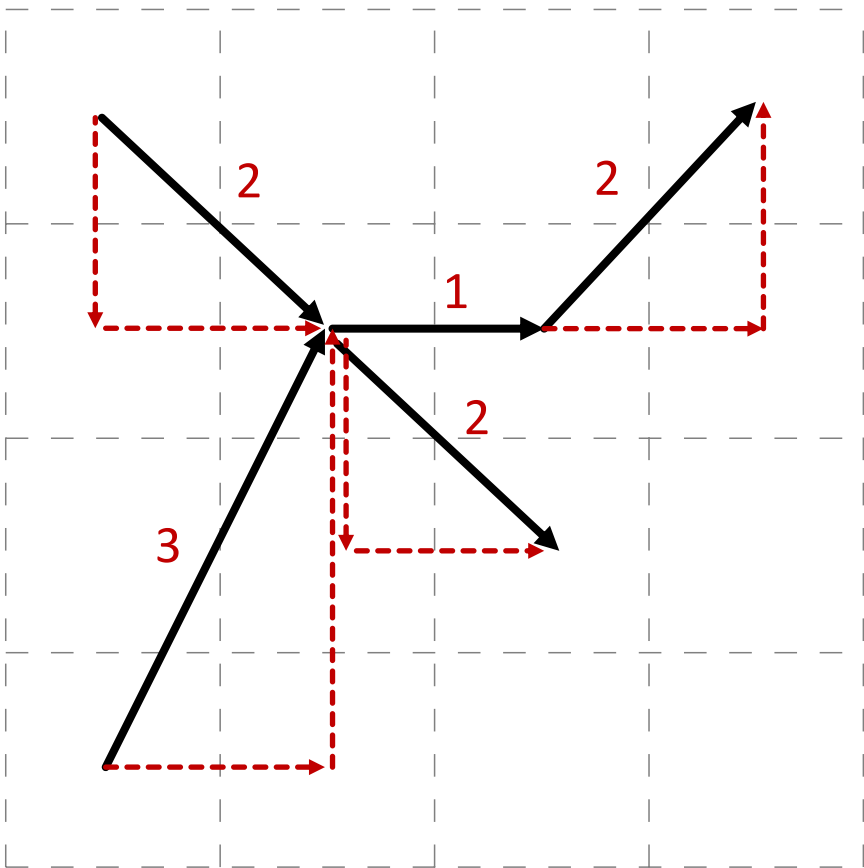


Distance

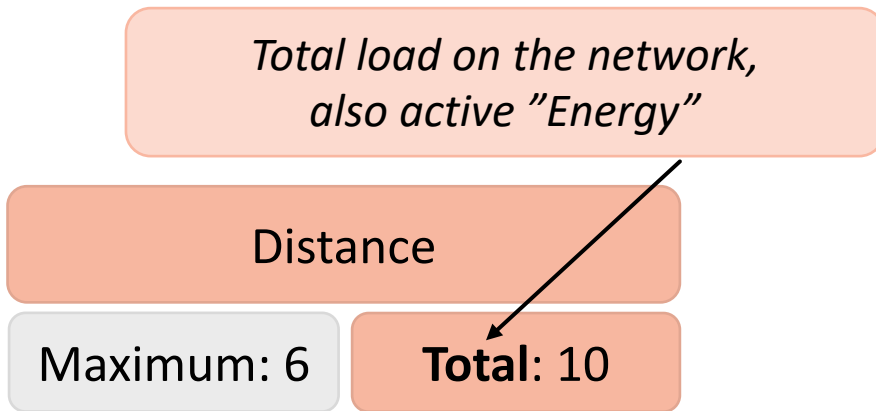
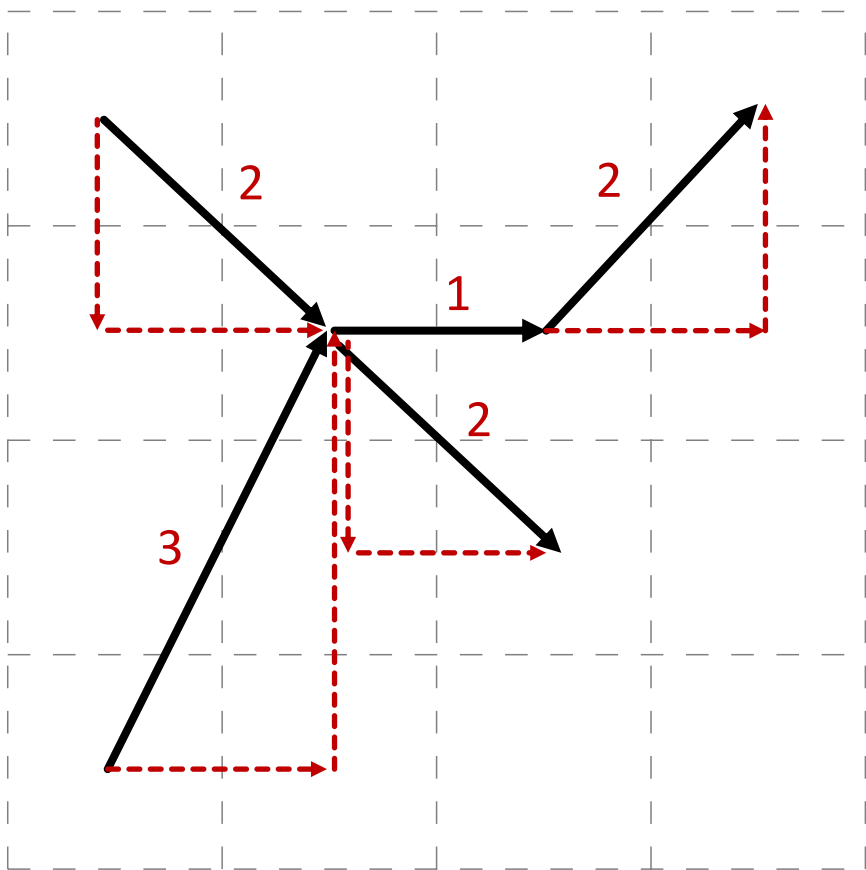
Maximum: 6

Total: 10

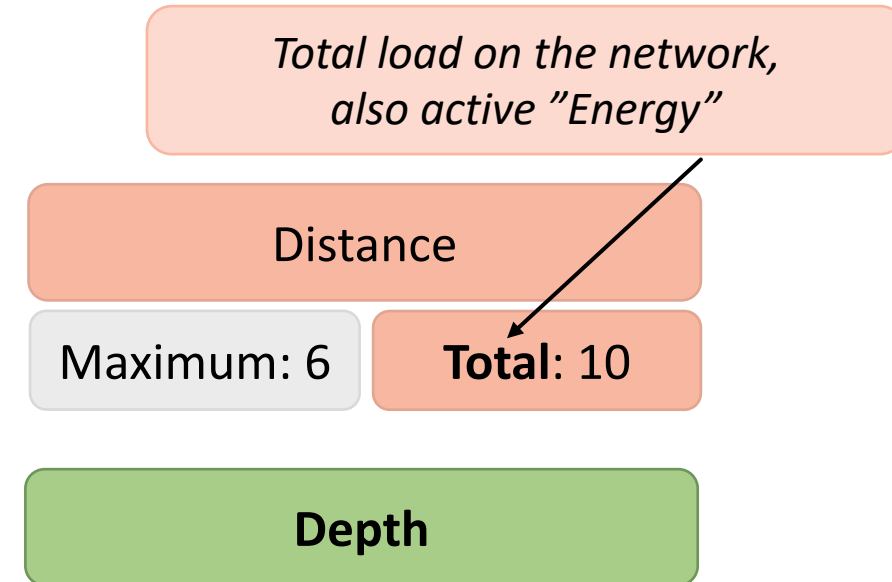
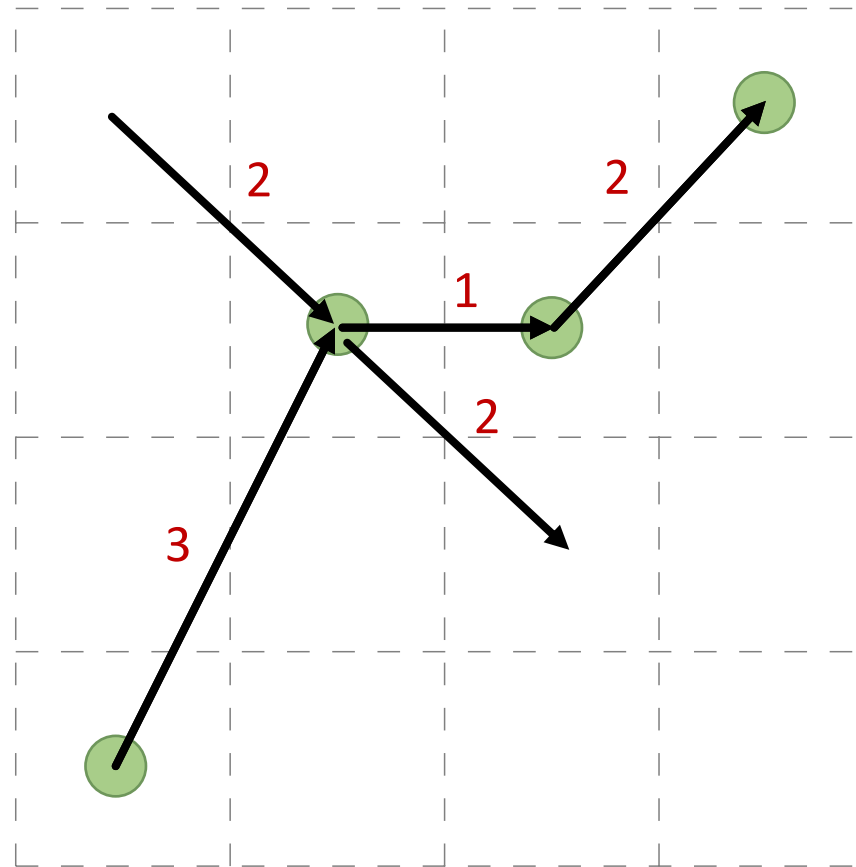
Model of Computation



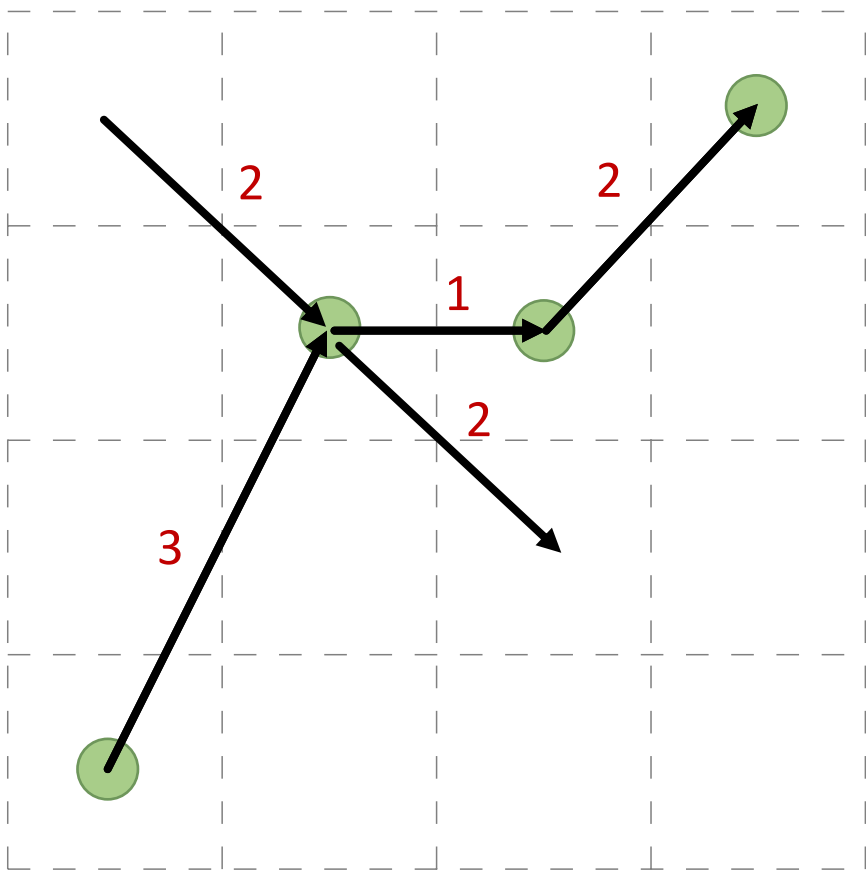
Model of Computation



Model of Computation



Model of Computation



*Total load on the network,
also active "Energy"*

Distance

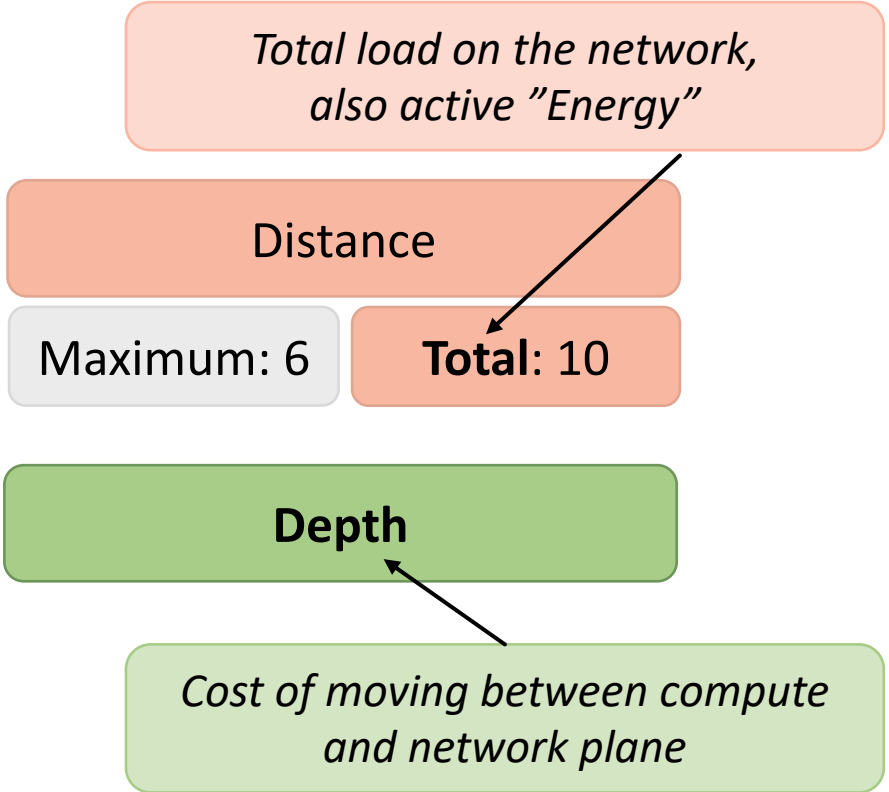
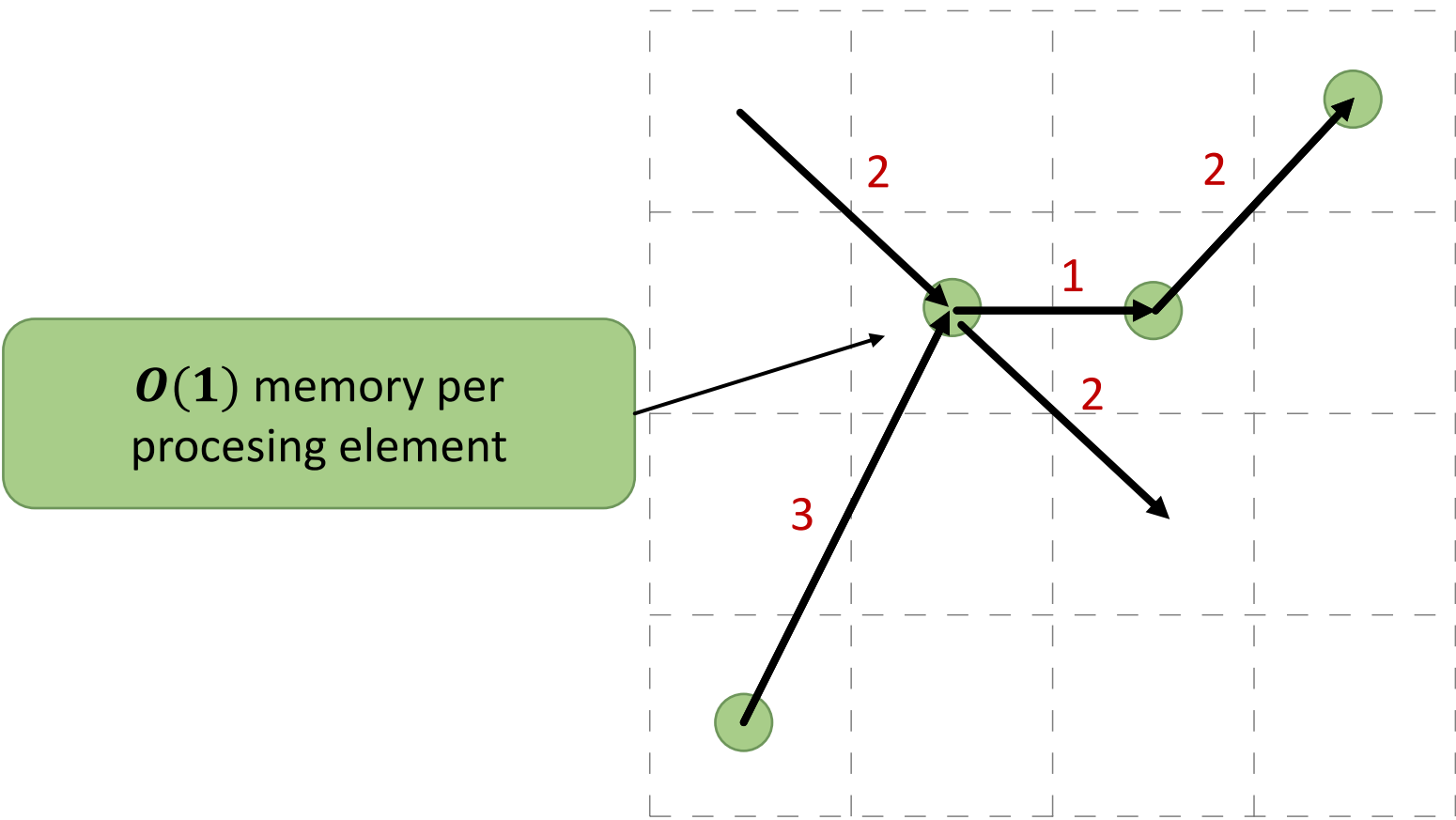
Maximum: 6

Total: 10

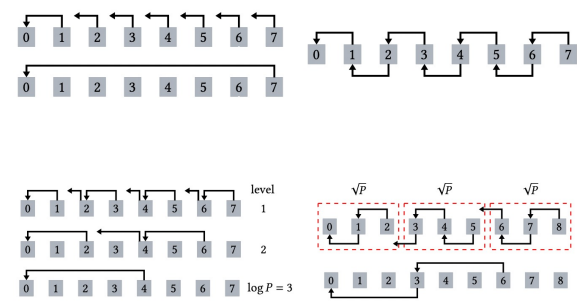
Depth

*Cost of moving between compute
and network plane*

Model of Computation – Spatial Computer Model

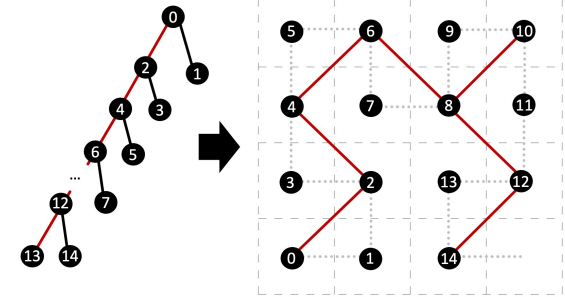


Reduce and AllReduce in Practice



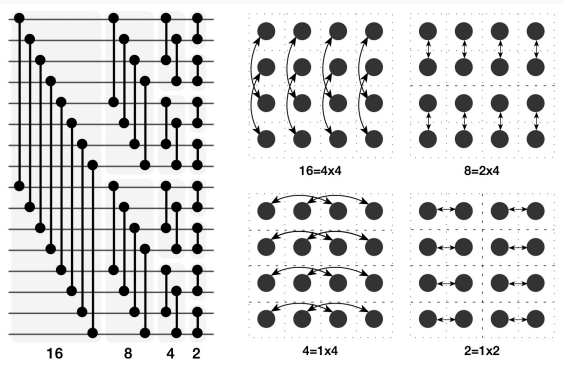
HPDC 2024

Tree Algorithms



IPDPS 2024

Sorting, Selection, Collectives, and more



IPDPS 2025

Sorting in 2D

14	6	1	12
7	0	5	9
3	4	2	8
15	10	11	13



0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

n numbers

Sorting in 2D

14	6	1	12
7	0	5	9
3	4	2	8
15	10	11	13



~~$O(n \log n)$~~ Energy?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

n numbers

Permutation Lower Bound

w

h

6	7	8
9	10	11
0	1	2
3	4	5

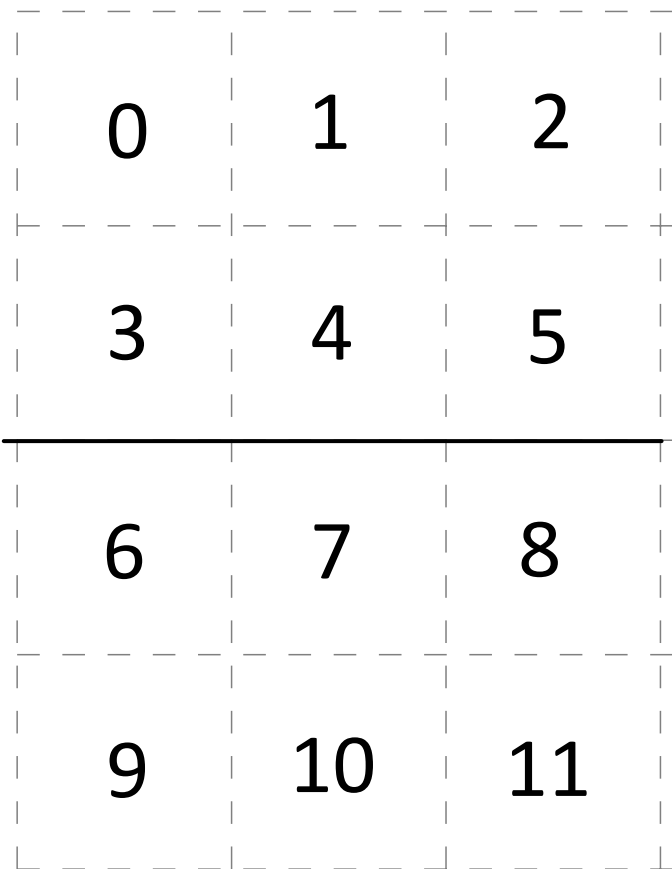
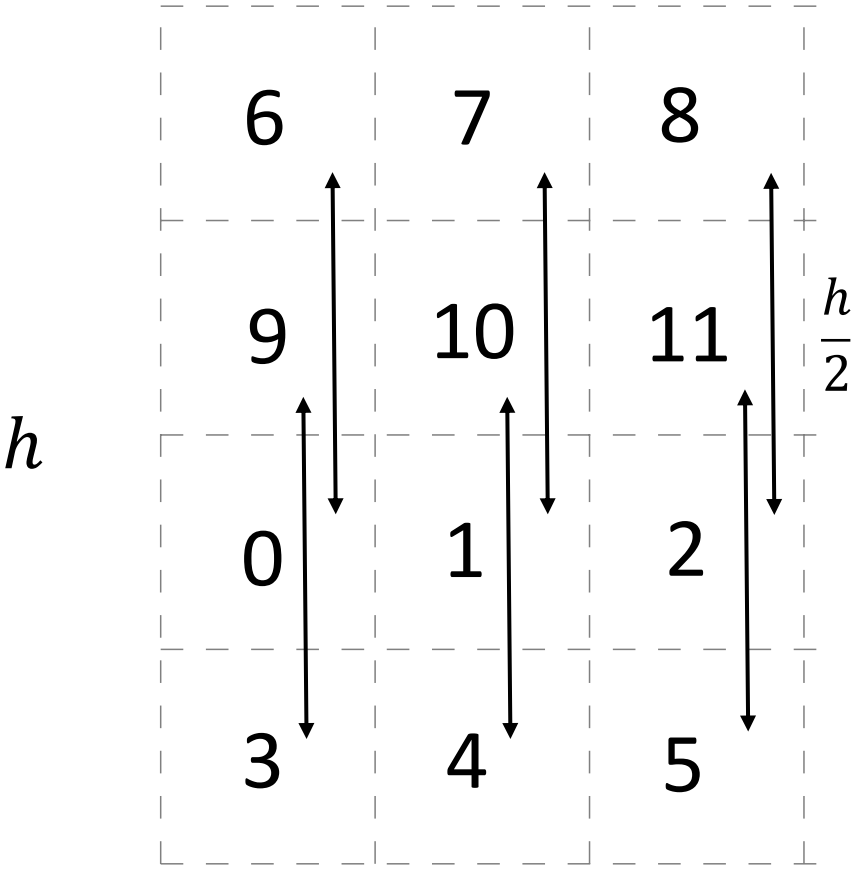


0	1	2
3	4	5
6	7	8
9	10	11

$$n = w \cdot h$$

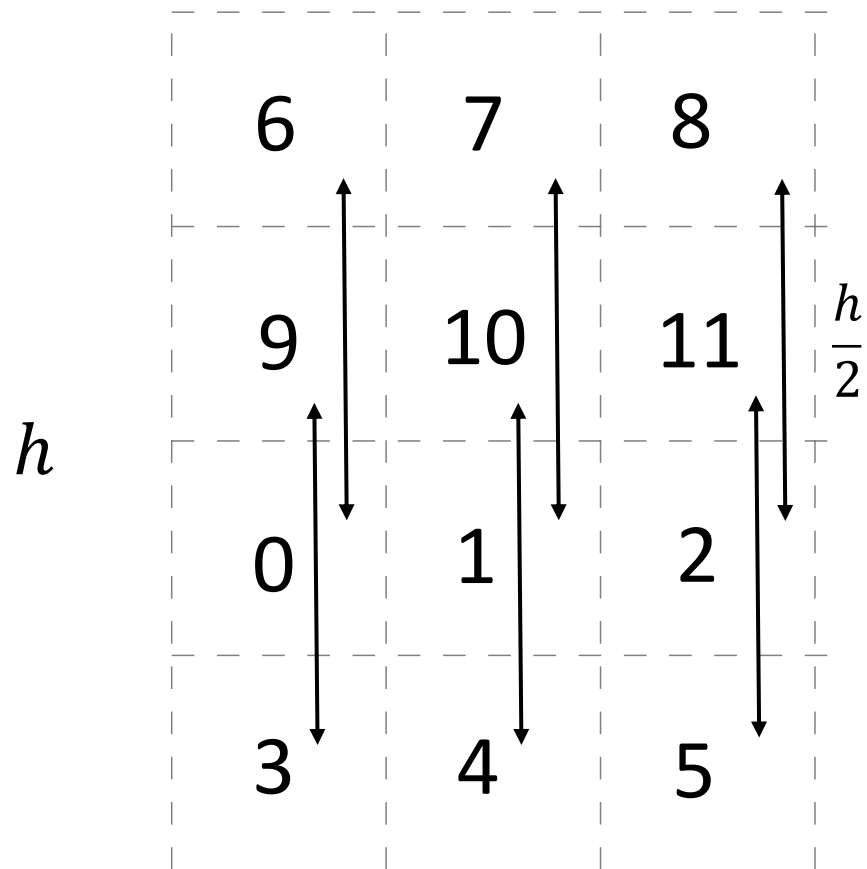
Permutation Lower Bound

w

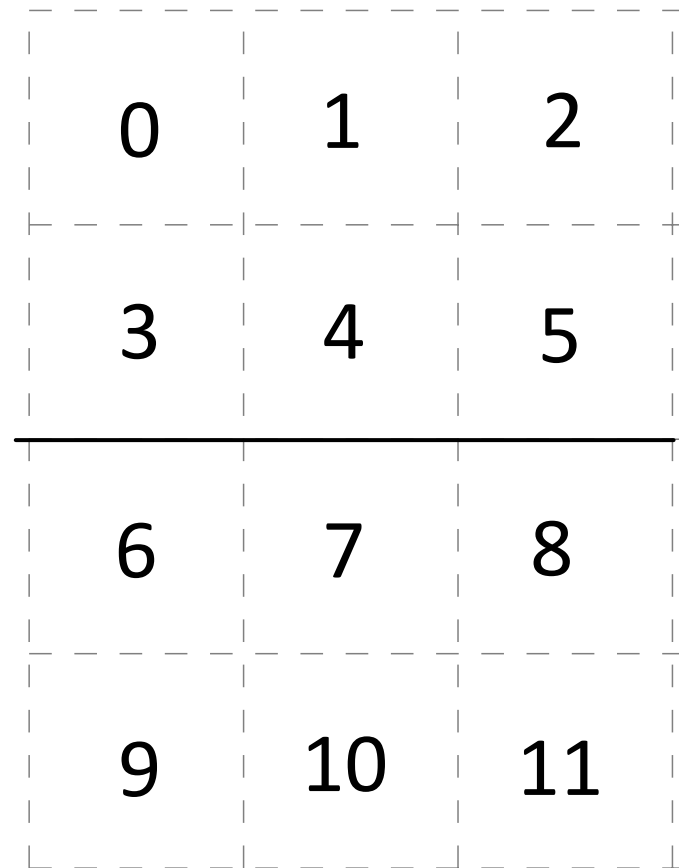


$$n = w \cdot h$$

Permutation Lower Bound

 w

 $\frac{h}{2}wh$ Energy

 1D ($w = 1$): $\Omega(n^2)$

 Square ($w = h$): $\Omega(n^{3/2})$


$$n = w \cdot h$$

Sorting Results

14	6	1	12
7	0	5	9
3	4	2	8
15	10	11	13

n numbers

Polylog Depth

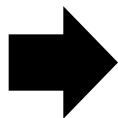
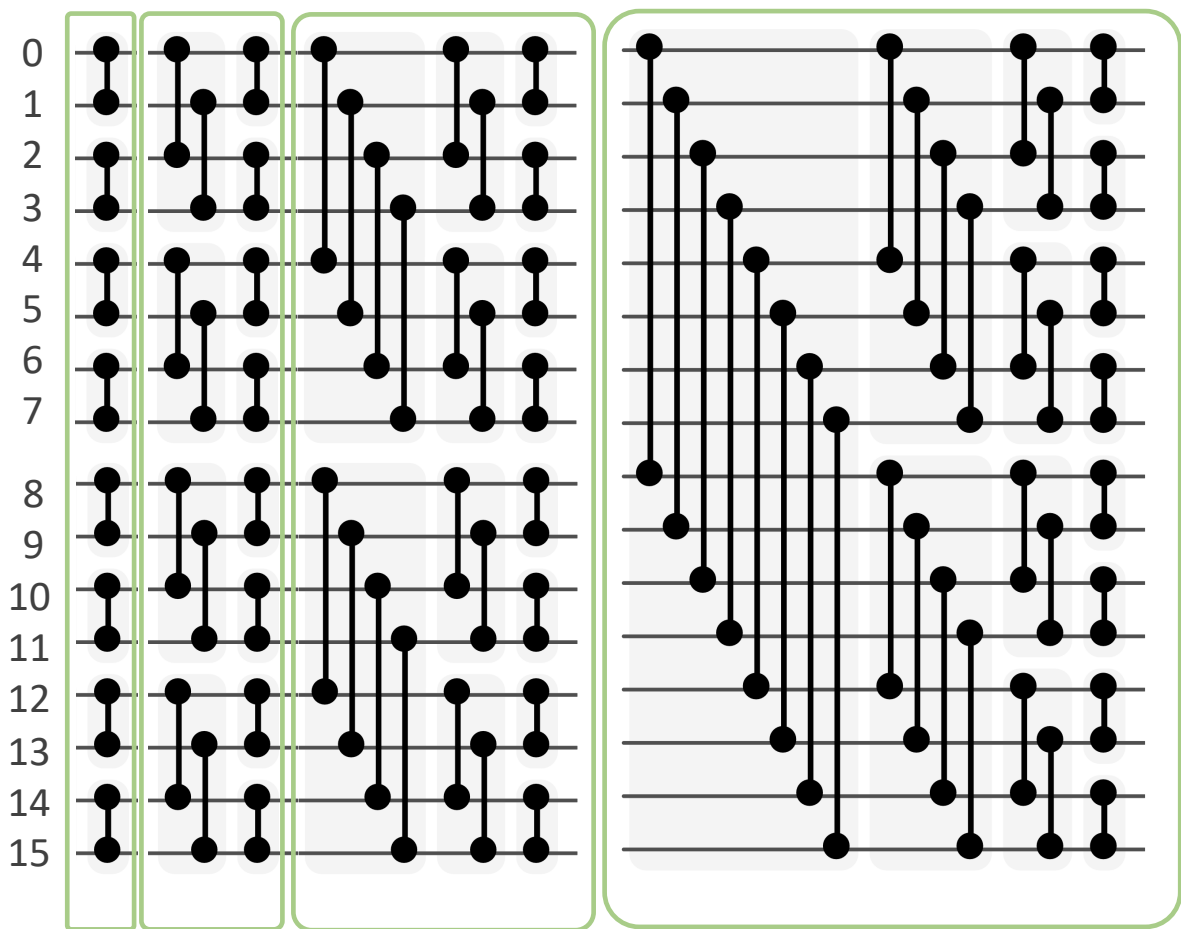


$\Theta(n^{3/2})$ Energy

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Optimal

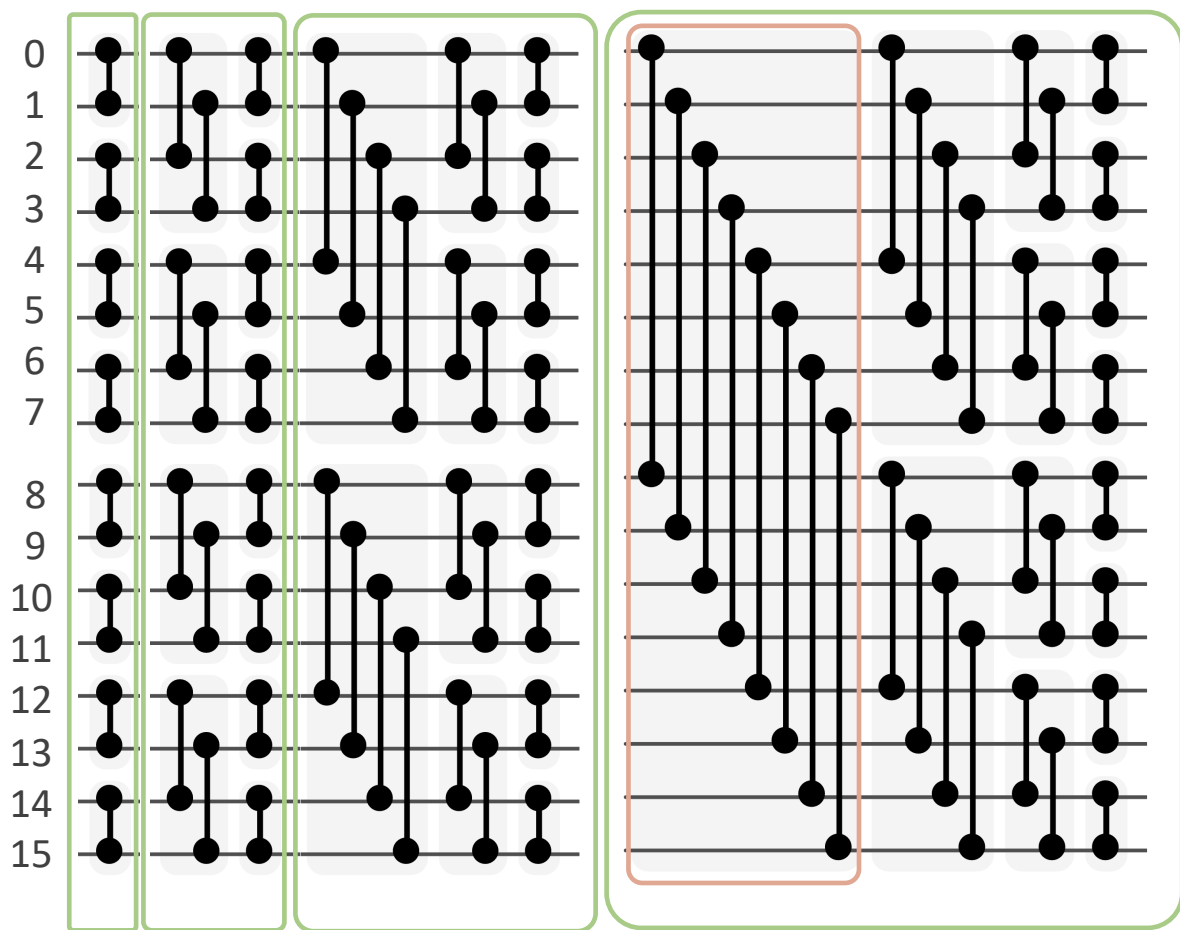
Sorting Networks



0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

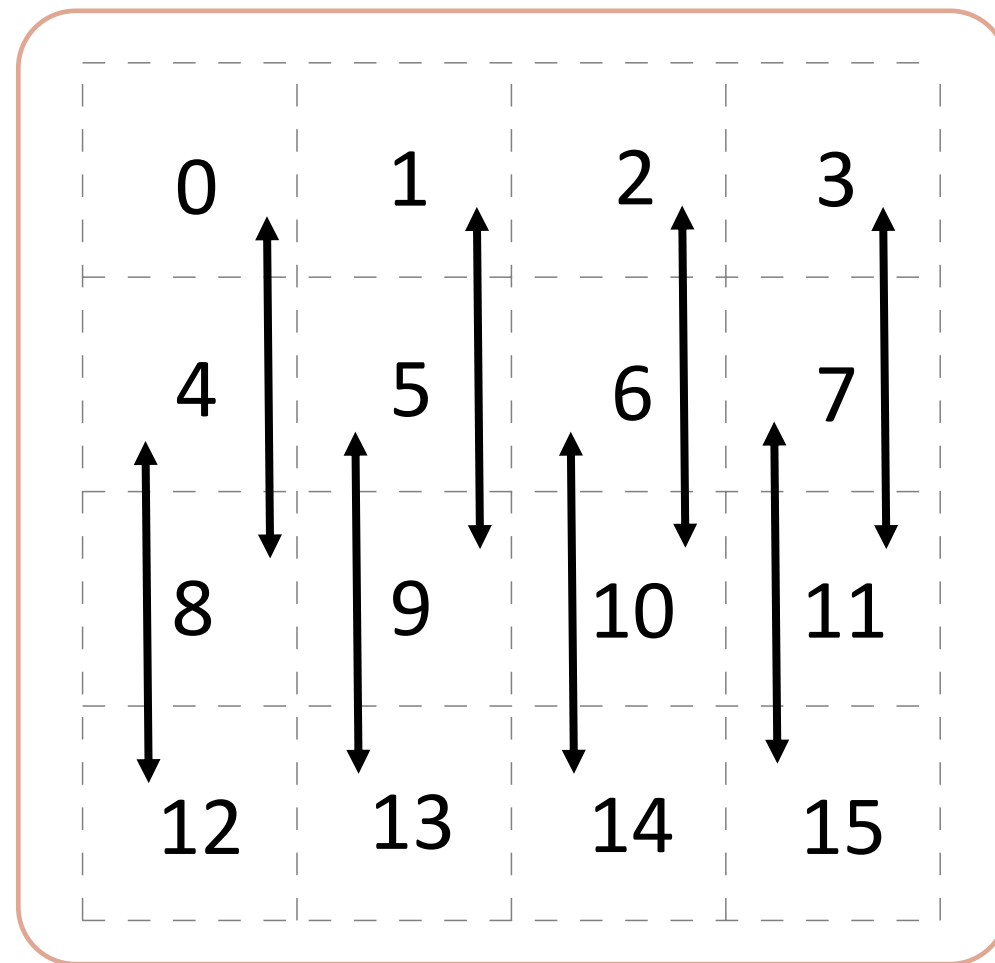
Bitonic Sorting Network

Sorting Networks

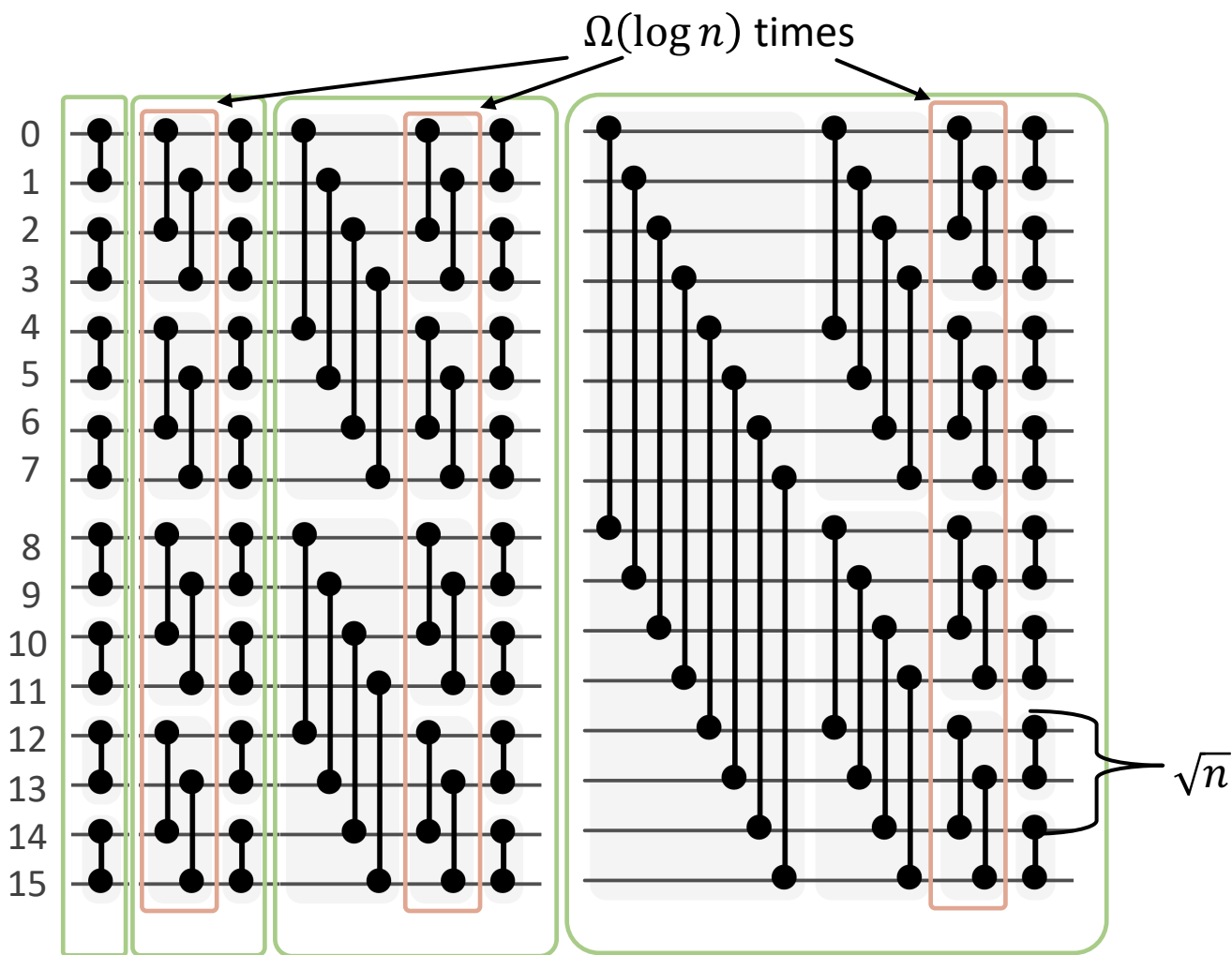


Bitonic Sorting Network

$\Theta(n^{3/2})$ Energy

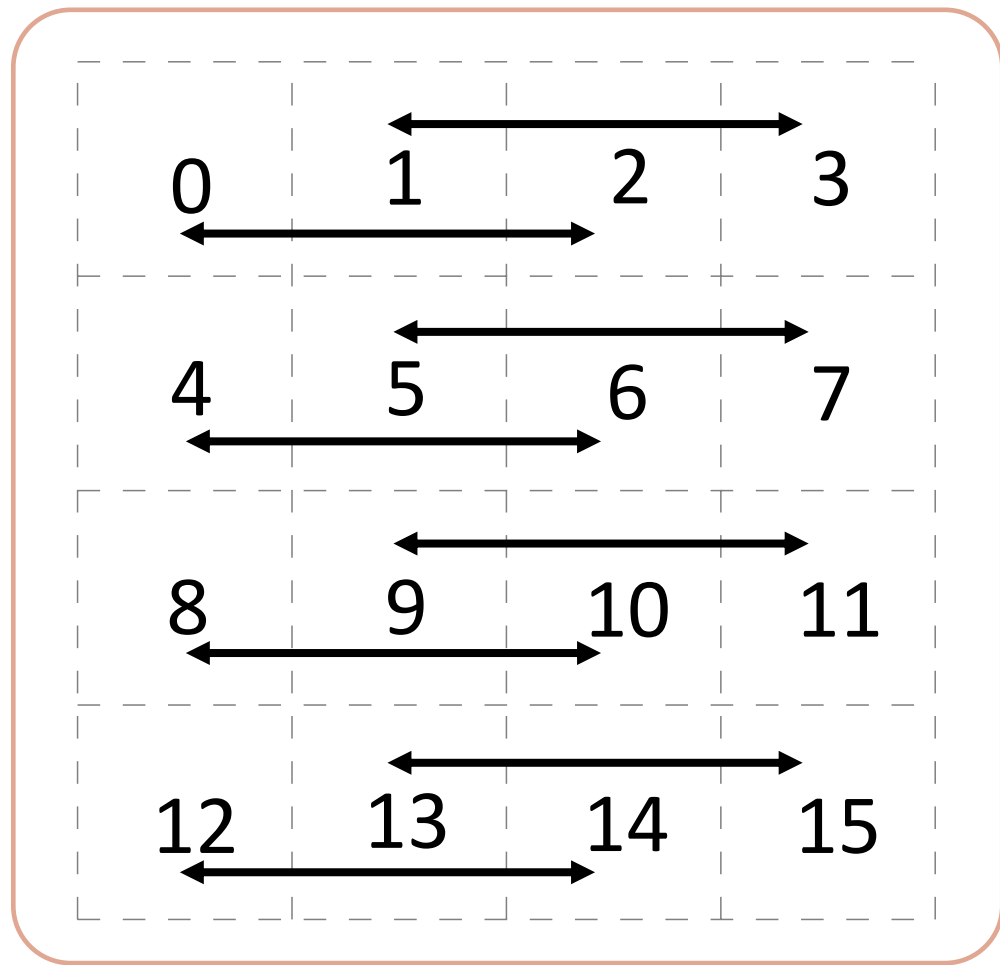


Sorting Networks

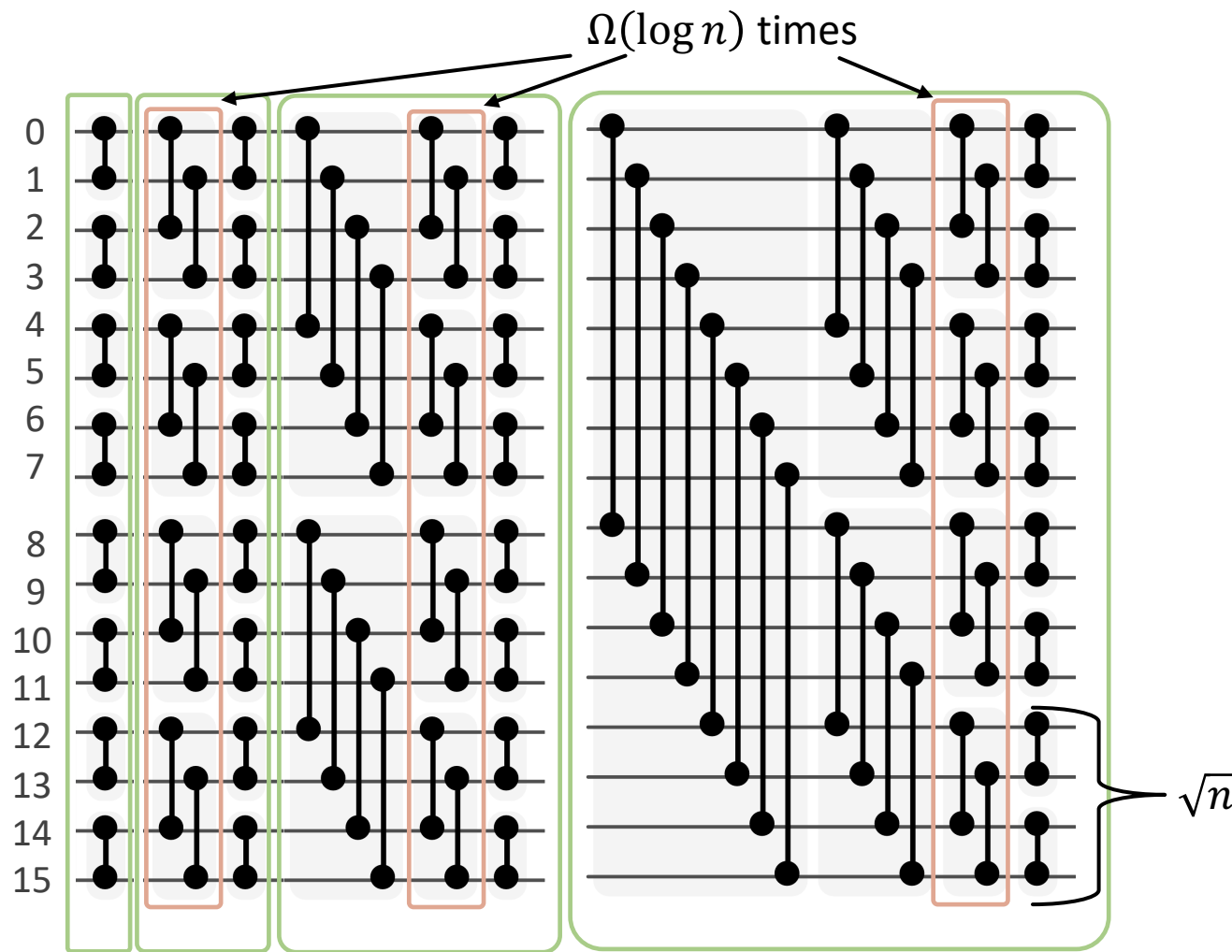


Bitonic Sorting Network

$\Theta(n^{3/2})$ Energy

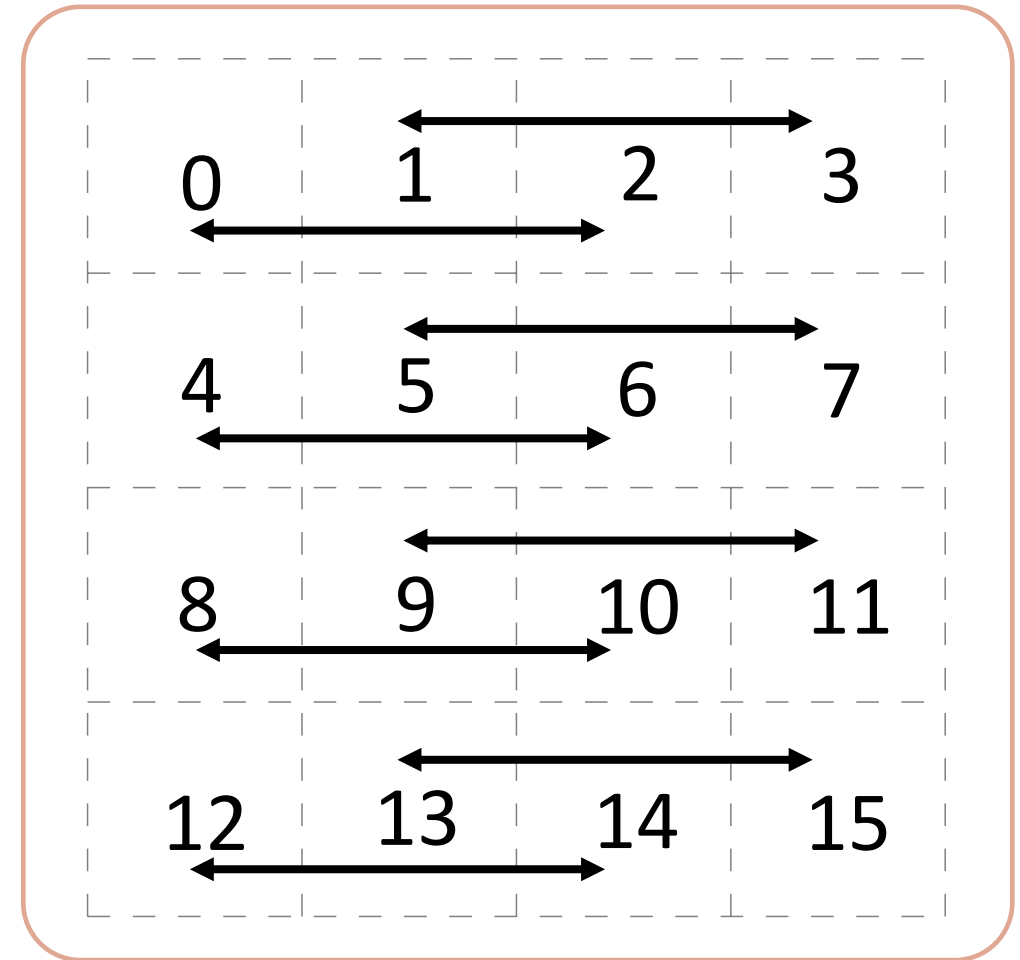


Sorting Networks



Bitonic Sorting Network
 $\Theta(n^{3/2} \log n)$ Energy
 $\Theta(\log^2 n)$ Depth

$\Theta(n^{3/2})$ Energy



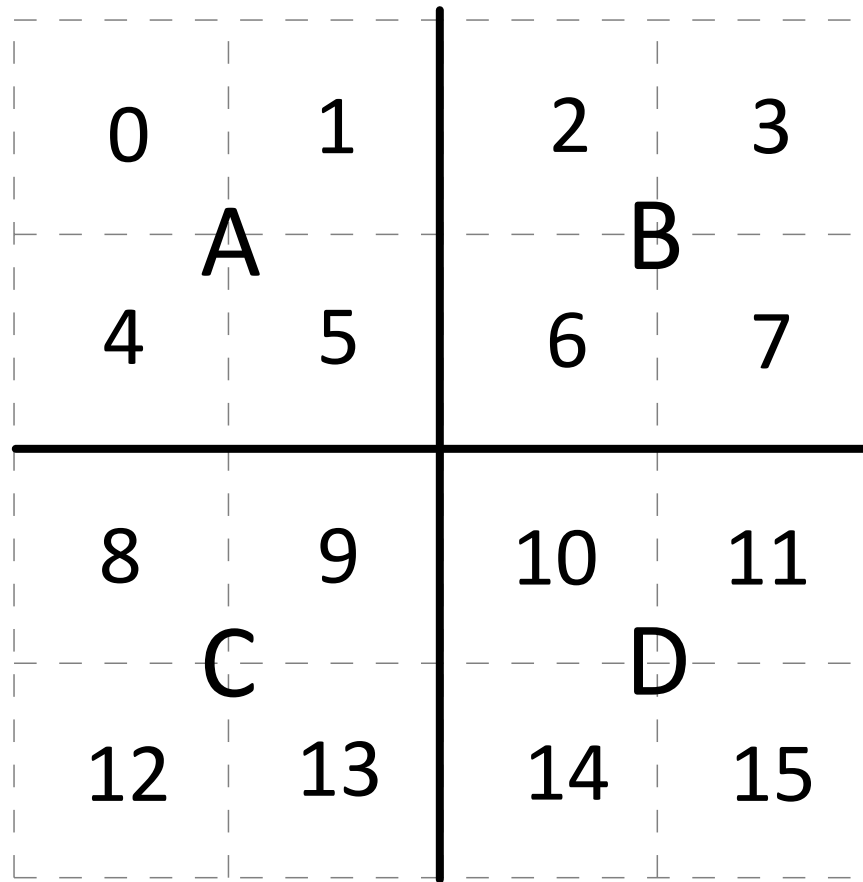
2D Merge Sort

*Idea: Reduce size of **both** dimensions of the subgrid in every step*

We show:

how to merge in $\Theta(n^{3/2})$ Energy

$\Theta(n^{3/2})$ Energy overall

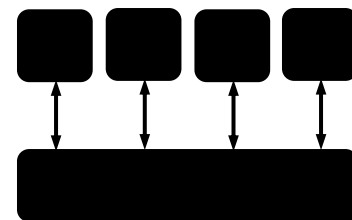


Implications of the Sorting Results

1. PRAM Simulation

Idea: Simulate each step of a PRAM algorithm using sorting to resolve concurrent reads

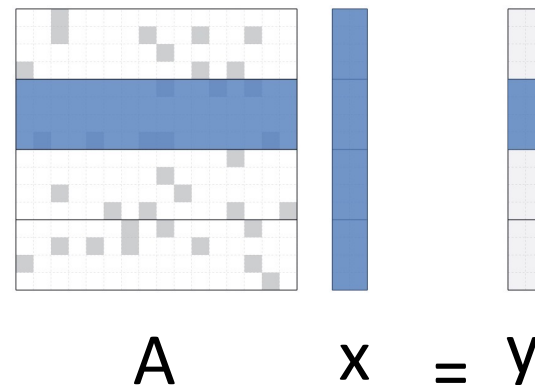
Implication: Easy upperbounds for many problems



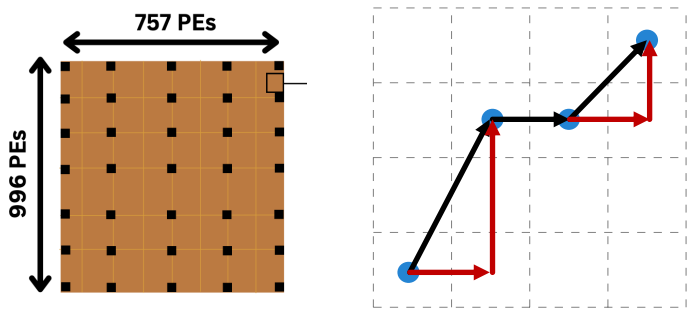
2. Sparse Matrix-Vector Multiplication (SpMV)

Idea: Use sorting to move data to where it needs to go

Implication: Energy-Optimal low depth SpMV (when $nnz \in O(n)$)

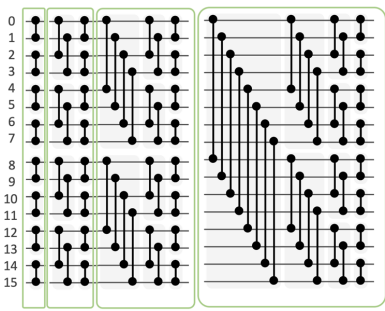


Conclusions



A diagram showing a 757x996 grid of Processing Elements (PEs) represented as small brown squares. To the right, a smaller grid illustrates a path of computation with blue dots at each step, connected by black and red arrows, showing a non-linear, zig-zagging trajectory across the grid.

Spatial model of computation

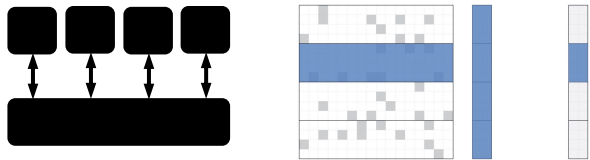


A diagram illustrating sorting and selection algorithms. It features a grid of nodes with a vertical index on the left ranging from 0 to 15. Black lines represent data paths or comparisons between nodes, showing a complex, branching structure that resembles a search tree or a sorting network.

Sorting, Selection, and more

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Energy-Optimal low depth sorting



A diagram illustrating PRAM simulation and Sparse Matrix-Vector multiplication (SpmV). On the left, four small black squares are connected by arrows to a single larger black rectangle below them. To the right, a grid of small squares is shown, with a horizontal band of blue squares, followed by a vertical bar and a single column of squares, representing data structures in a simulation.

PRAM simulation, SpmV

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