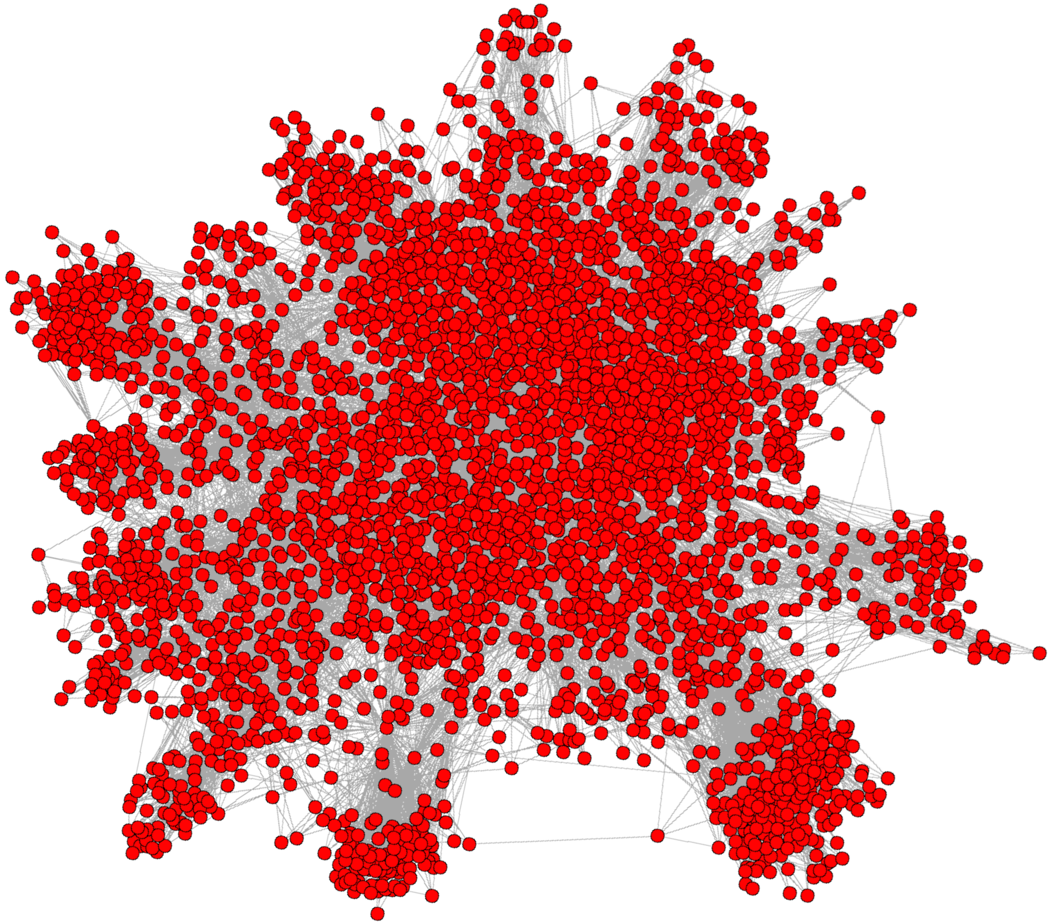
MACIEJ BESTA, DIMITRI STANOJEVIC, TIJANA ZIVIC, JAGPREET SINGH, MAURICE HOEROLD, TORSTEN HOEFLER

Log(Graph): A Near-Optimal High-Performance Graph Representation

Large graphs…

Large graphs…



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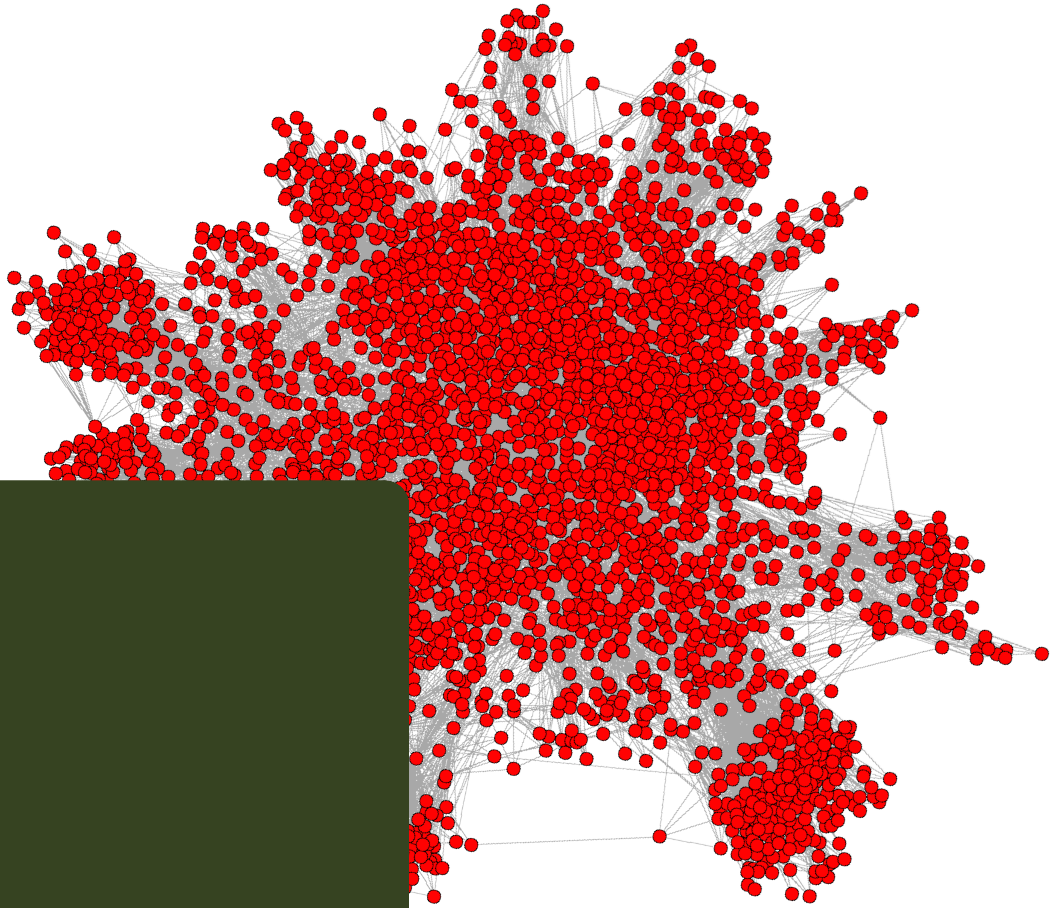

Running on…

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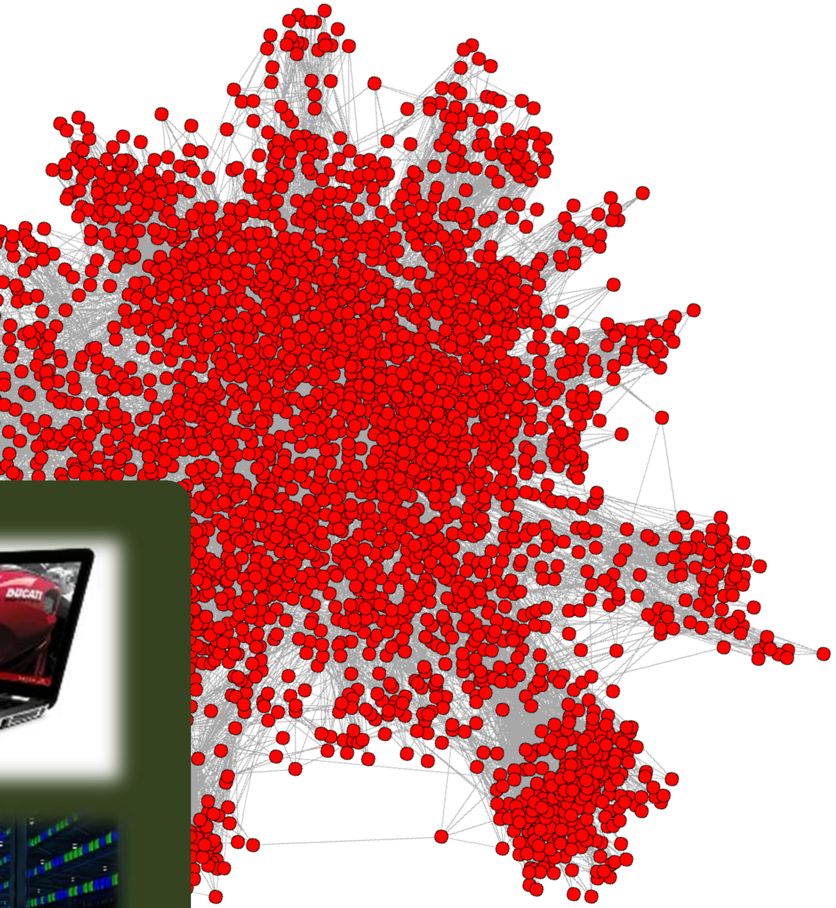

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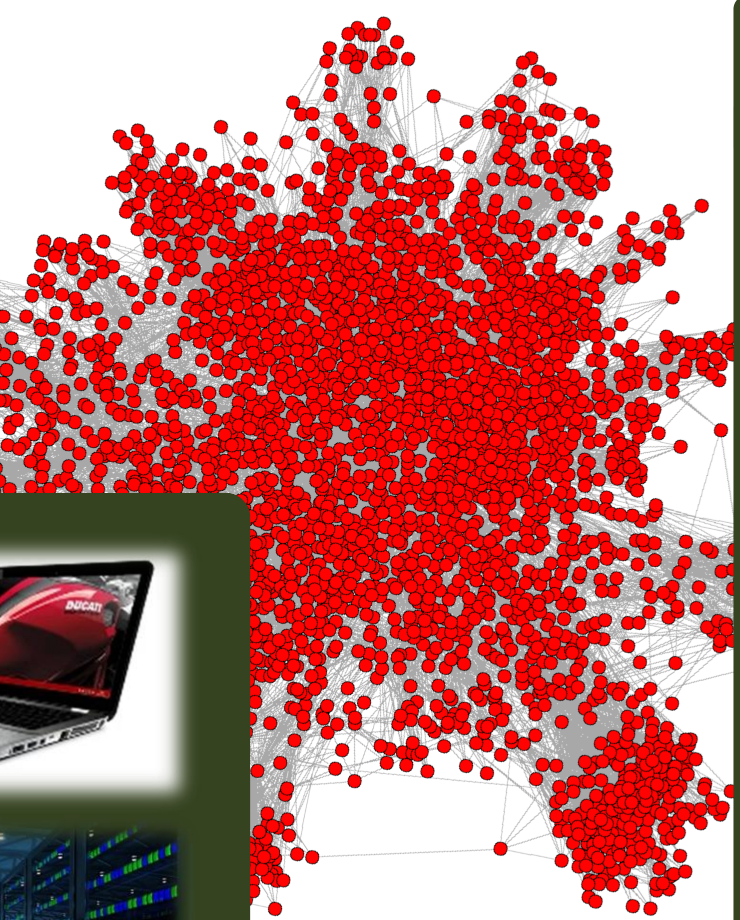

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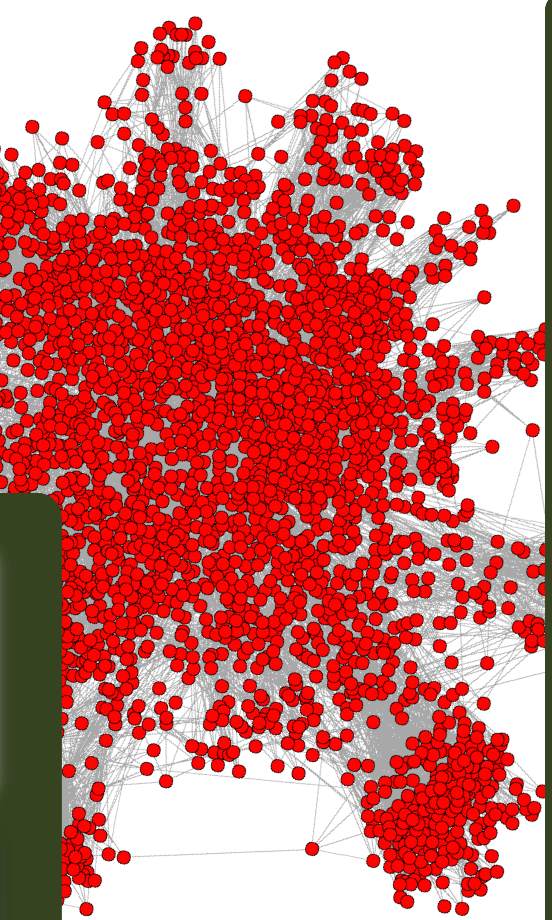

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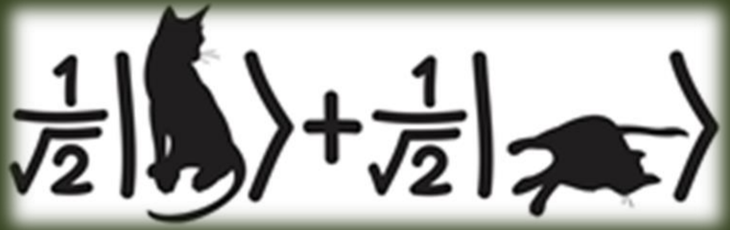

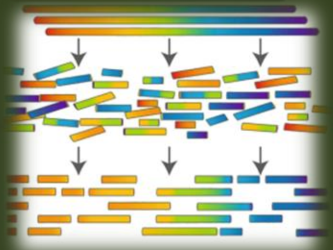

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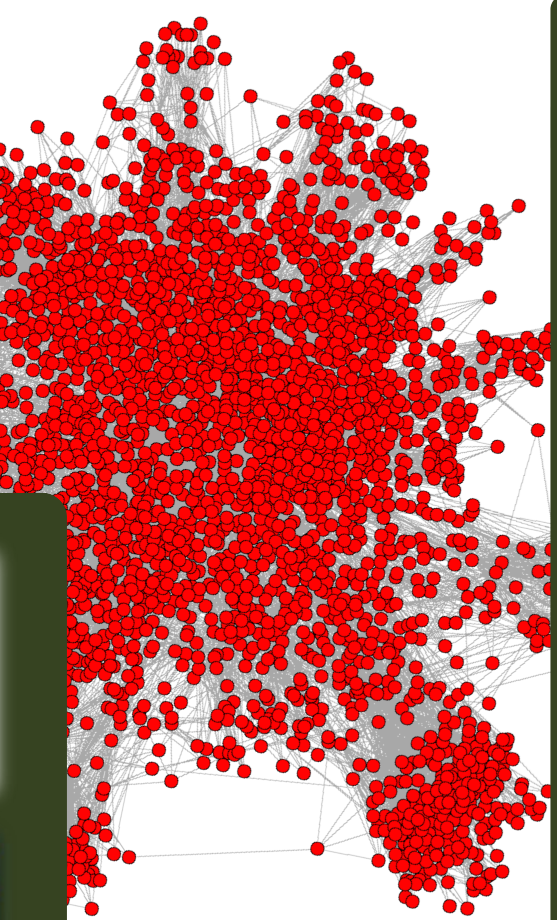
Huge



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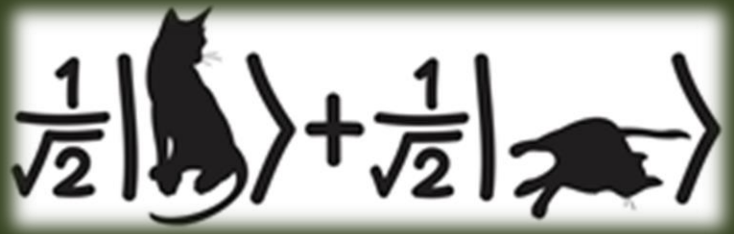

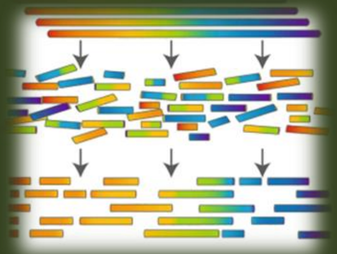

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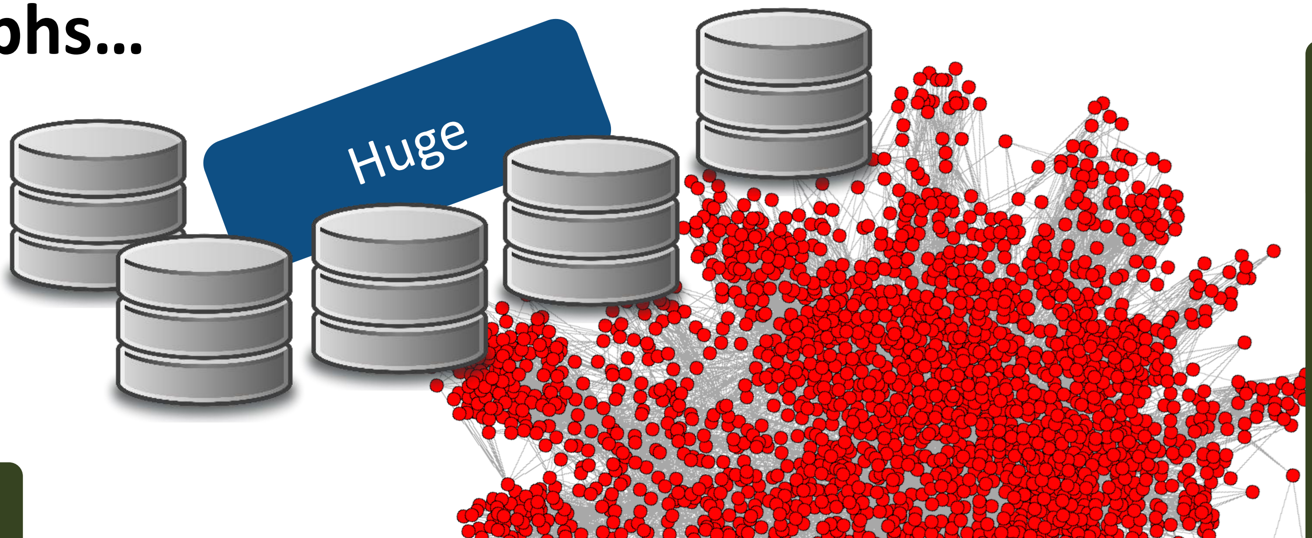

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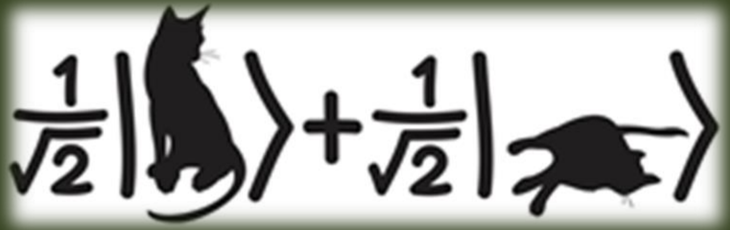

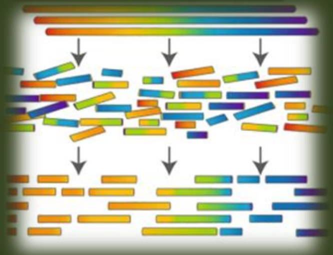

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TF	Twitter (MPI)	D	52,579,682	1,963,263,821
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KONECT graph datasets

Graph500 Benchmark

Top Ten from June 2018 BFS

RANK	MACHINE	VENDOR	INSTALLATION SITE	LOCATION	COUNTRY	YEAR	NUMBER OF NODES	NUMBER OF CORES	SCALE	GTEPS
1	K computer	Fujitsu	RIKEN Advanced Institute for Computational Science (AICS)	Kobe Hyogo	Japan	2011	82944	663552	40	38621.4
2	Sunway TaihuLight	NRCPC	National Supercomputing Center in Wuxi	Wuxi	China	2015	40768	10599680	40	23755.7
3	DOE/NNSA/LLNL Sequoia	IBM	Lawrence Livermore National Laboratory	Livermore CA	USA	2012	98304	1572864	41	23751
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Webgraph datasets

Graph	Crawl date	Nodes	Arcs
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gsh-2015	2015	988 490 691	33 877 399 152
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uk-2014-tpd	2014	1 766 010	18 244 650
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Web data commons datasets

Granularity	#Nodes	#Arcs
Page	3,563 million	128,736 million
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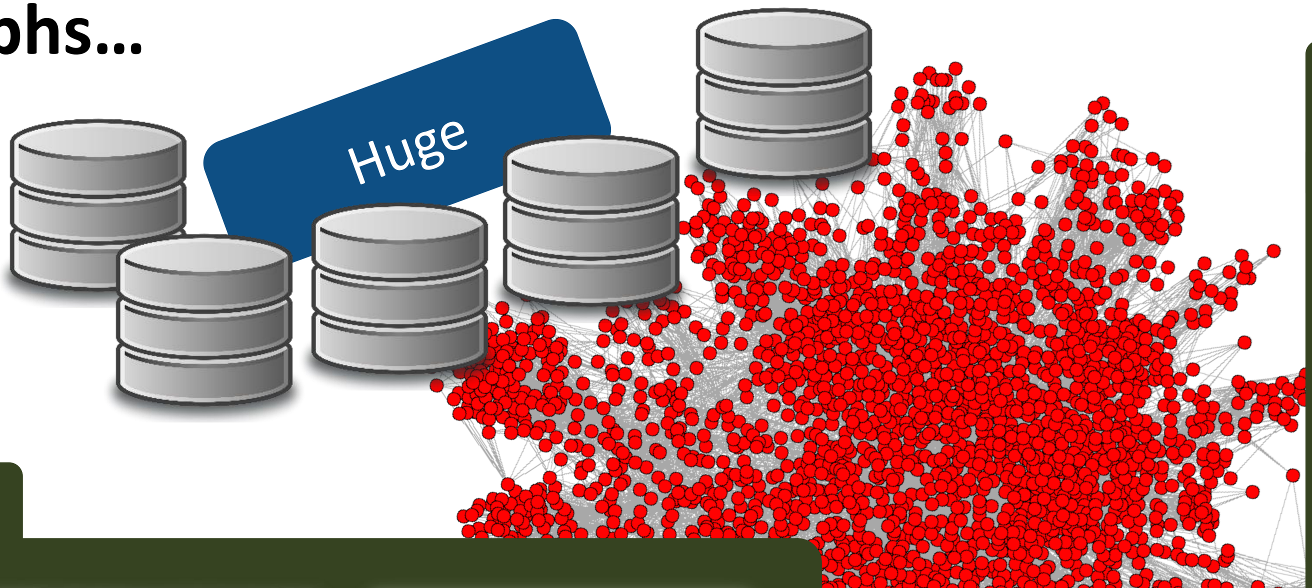
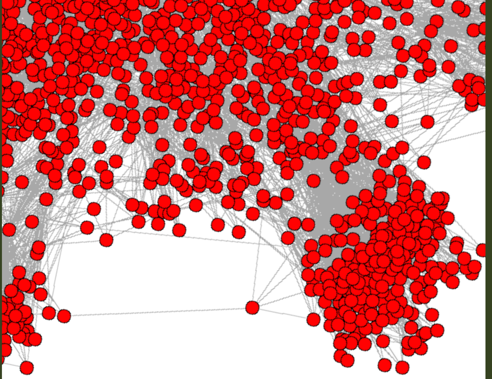
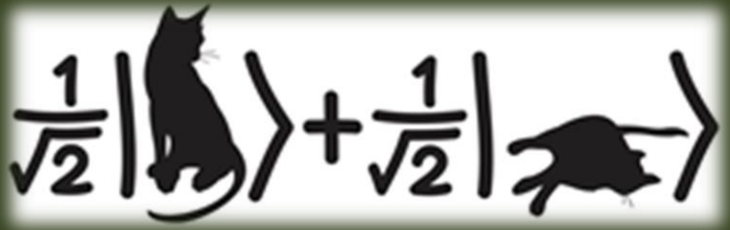
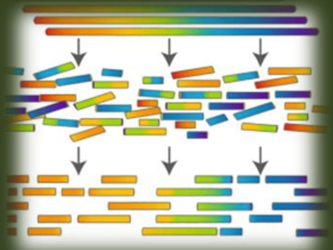




Running on…

Used in…





Large graphs…

Huge

Running on…

Compression incurs expensive decompression

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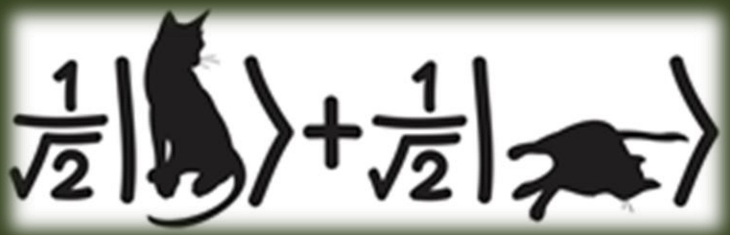

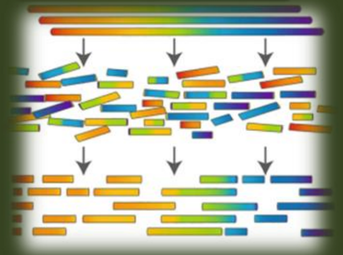

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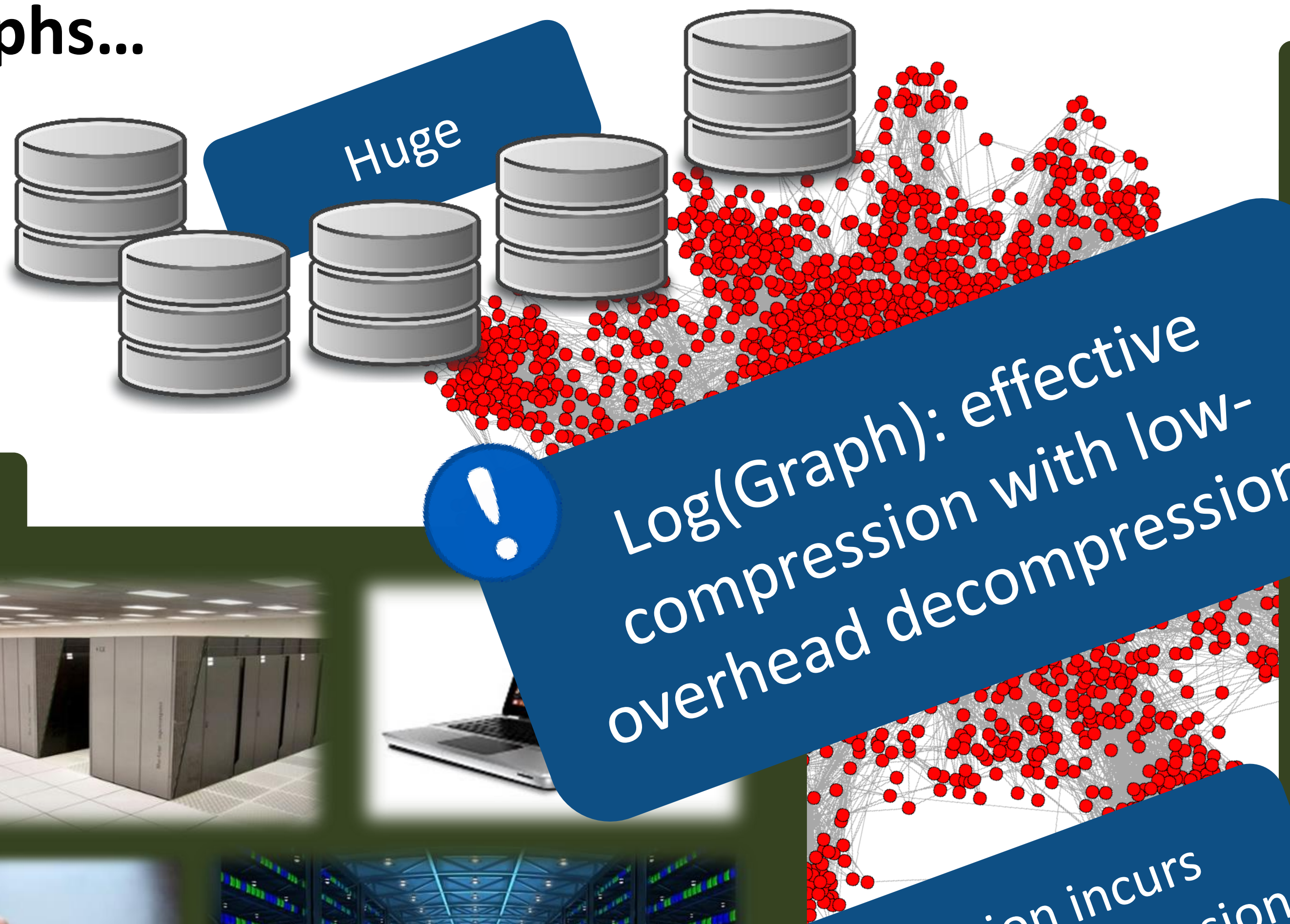

Huge

Log(Graph): effective compression with low-overhead decompression!

Running on…

Compression incurs expensive decompression

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The storage **lower bound** Ω

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$S = \{x_1, x_2, x_3, \dots\}$

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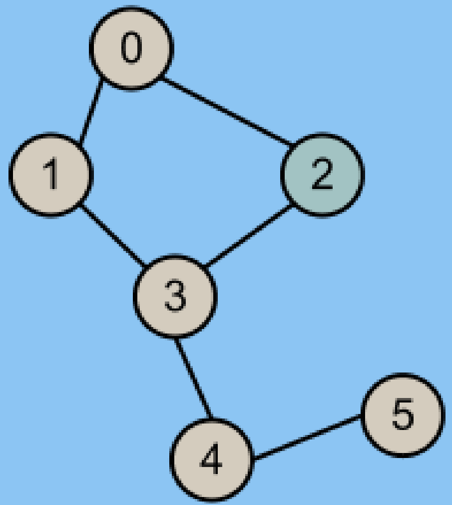
Encode different parts of a graph representation using (logarithmic) **storage lower bounds**



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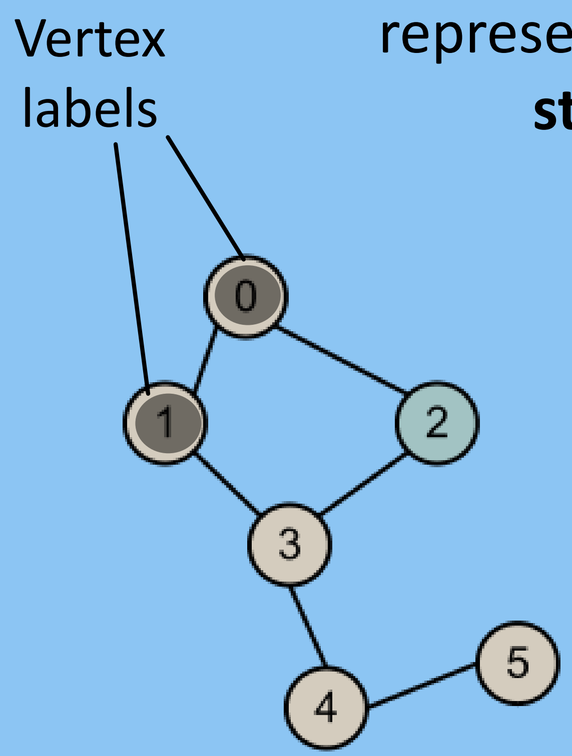

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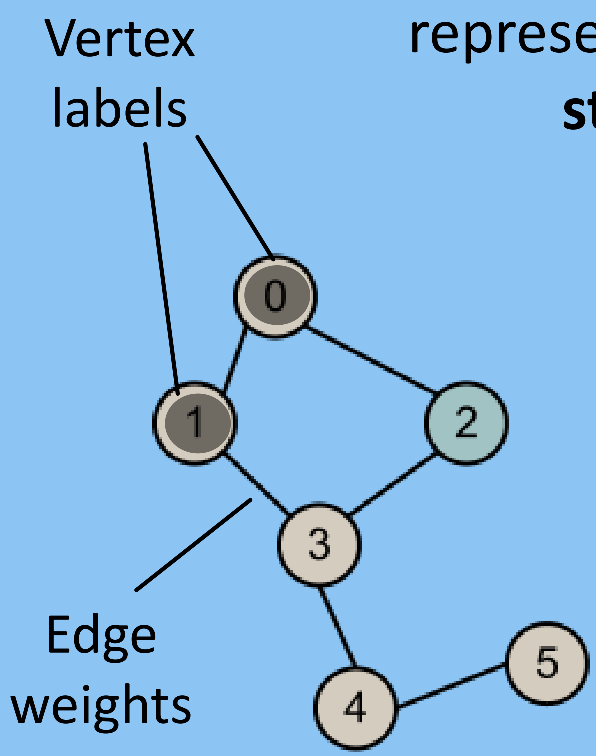

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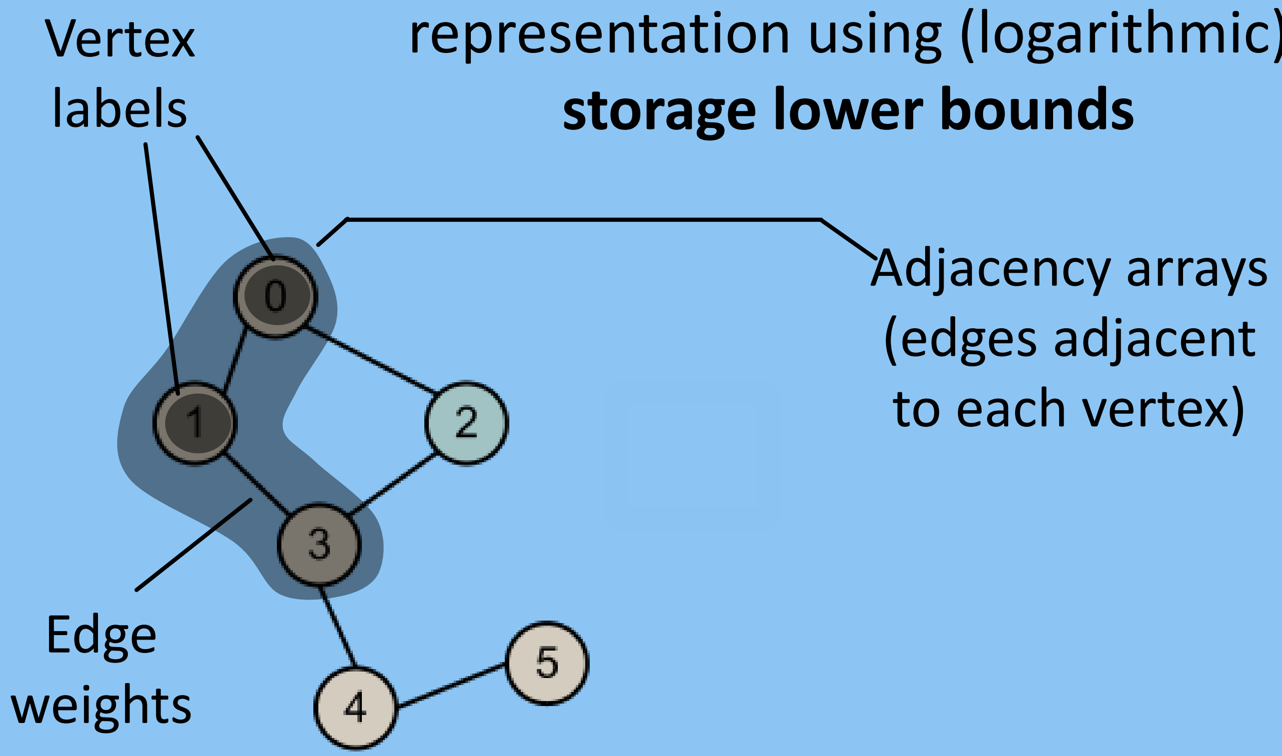

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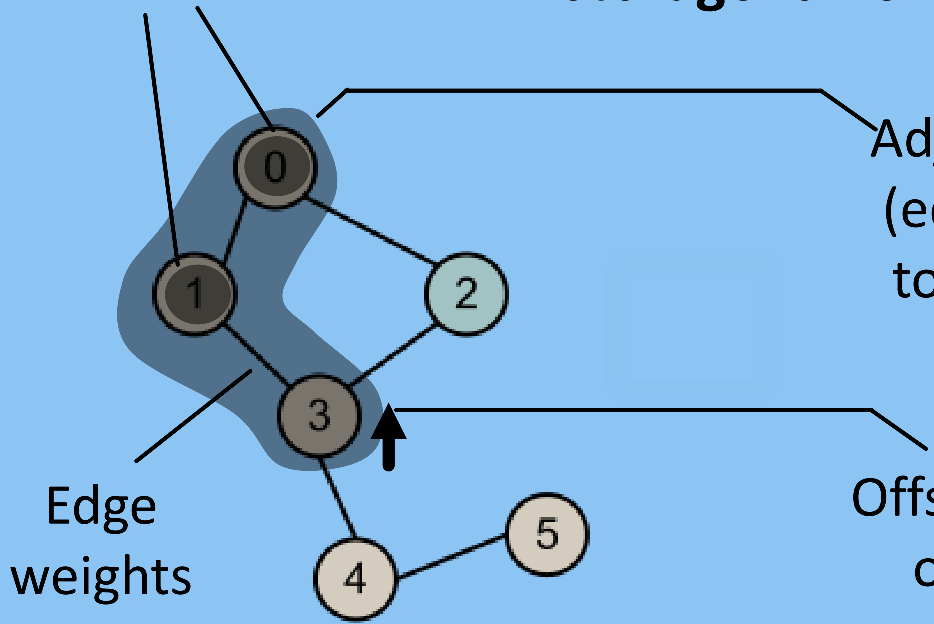

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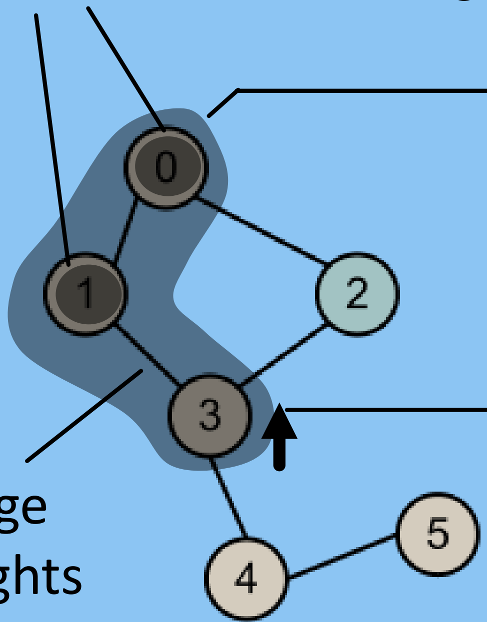
Encode different parts of a graph representation using (logarithmic) **storage lower bounds**

Log (Vertex labels **)**



Adjacency arrays (edges adjacent to each vertex)

Edge weights

Offsets (locations) of adj. arrays

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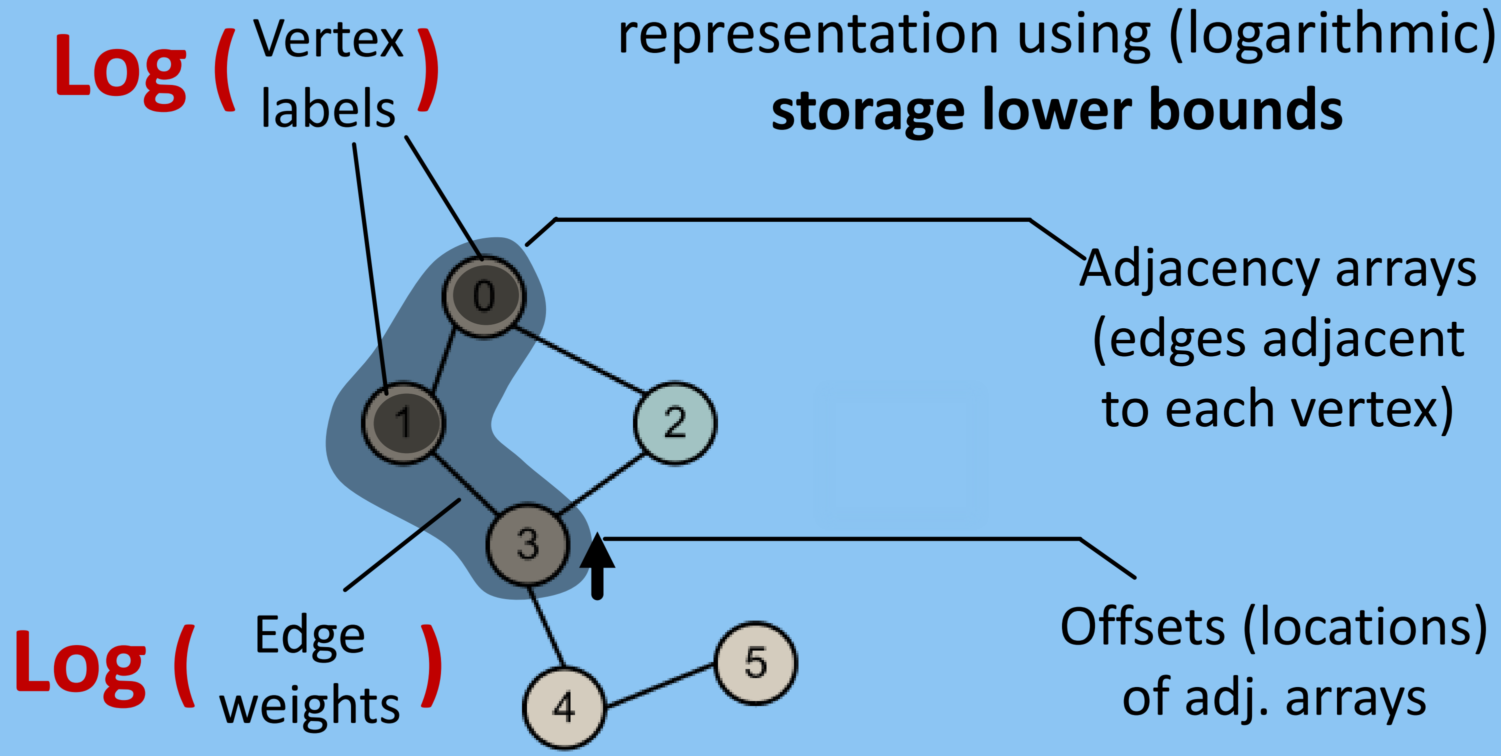

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$$S = \{x_1, x_2, x_3, \dots\} \quad \begin{matrix} x_1 \rightarrow 0 \dots 01 \\ x_2 \rightarrow 0 \dots 10 \\ x_3 \rightarrow 0 \dots 11 \\ \dots \end{matrix}$$

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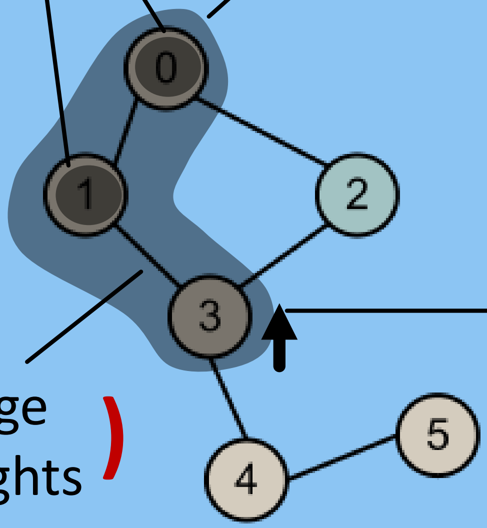

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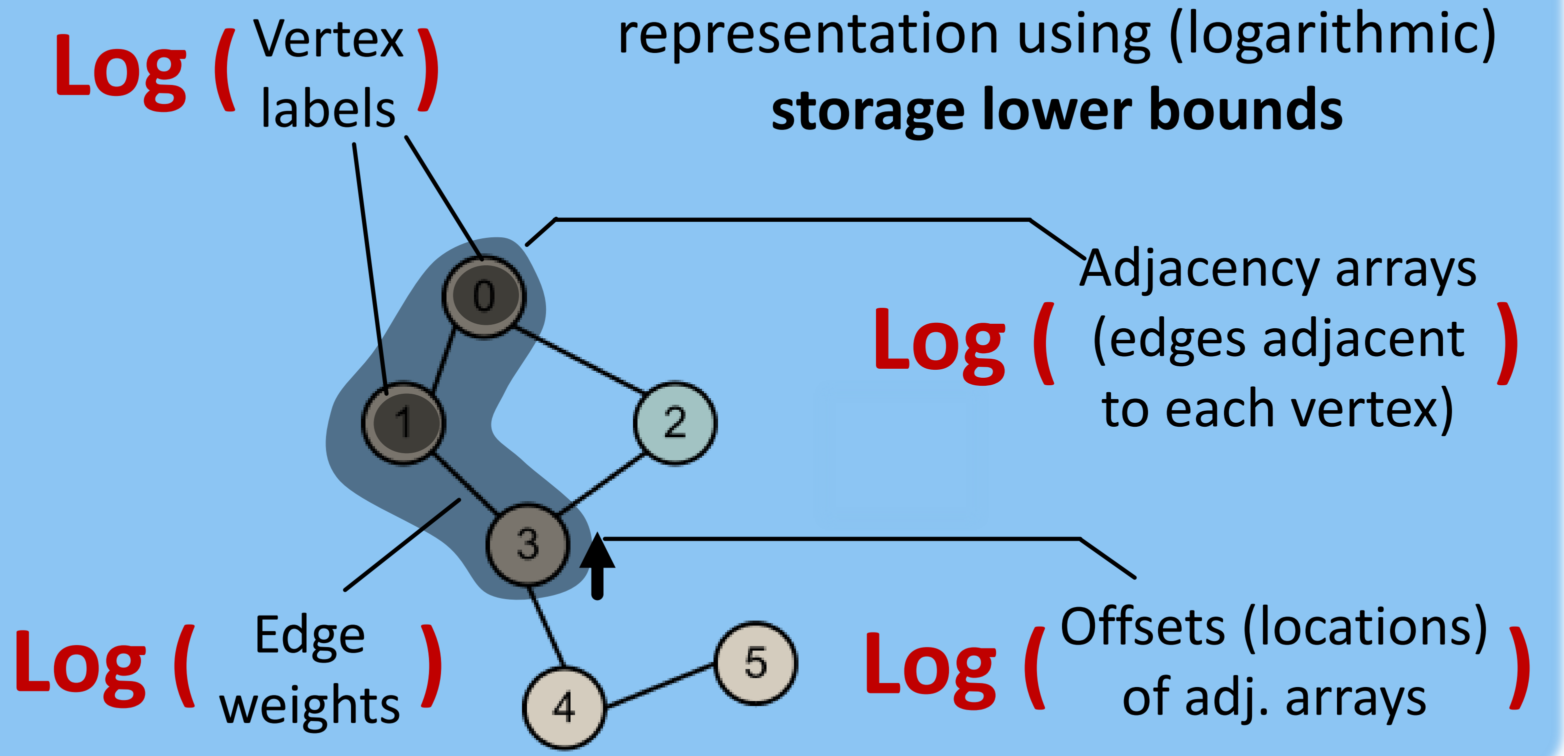
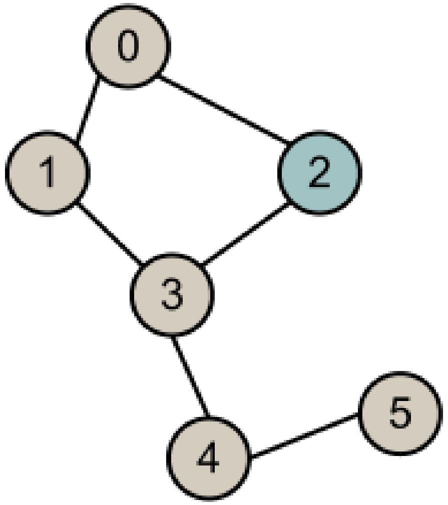
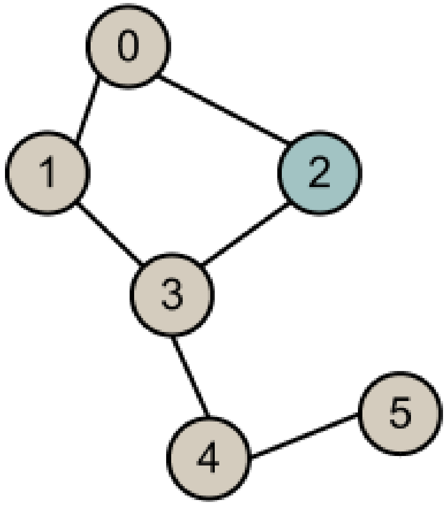


Adjacency Array Graph Representation



ADJACENCY ARRAY GRAPH REPRESENTATION

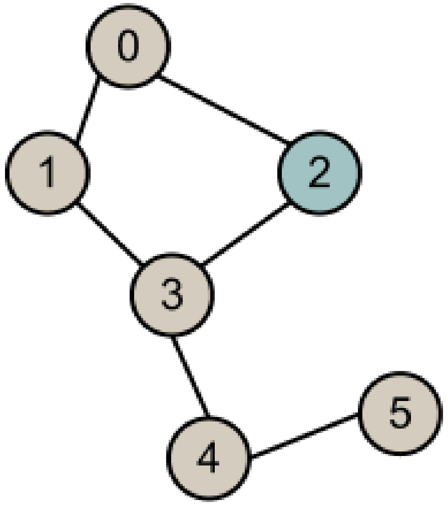
Representation



Adjacency Array Graph Representation

Representation

0
1
2
3
4
5



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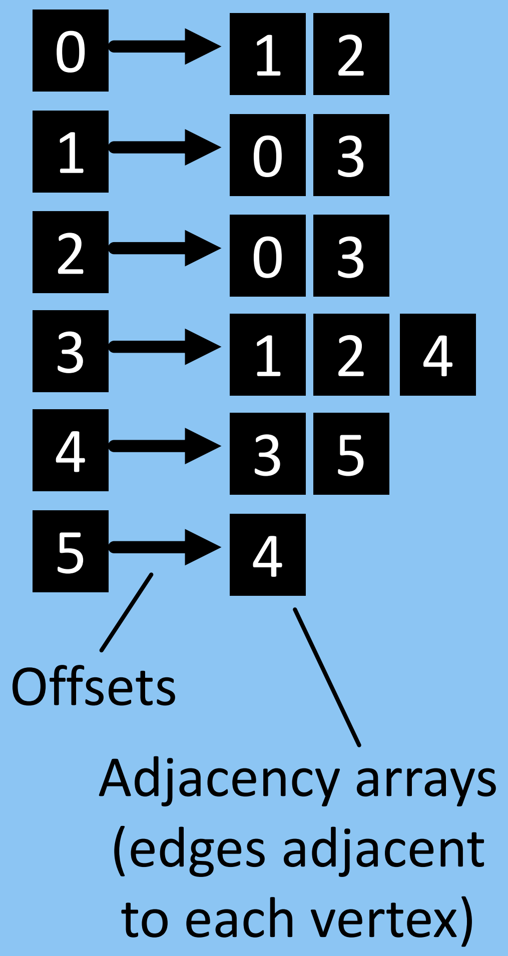

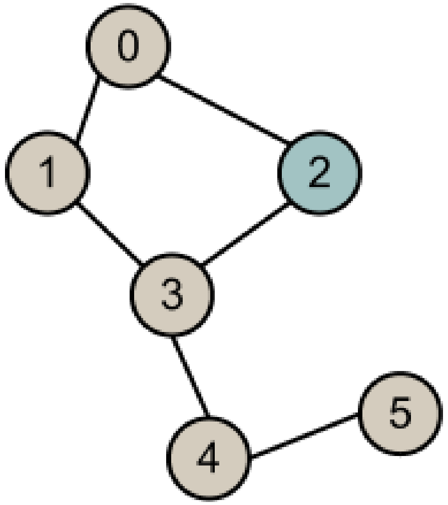

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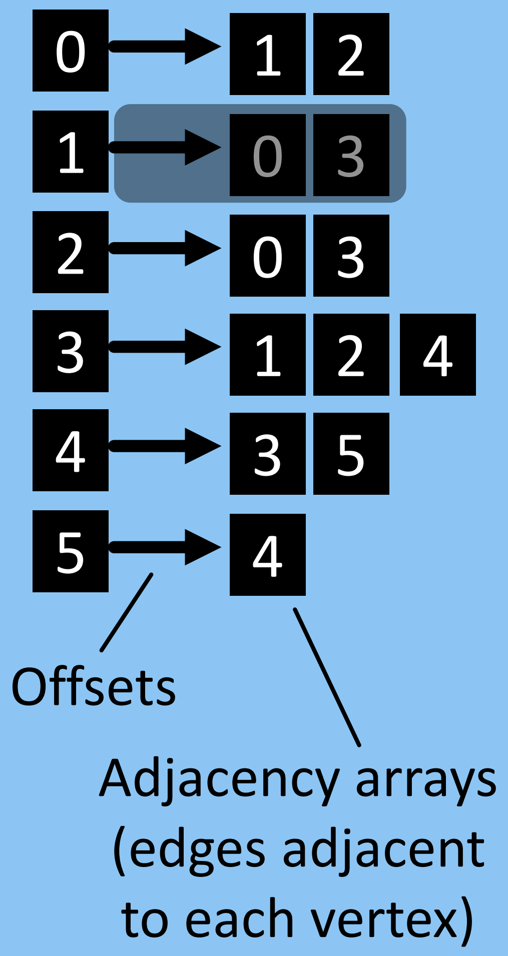

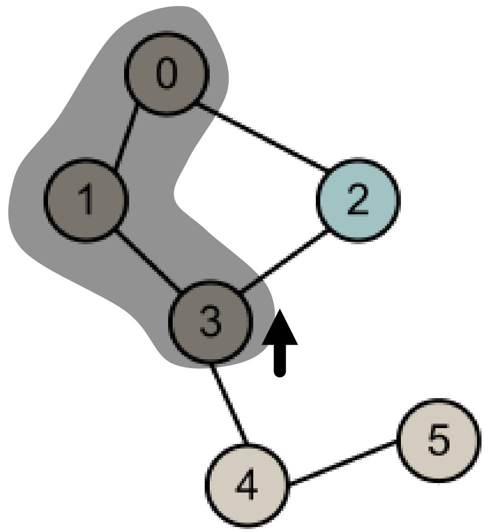

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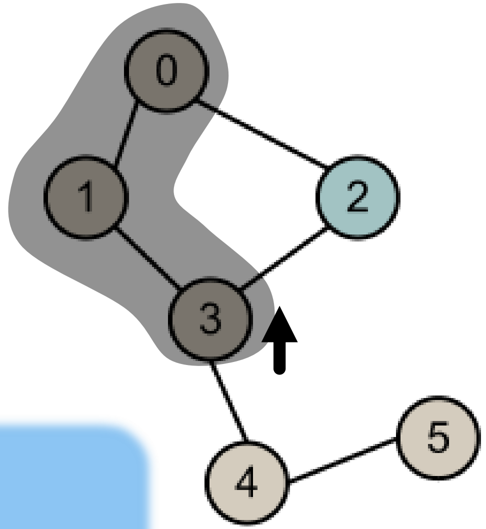
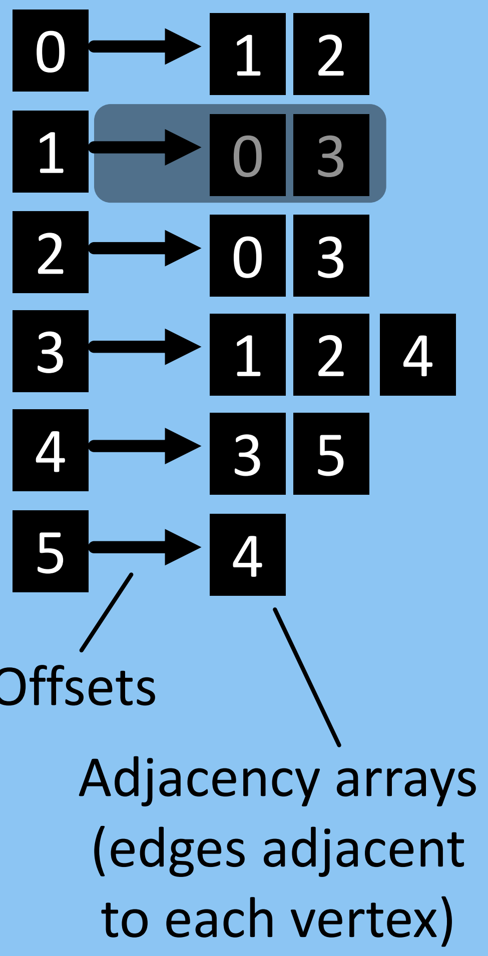
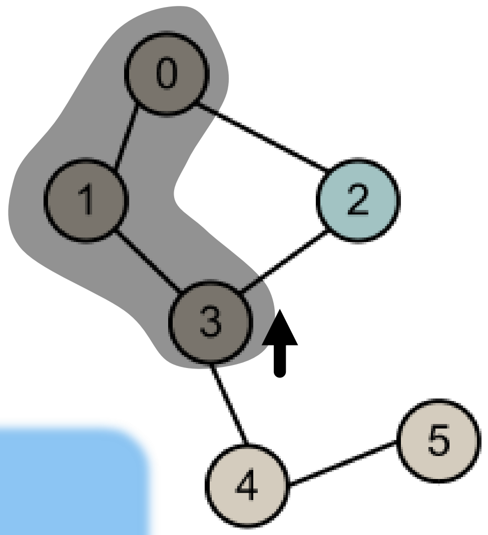


Representation

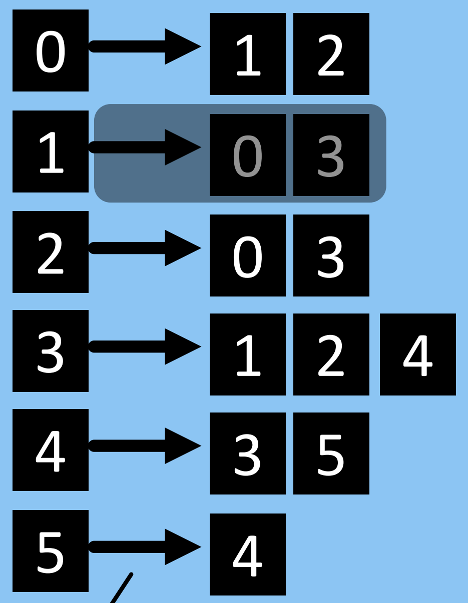


Physical realization

Adjacency Array Graph Representation



Representation



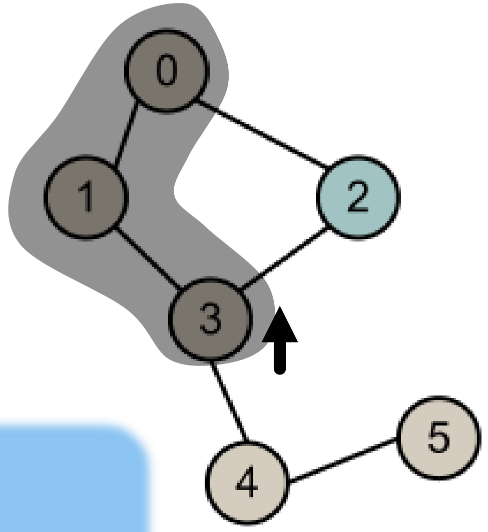
Offsets

Adjacency arrays (edges adjacent to each vertex)

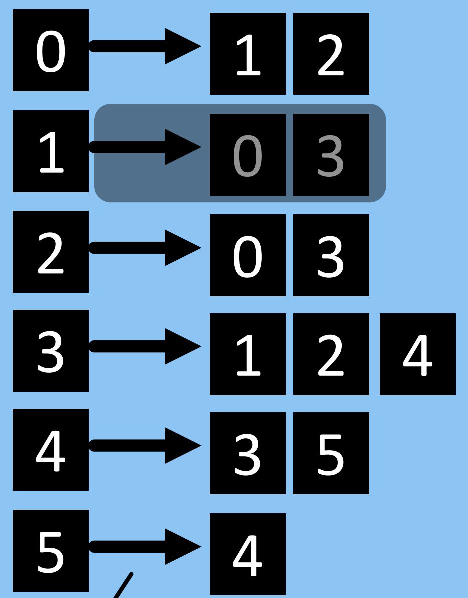
Physical realization

Adjacency arrays (one contiguous array)

Adjacency Array Graph Representation



Representation

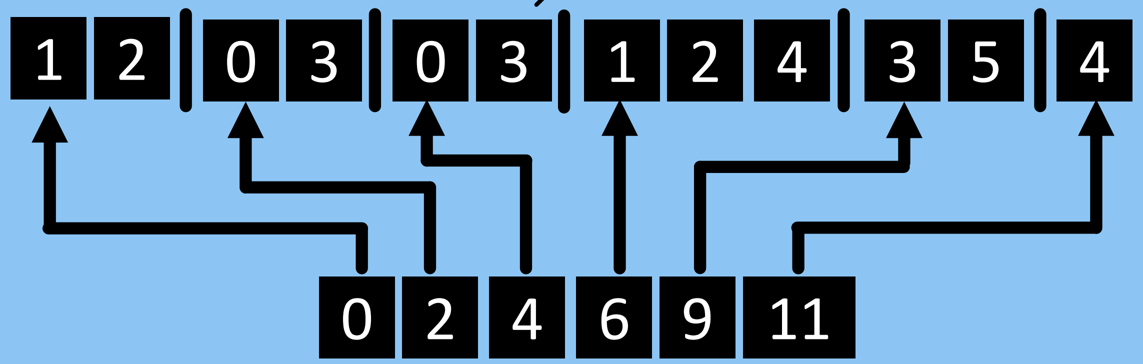


Offsets

Adjacency arrays (edges adjacent to each vertex)

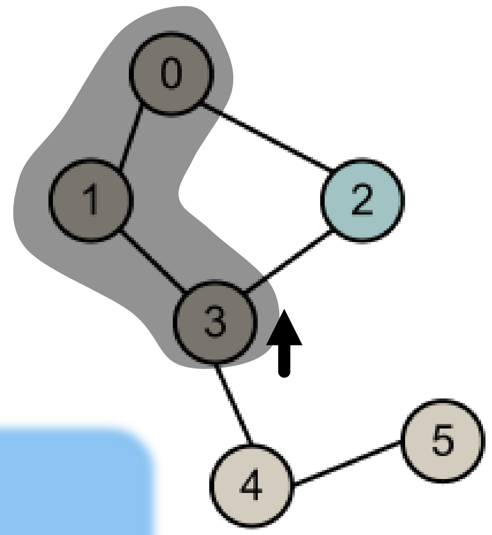
Physical realization

Adjacency arrays (one contiguous array)

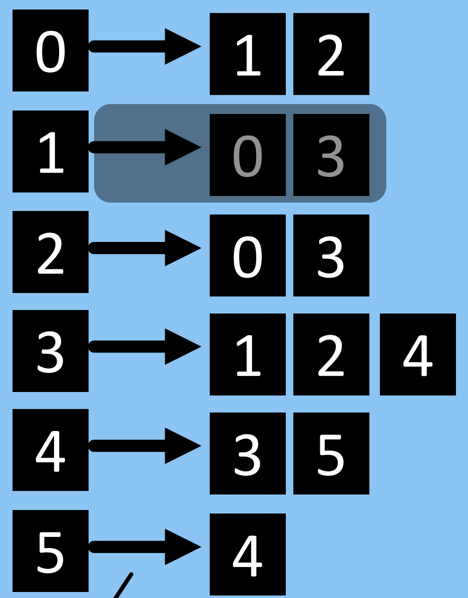


Offsets (another contiguous array)

ADJACENCY ARRAY GRAPH REPRESENTATION



Representation

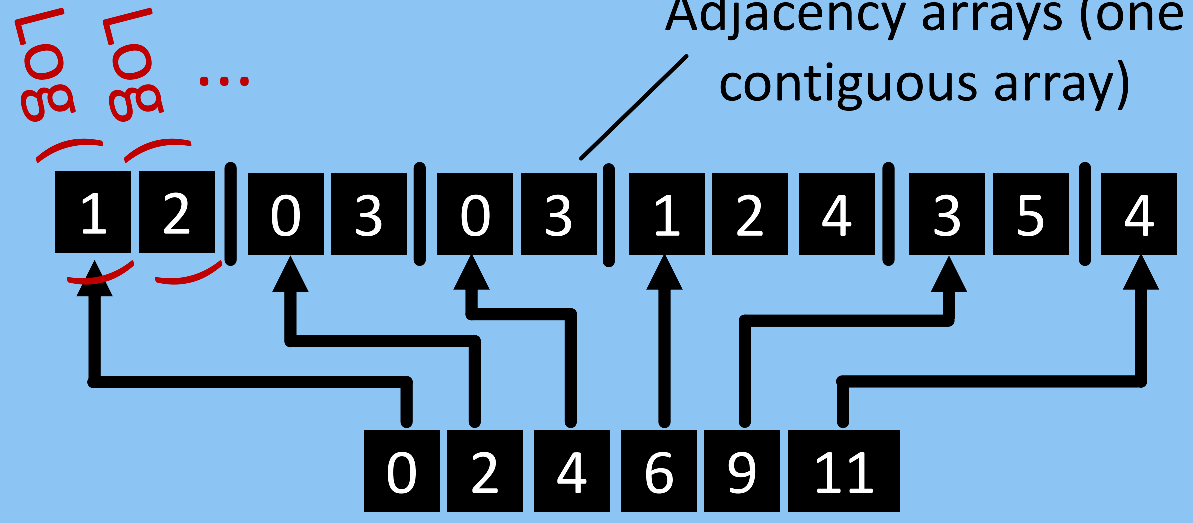


Offsets

Adjacency arrays (edges adjacent to each vertex)

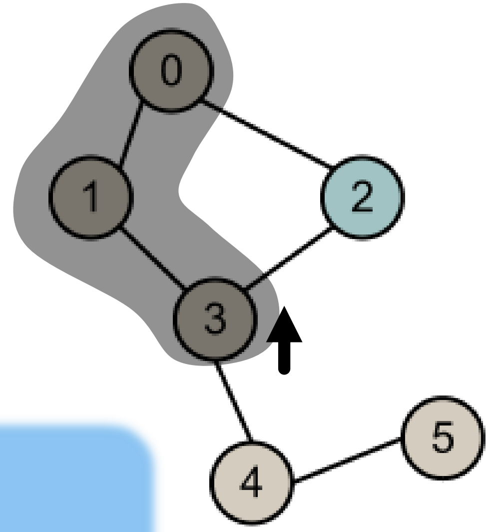
Physical realization

Adjacency arrays (one contiguous array)

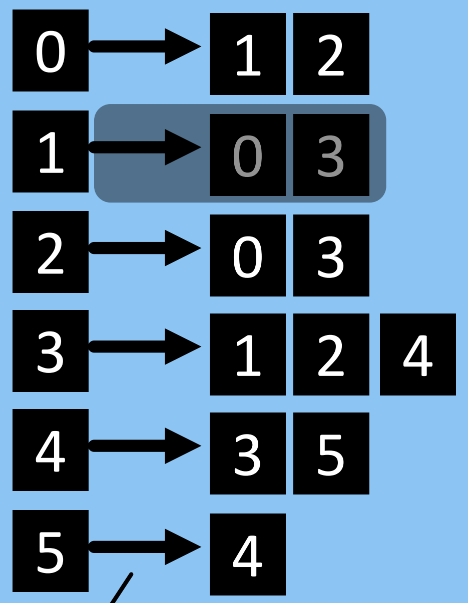


Offsets (another contiguous array)

Adjacency Array Graph Representation



Representation

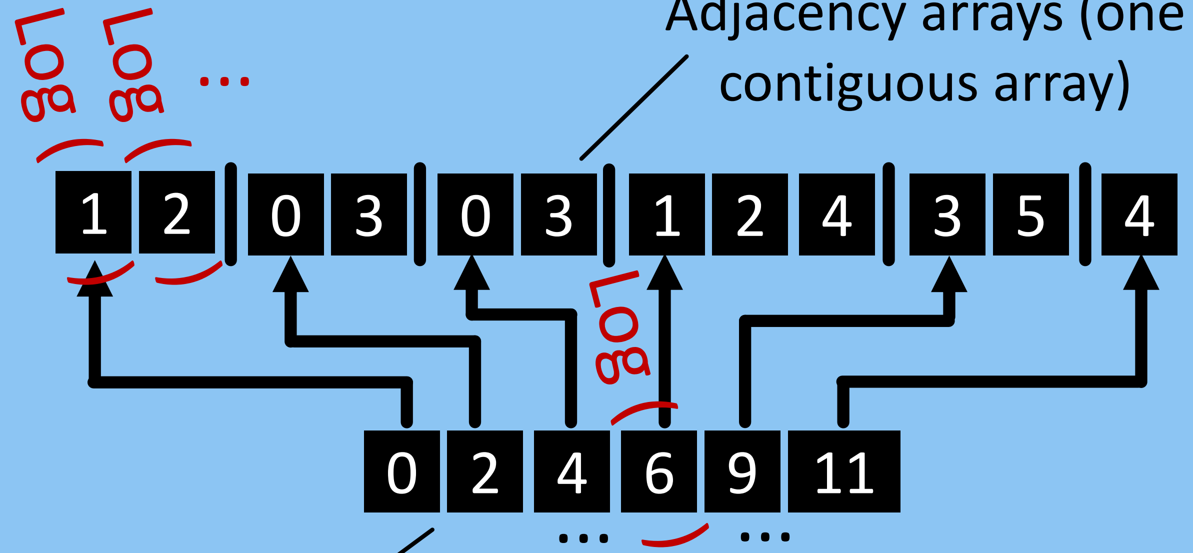


Offsets

Adjacency arrays (edges adjacent to each vertex)

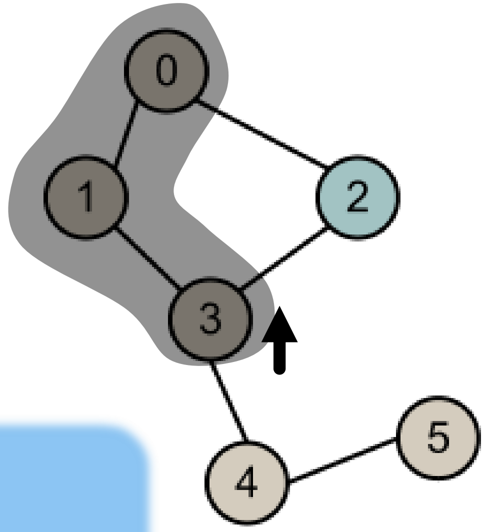
Physical realization

Adjacency arrays (one contiguous array)

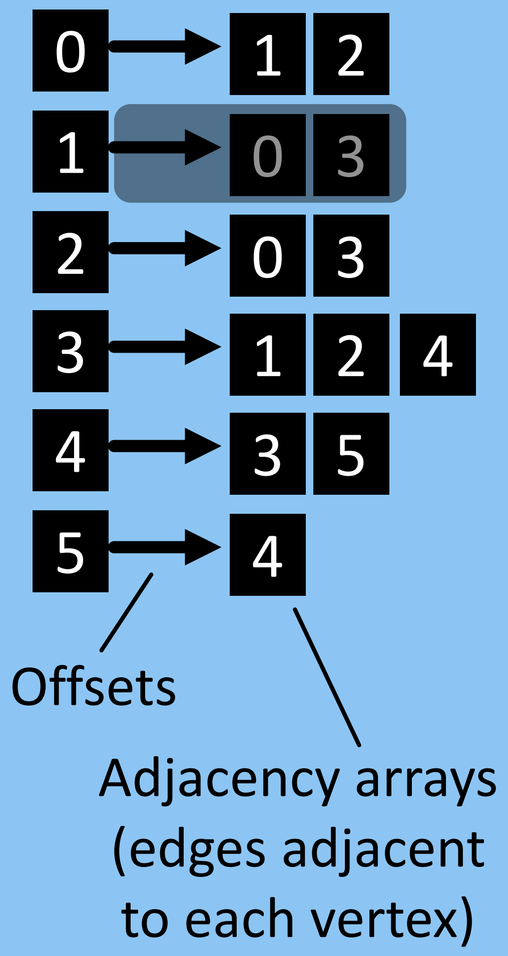


Offsets (another contiguous array)

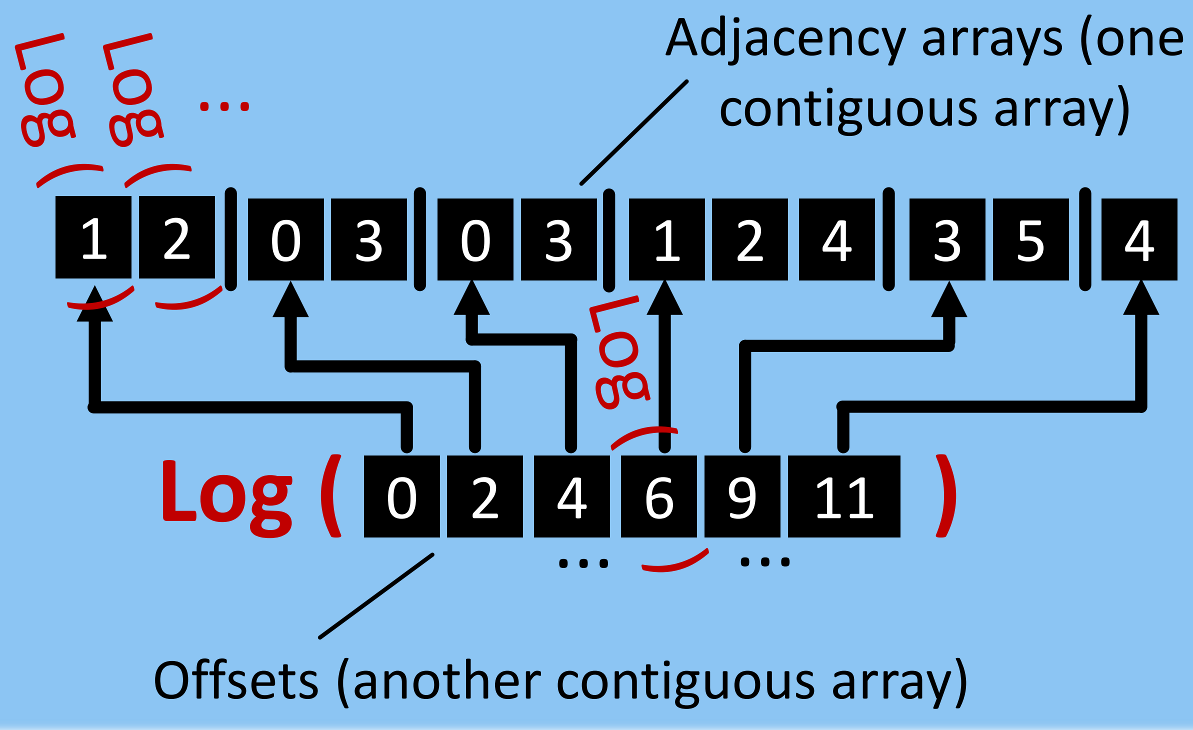
Adjacency Array Graph Representation



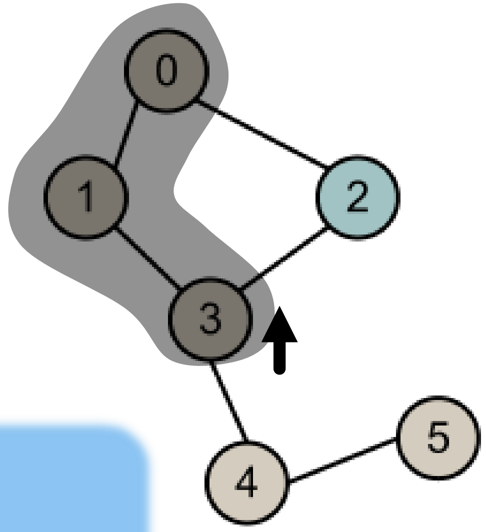
Representation



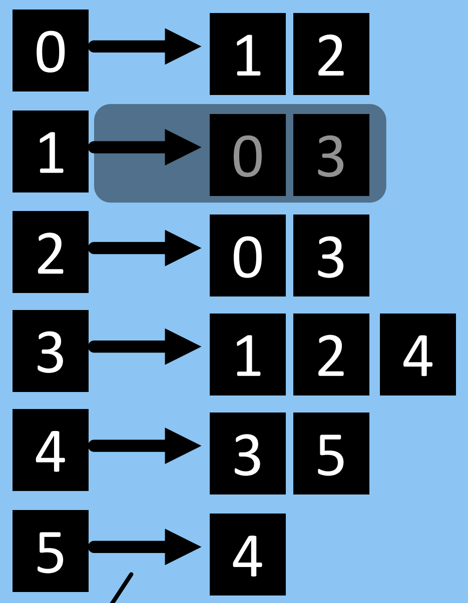
Physical realization



Adjacency Array Graph Representation



Representation

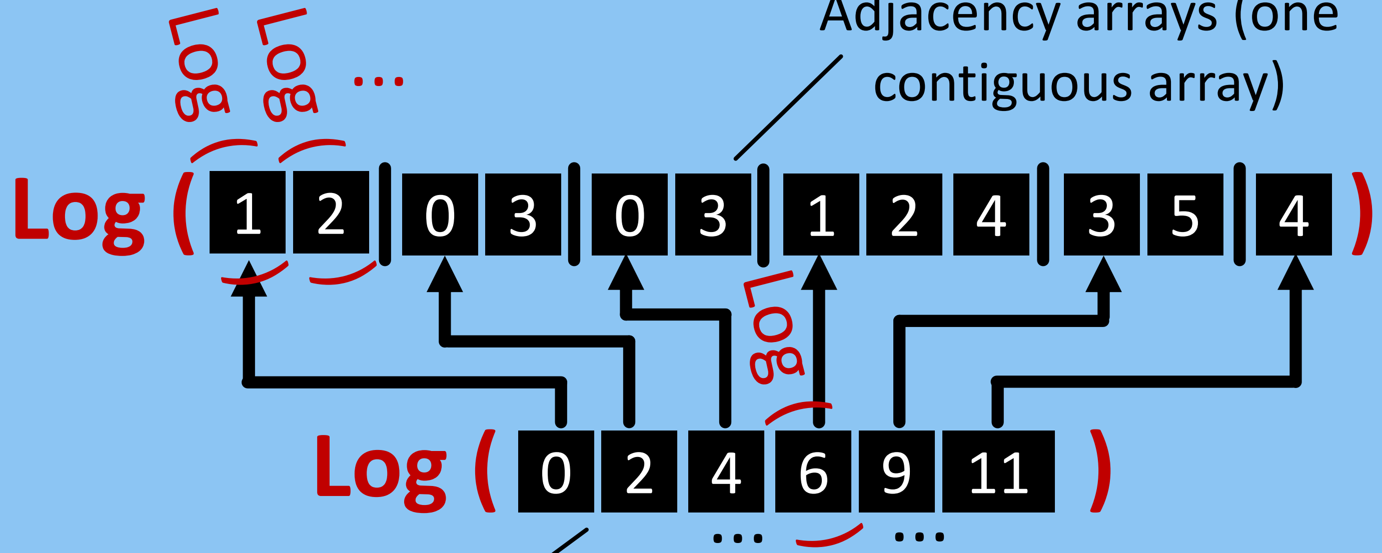


Offsets

Adjacency arrays (edges adjacent to each vertex)

Physical realization

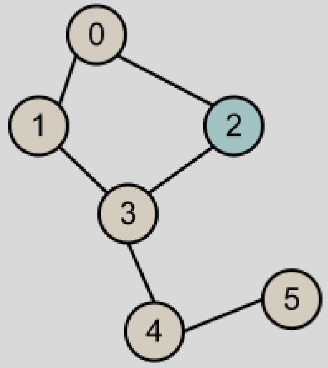
Adjacency arrays (one contiguous array)



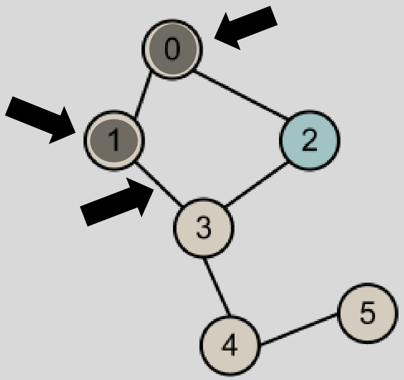
Offsets (another contiguous array)

1 **Log (** Vertex labels **), Log (** Edge weights **)**

1 **Log (** Vertex labels **), Log (** Edge weights **)**



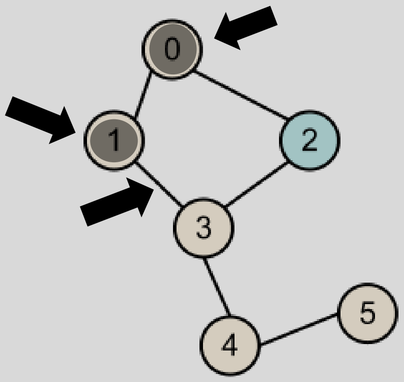
1 **Log (** Vertex labels **), Log (** Edge weights **)**



1 **Log (Vertex labels), Log (Edge weights)**

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v

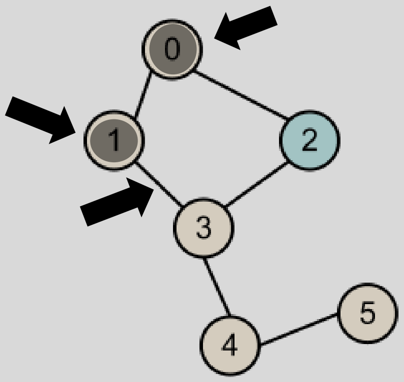


1 Log (Vertex labels), Log (Edge weights)

Lower bounds (global)

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v

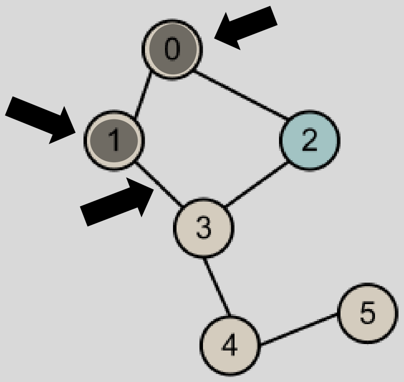


1 **Log (Vertex labels), Log (Edge weights)**

Lower bounds (global)

$$\lceil \log n \rceil$$

Symbols



n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v

1 **Log (Vertex labels), Log (Edge weights)**

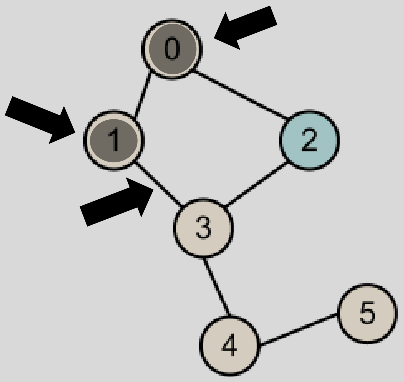
Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

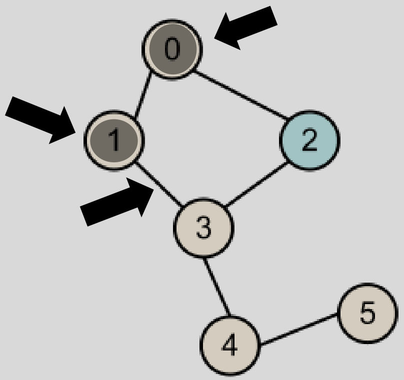
Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



1 Log (Vertex labels), Log (Edge weights)

Symbols



n : #vertices,
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d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v

Lower bounds (global)

$$\lceil \log n \rceil$$

This is it?
Not really ☺

Lower bounds (local)

1 **Log (Vertex labels), Log (Edge weights)**

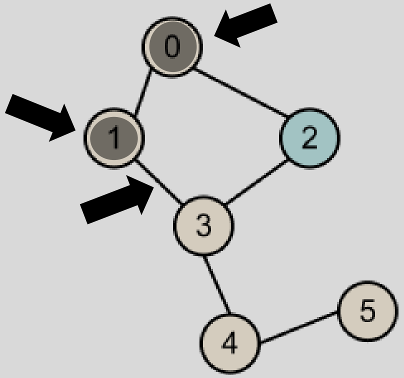
Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (local)

Assume:

1 Log (Vertex labels), Log (Edge weights)

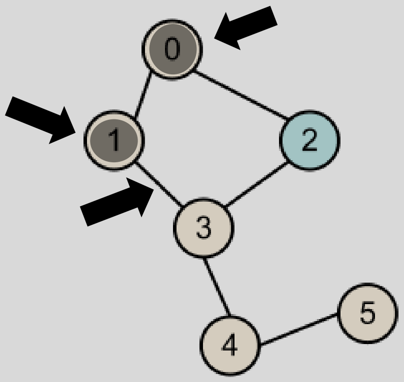
Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$

1 Log (Vertex labels), Log (Edge weights)

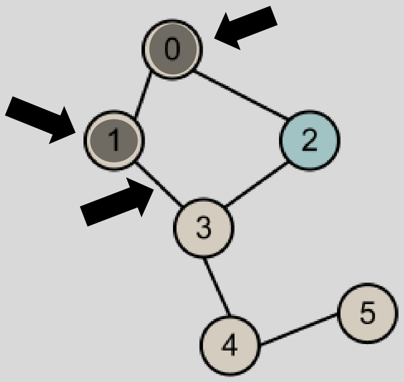
Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



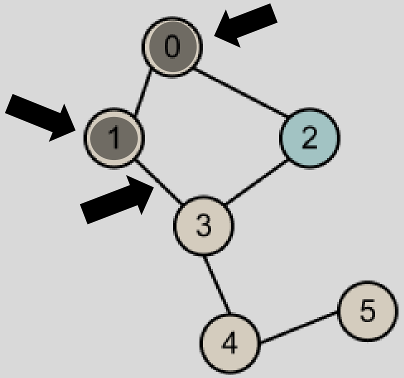
Lower bounds (local)

Assume:
- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$

1 Log (Vertex labels), Log (Edge weights)

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

1 Log (Vertex labels), Log (Edge weights)

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
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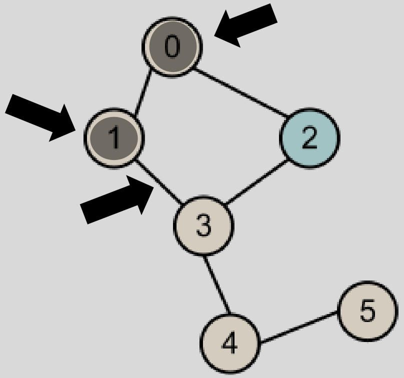


Lower bounds (global)

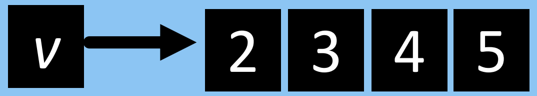
$\lceil \log n \rceil$

This is it?
Not really ☺

Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$



1 Log (Vertex labels), Log (Edge weights)

Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
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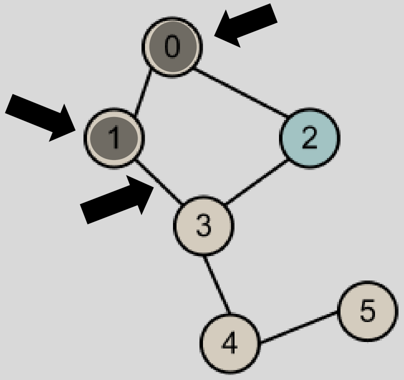
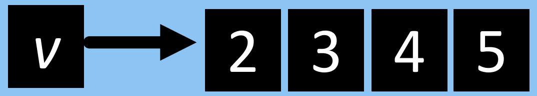


Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

$\lceil \log 2^{22} \rceil = 22$



1 Log (Vertex labels), Log (Edge weights)

Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Symbols

n : #vertices,
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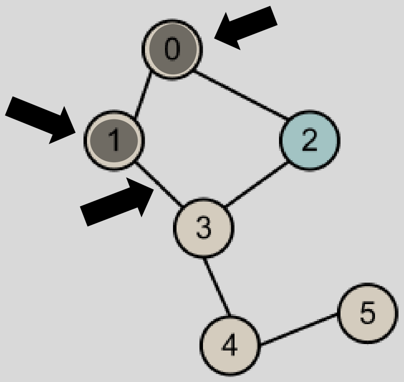
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

v → 2 | 3 | 4 | 5

$\lceil \log 2^{22} \rceil = 22$

v → 0...10 | 0...11 | 0...100 | 0...101

1 Log (Vertex labels), Log (Edge weights)

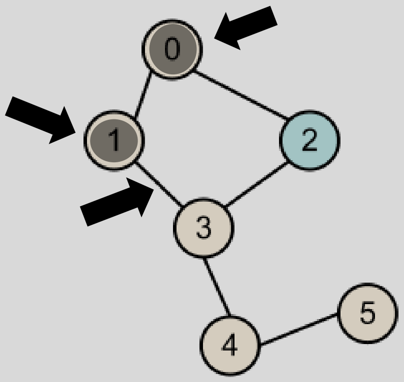
Lower bounds (global)

$$\lceil \log n \rceil$$

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

v → 2 | 3 | 4 | 5

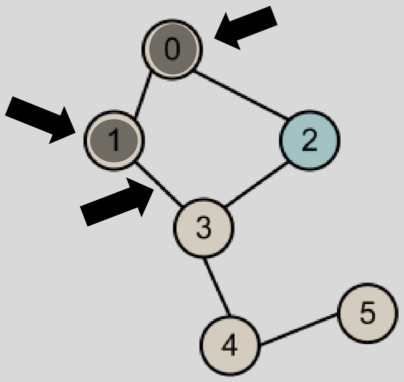
$$\lceil \log 2^{22} \rceil = 22$$

v → 0...10 | 0...11 | 0...100 | 0...101

19 zeros!

1 Log (Vertex labels), Log (Edge weights)

Symbols



n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v

Lower bounds (global)

$\lceil \log n \rceil$

This is it?
Not really ☺

Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

v → 2 | 3 | 4 | 5

$\lceil \log 2^{22} \rceil = 22$

v → 0...10 | 0...11 | 0...100 | 0...101

19 zeros!

Thus, use the local bound $\lceil \log \widehat{N_v} \rceil$

1 Log (Vertex labels), Log (Edge weights)

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
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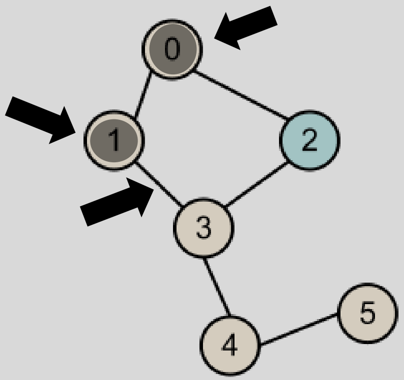

Lower bounds (local): problem

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

1 **Log (Vertex labels), Log (Edge weights)**

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
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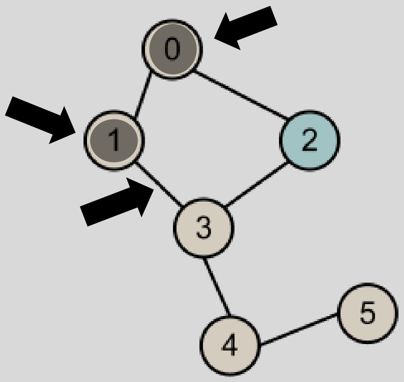


Lower bounds (local): problem

What if:

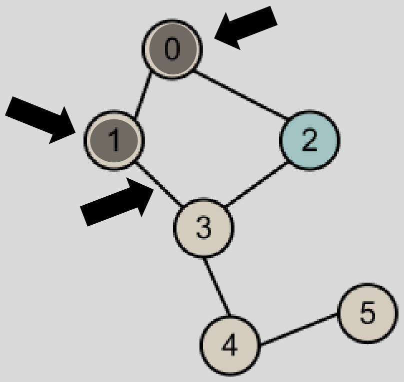
- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

1 Log (Vertex labels), Log (Edge weights)

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



Lower bounds (local): problem

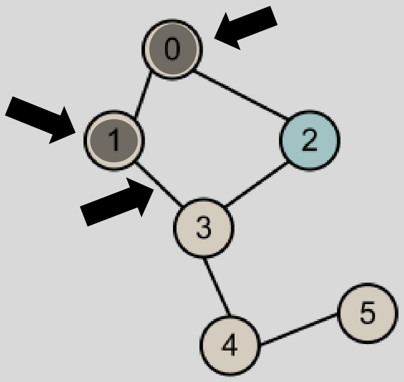
What if:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$
- ...one neighbor has a large ID:

1 **Log (Vertex labels), Log (Edge weights)**

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



This is it?
Not really ☺

Lower bounds (local): problem

What if:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$
- ...one neighbor has a large ID:

v → | 2 | 3 | 4 | 5 | 1M |

v → | 0...10 | 0...11 | 0...100 | 0...101 |

1 Log (Vertex labels), Log (Edge weights)

This is it?
Not really ☺

Symbols

n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
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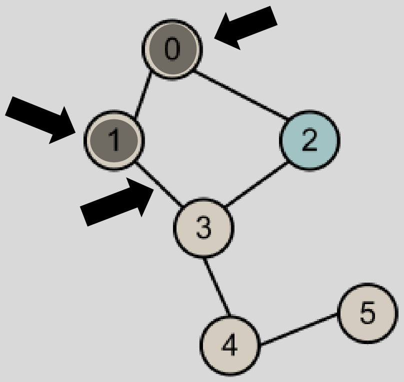
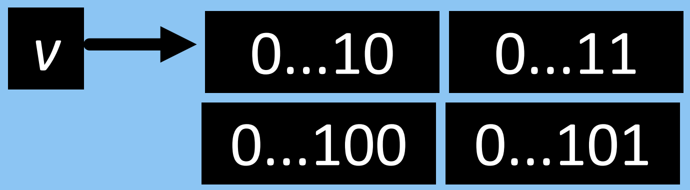


Lower bounds (local): problem

What if:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$
- ...one neighbor has a large ID:

$$\lceil \log 2^{20} \rceil = 20$$



1 Log (Vertex labels), Log (Edge weights)

This is it?
Not really ☺

Symbols

n : #vertices,
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d_v : degree of vertex v,
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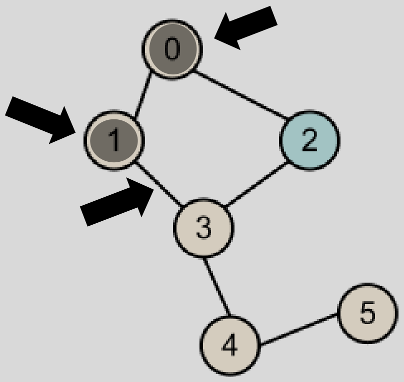


Lower bounds (local): problem

What if:

- a graph, e.g., $V = \{1, \dots, 2^{22}\}$
- A vertex v with few neighbors: $d_v \ll n$
- ...all these neighbors have small labels: $\widehat{N_v} \ll n$
- ...one neighbor has a large ID:

v → | 2 | 3 | 4 | 5 | 1M |

$$\lceil \log 2^{20} \rceil = 20$$

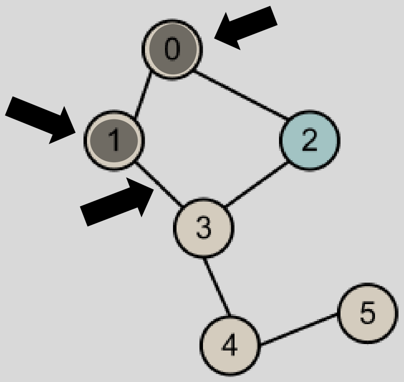
v → | 0...10 | 0...11 |
| 0...100 | 0...101 |

17 zeros!

1 **Log (** Vertex labels **), Log (** Edge weights **)**

Symbols

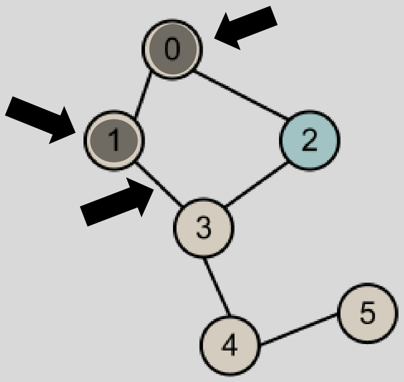
n : #vertices,
m : #edges,
d_v : degree of vertex v,
N_v : neighbors (adj. array) of vertex v,
$\widehat{N_v}$: maximum among N_v



1 **Log (Vertex labels), Log (Edge weights)**

...Use **Integer Linear Programming (ILP)**!

Symbols



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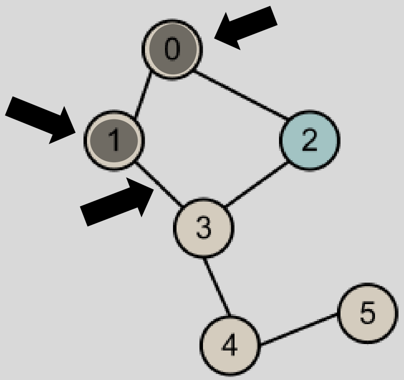

Lower bounds (local) enhanced with ILP

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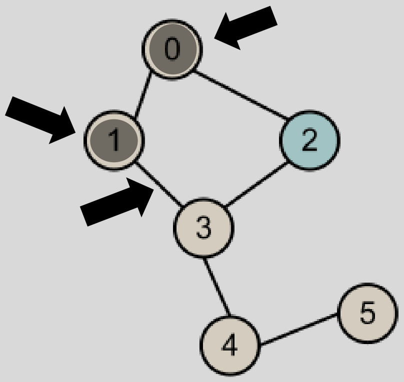

Lower bounds (local) enhanced with ILP

Permute vertex labels **to reduce such maximum labels** in as many neighborhoods as possible

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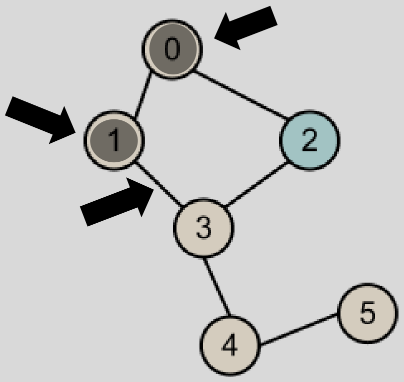

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v	→	2	3	4	5	1M

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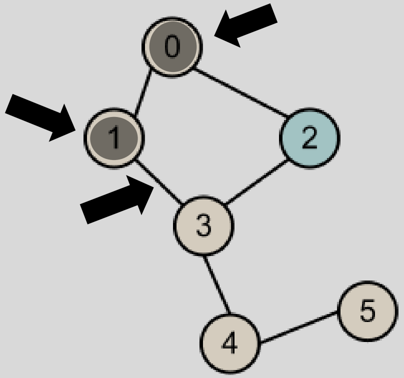

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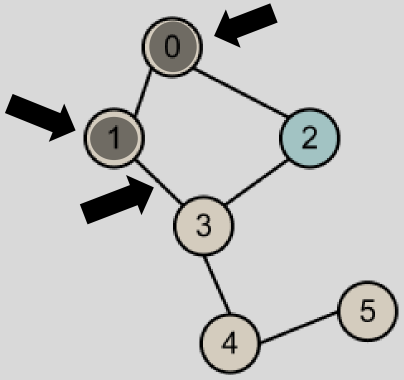

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Permute(2 3 4 5 1M) =

(simultaneously for all other neighborhoods)

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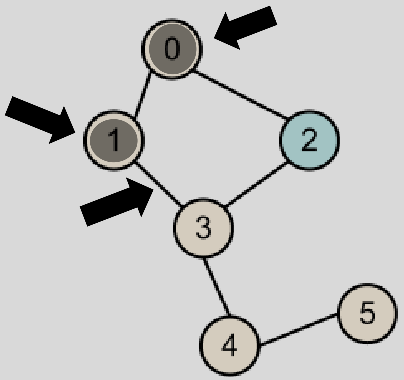

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$\leq 100?$

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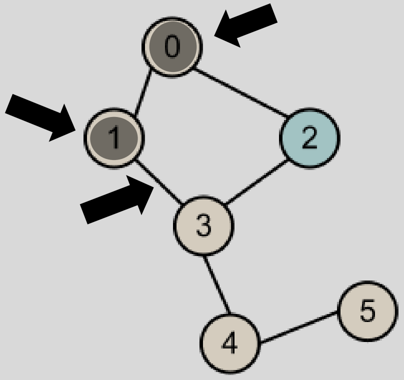

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Heuristics:

$$\min \sum_{v \in V} \widehat{N_v} \frac{1}{d_v}$$

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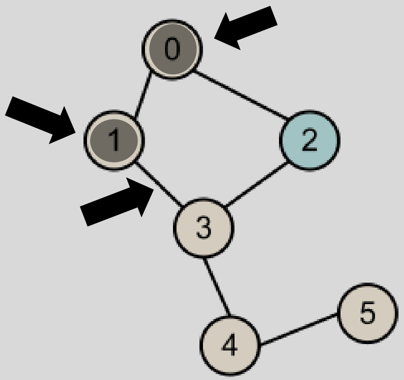

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Inverse of the neighborhood size

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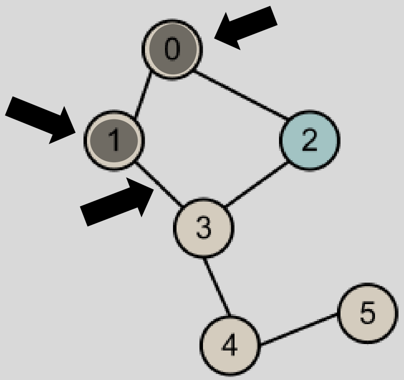

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Intuition: maximum labels in **new** neighborhoods will be **smaller**

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Inverse of the neighborhood size

1 **Log (** Vertex labels **), Log (** Edge weights **)**

∰ Formal analyses

Symbols

$\widehat{W}$: max edge weight,
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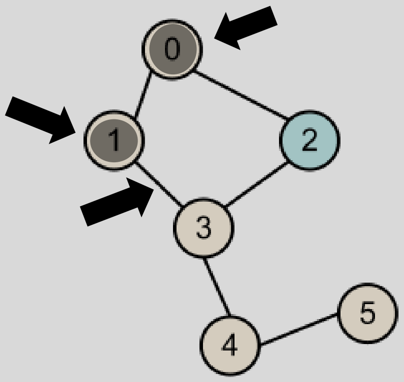

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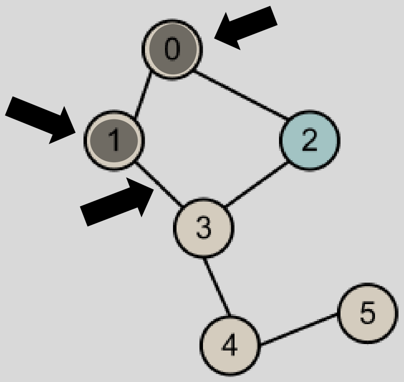

Power-law graphs

Random uniform graphs

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Power-law graphs

The probability that a vertex has degree *d* is:

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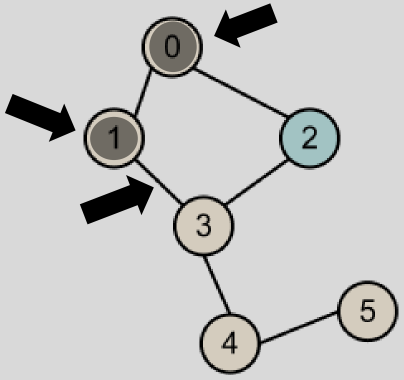

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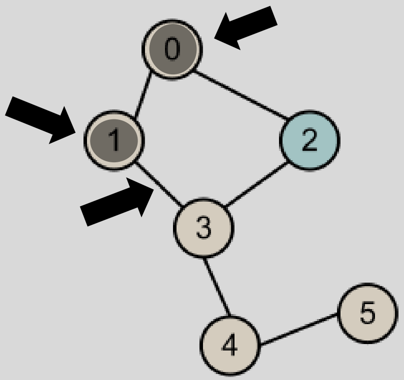

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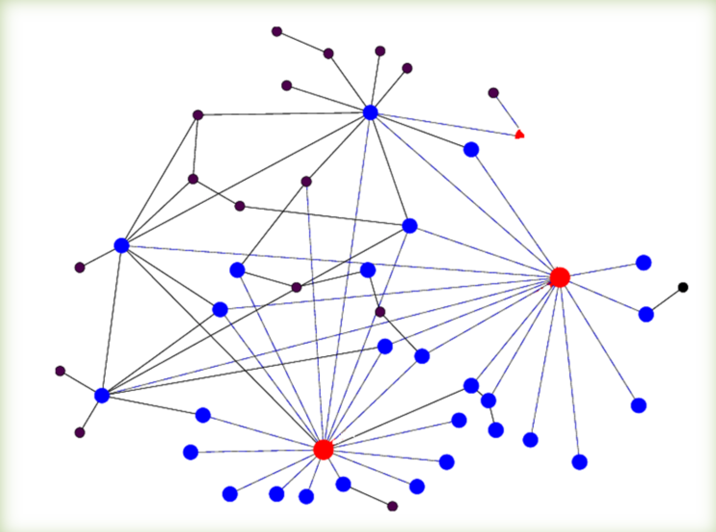

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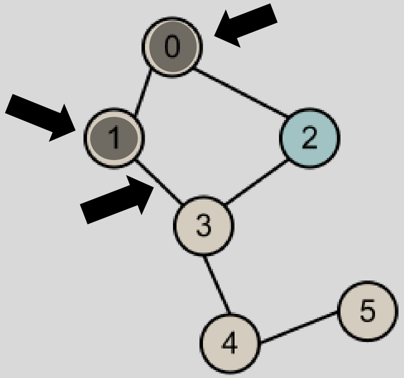

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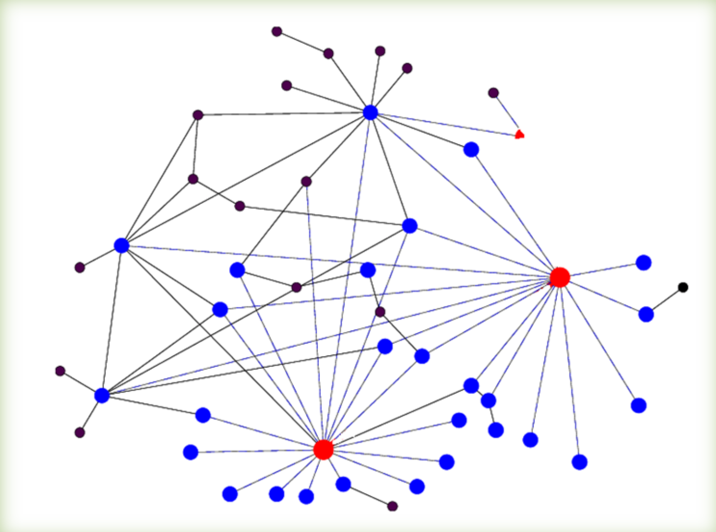

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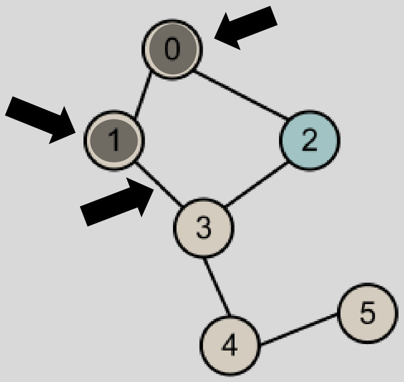

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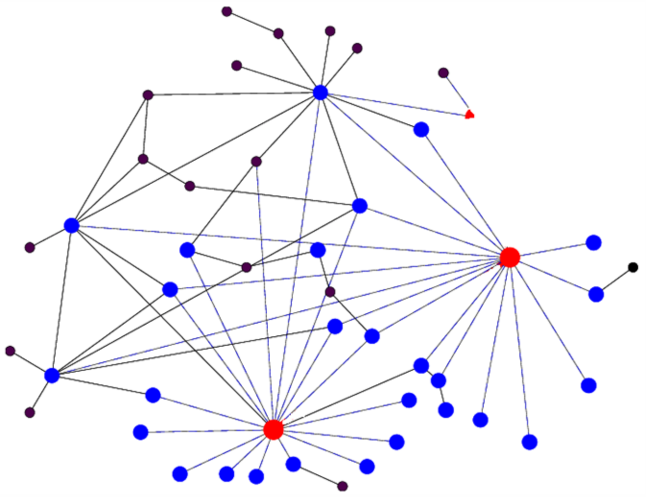

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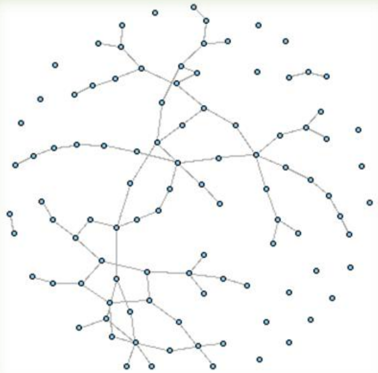

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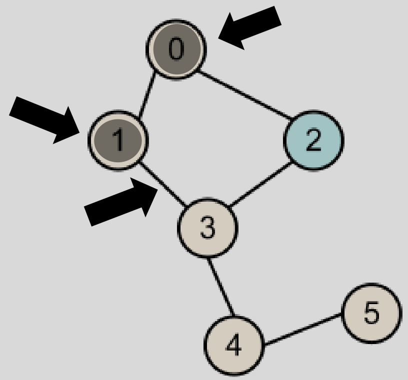

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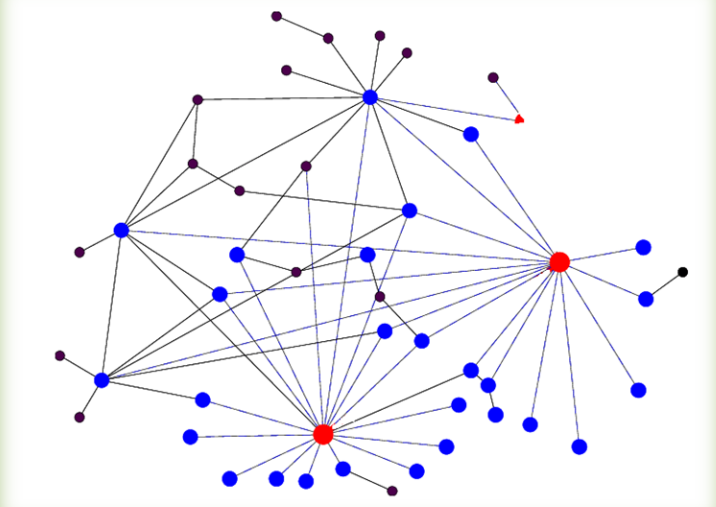

Expected size of the adjacency array

$$E[|\mathcal{A}|] \approx \frac{\alpha}{2-\beta}\left(\left(\frac{\alpha n \log n}{\beta - 1}\right)^{\frac{2-\beta}{\beta-1}} - 1\right)\left(\lceil \log n \rceil + \left\lceil \log \widehat{W} \right\rceil\right)$$

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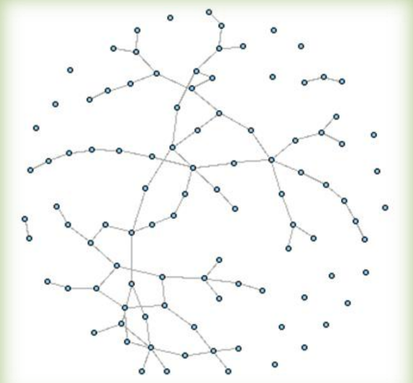

Expected size of the adjacency array

$$E[|\mathcal{A}|] = \left(\lceil \log n \rceil + \left\lceil \log \widehat{W} \right\rceil\right) pn^2$$

1 **Log (Vertex labels), Log (Edge weights)**

∰ Formal analyses: more (check the paper ☺)

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$$E[|\mathcal{O}|] = n\left\lceil \log\left(2pn^2\right)\right\rceil = n\lceil \log 2p + 2\log n\rceil$$

$$\forall_{v,u\in V}\ (u \in N_v) \ \Rightarrow \left[\mathcal{N}(u) \le \widehat{N}_v\right]$$

$$|\mathscr{A}| = \sum_{v\in V}\left(d_v\left\lceil \log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{N}_v\right\rceil\right)$$

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$$|\mathcal{A}| = 2m\left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right)$$

$$|\mathcal{A}| = \sum_{v\in V}\left(d_v\left(\left\lceil \log \widehat{N}_v\right\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right) + \left\lceil \log\log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{\mathcal{W}}\right\rceil\right)$$

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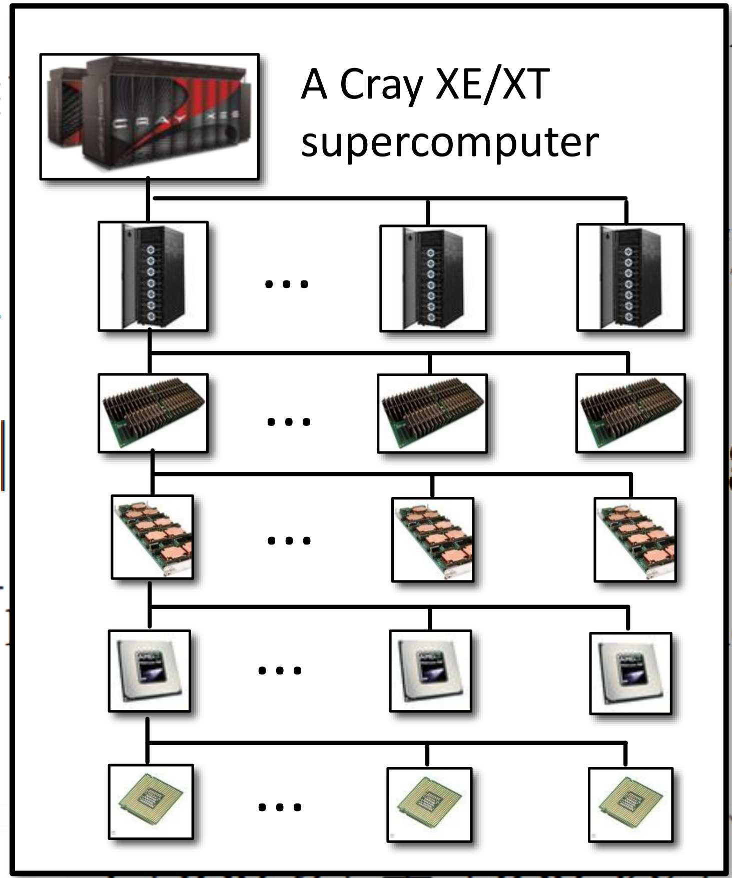
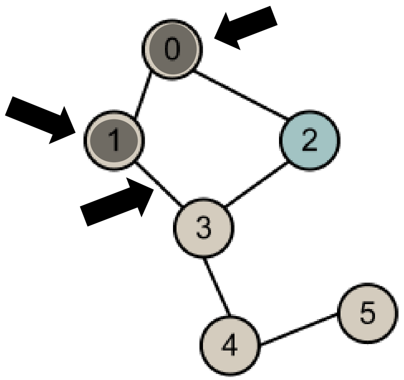


1 **Log (** Vertex labels **), Log (** Edge weights **)**

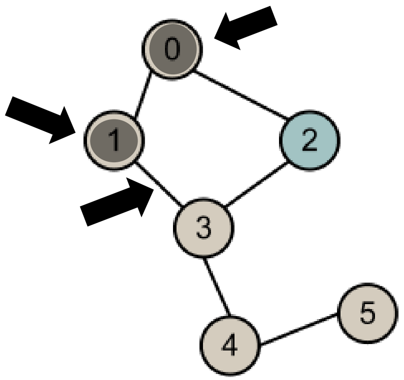
Key methods



1 **Log (** Vertex labels **), Log (** Edge weights **)**

Key methods

Use **the BEXTR bitwise operation** to help extract an arbitrary sequence of bits



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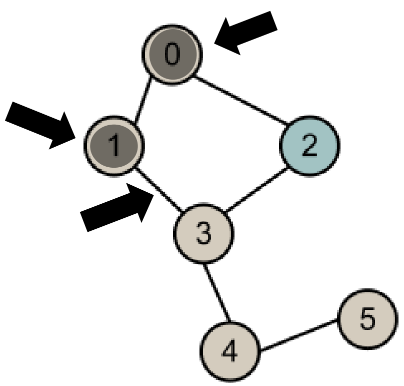


```
/* v_ID is an opaque type for IDs of vertices. */
v_ID N_{i,v}(v_ID v, int32_t i, int64_t* 𝒪, int64_t* 𝒜, int8_t s){
  int64_t exactBitOffset = s * (𝒪[v] + i);
  int8_t* address = (int8_t*) 𝒜 + (exactBitOffset >> 3);
  int64_t distance = exactBitOffset & 7;
  int64_t value = ((int64_t*) (address))[0];
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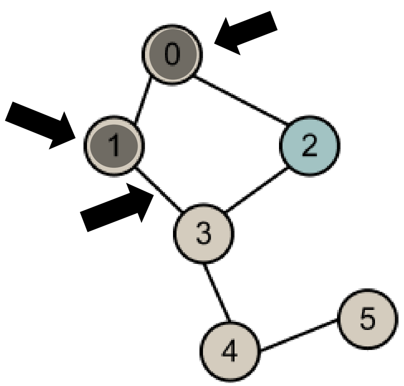
1 Log (Vertex labels), Log (Edge weights)

Key methods

Use **the BEXTR bitwise operation** to help extract an arbitrary sequence of bits **!**



Return *i*-th neighbor of vertex *v*

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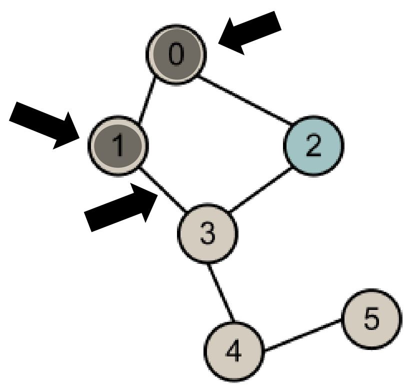

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Pointer to the offset array

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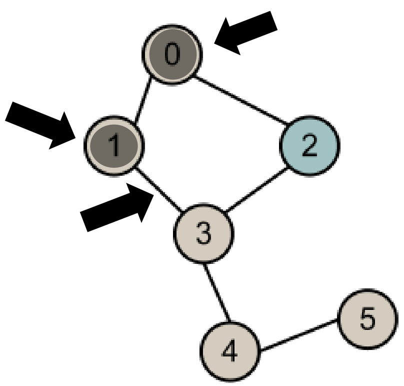

Return *i*-th neighbor of vertex *v*

Pointer to the offset array

Pointer to the adjacency array

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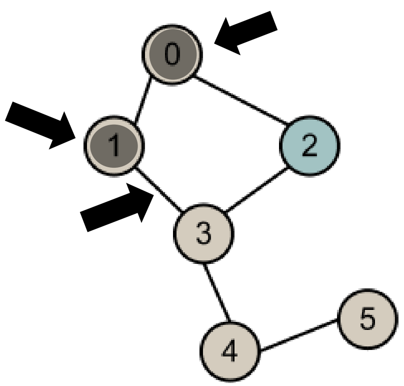

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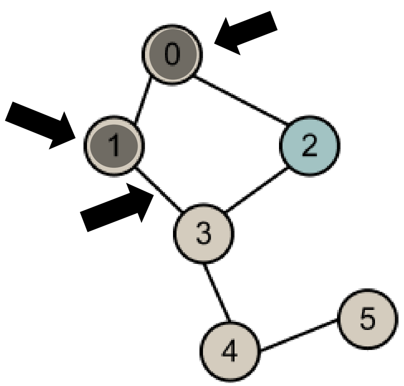

Return *i*-th neighbor of vertex *v*

Derive exact offset (in bits) to the neighbor label

Pointer to the offset array

Pointer to the adjacency array

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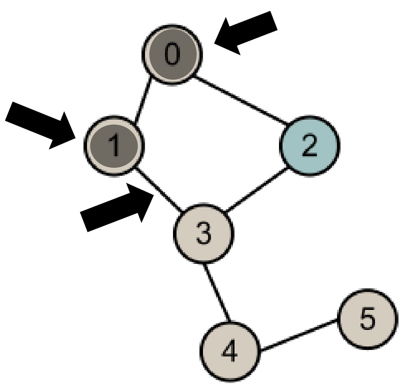
Key methods

Use the BEXTR bitwise operation to help extract an arbitrary sequence of bits !



Return *i*-th neighbor of vertex *v*

Derive exact offset (in bits) to the neighbor label

Pointer to the offset array

Pointer to the adjacency array

$s = \lceil \log n \rceil$

Get the closest byte alignment

```
/* v_ID is an opaque type for IDs of vertices. */
v_ID N_{i,v}(v_ID v, int32_t i, int64_t* O, int64_t* A, int8_t s){
  int64_t exactBitOffset = s * (O[v] + i);
  int8_t* address = (int8_t*) A + (exactBitOffset >> 3);
  int64_t distance = exactBitOffset & 7;
  int64_t value = ((int64_t*) (address))[0];
  return _bextr_u64(value, distance, s); }
```

1 Log (Vertex labels), Log (Edge weights)

Key methods

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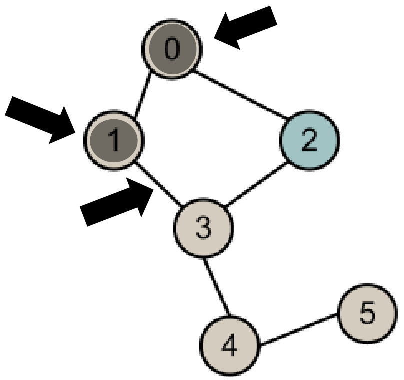

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Get the closest byte alignment

Get the distance from the byte alignment

1 **Log (Vertex labels), Log (Edge weights)**

Key methods

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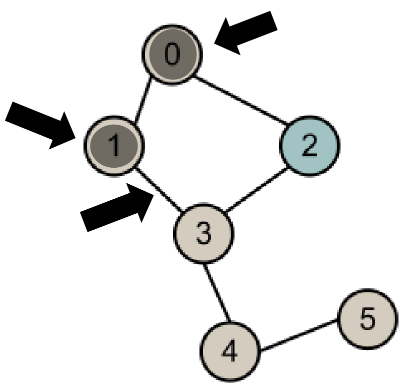

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Access the derived 64-bit value

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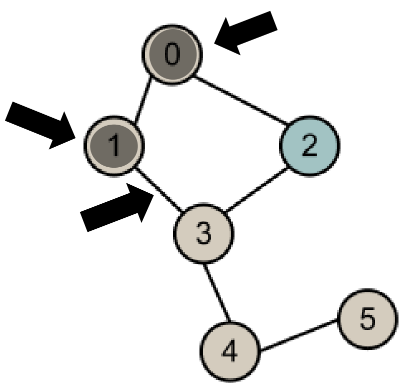

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Get the closest byte alignment

Get the distance from the byte alignment

Shift the derived 64-bit value by d bits and mask it with BEXTR

Access the derived 64-bit value

❷ Log (Offset structure)

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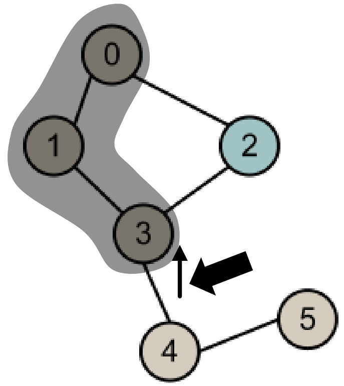
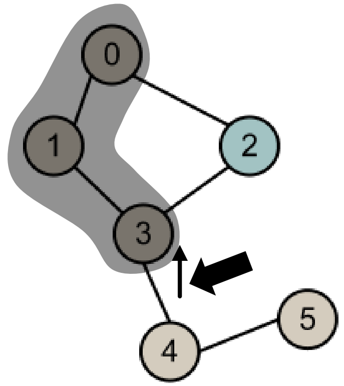


2 Log (Offset structure)

Use a **bit vector** instead of an array of offsets... !



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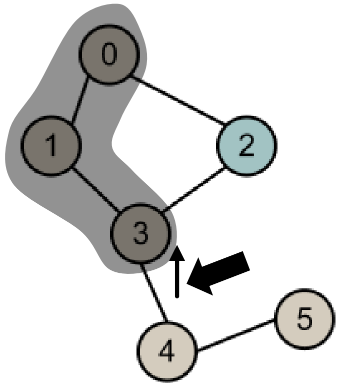

Bit vectors instead of offset arrays

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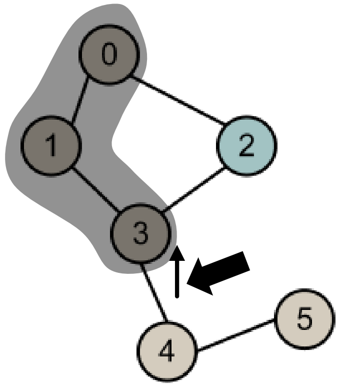

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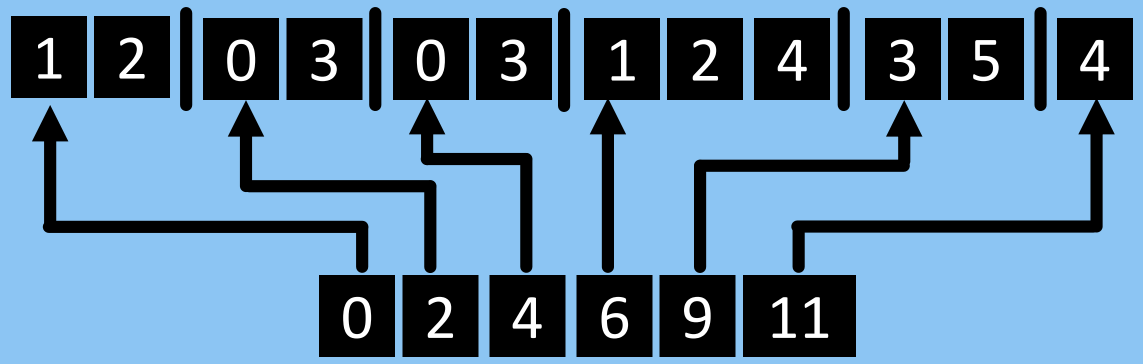

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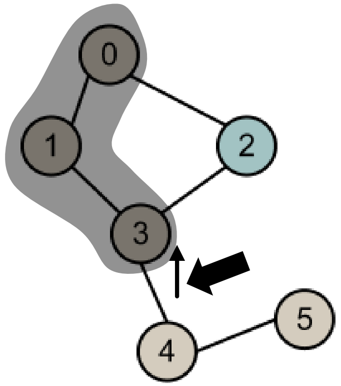

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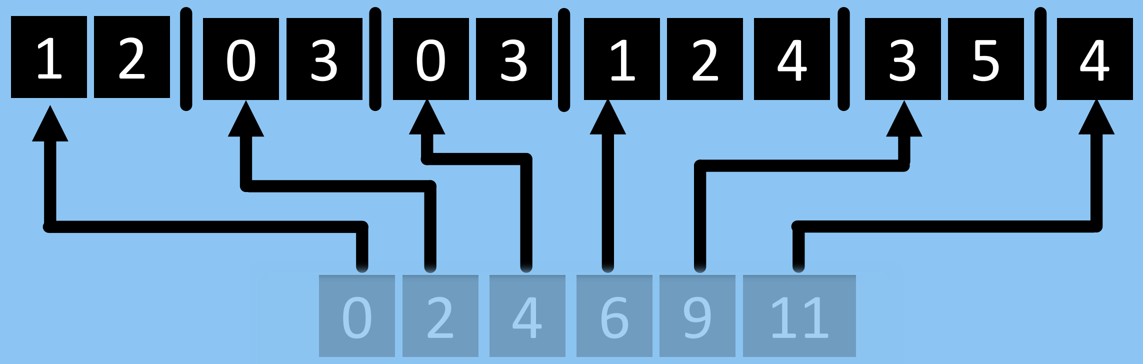

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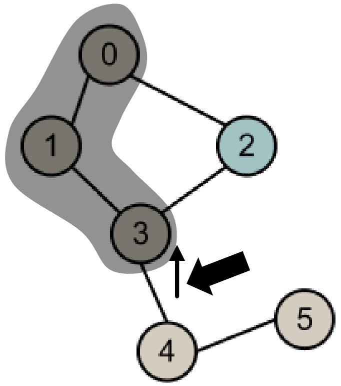

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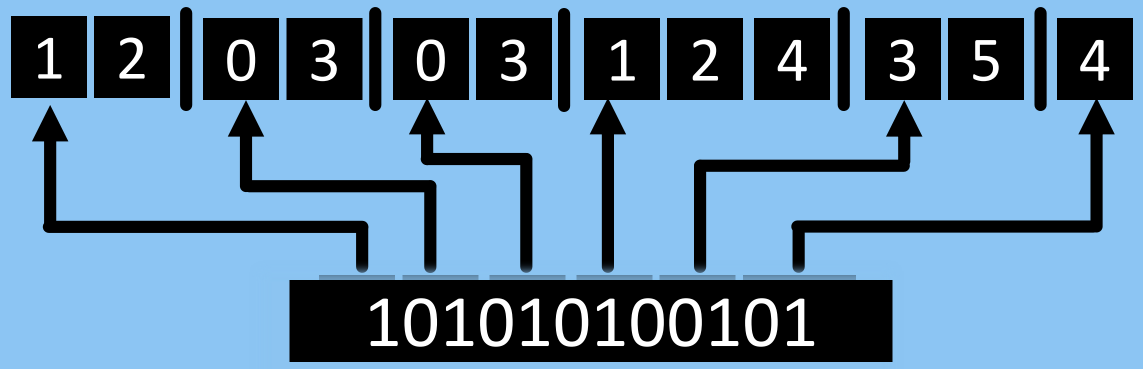

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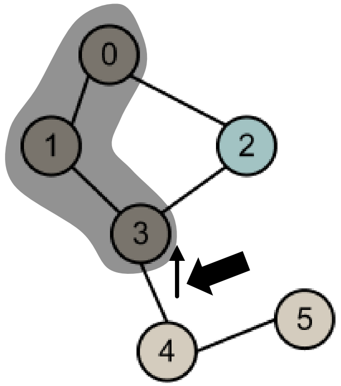

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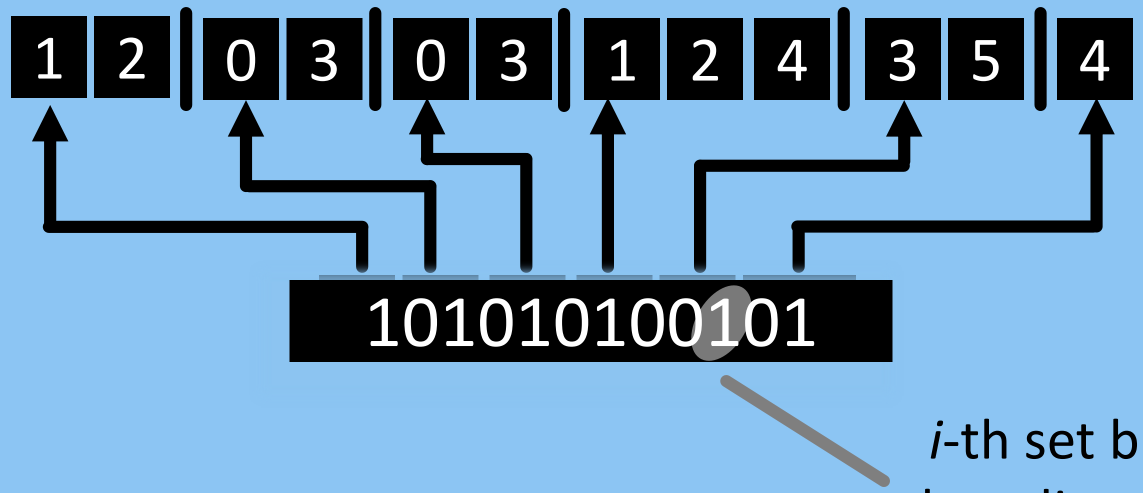

i-th set bit has a position *x* ➔ the adjacency array of a vertex *i* starts at a word *x*

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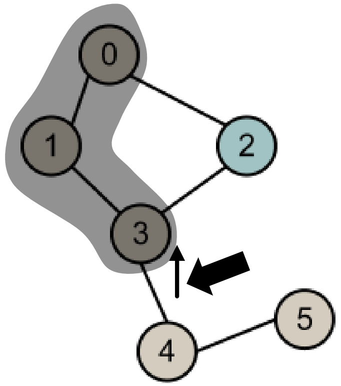

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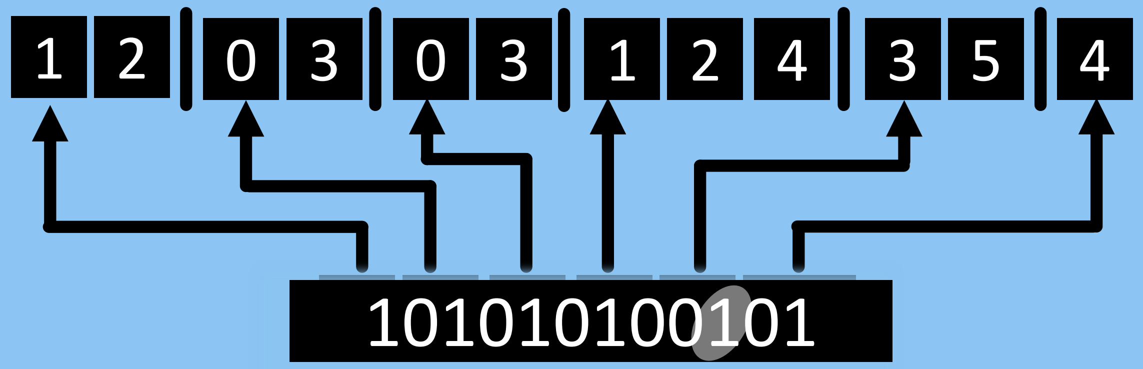
Bit vectors instead of offset arrays

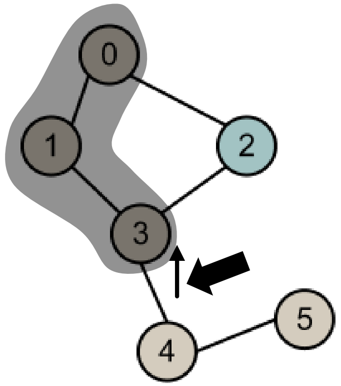


How many 1s are set before a given i-th bit?

i-th set bit has a position *x* ➔ the adjacency array of a vertex *i* starts at a word *x*

2 **Log (** Offset structure **)**

...Encode the resulting bit vectors as **succinct bit vectors [1]** !



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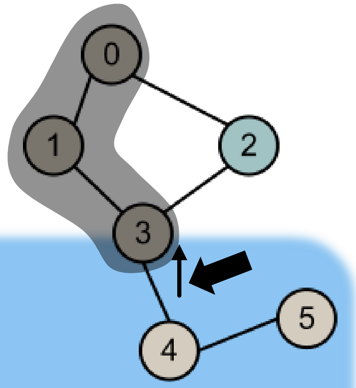

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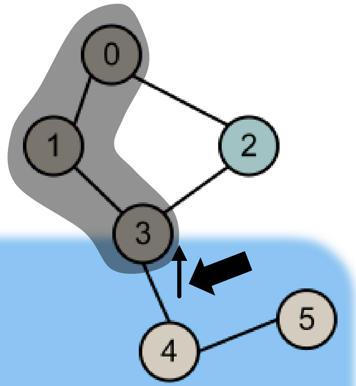

Succinct bit vectors

They use **⌈Q⌉ + *o*(Q) bits** (⌈Q⌉ - lower bound), they answer various queries in ***o*(Q) time**.

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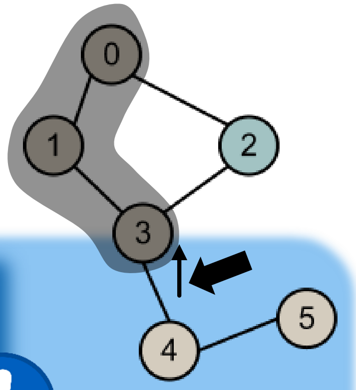

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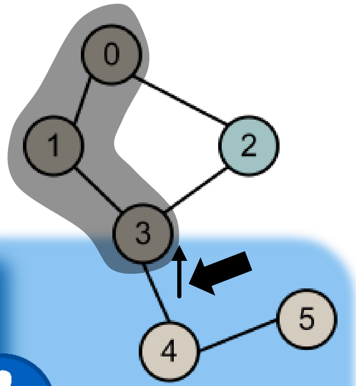

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101010100101000101010111110000001100001...

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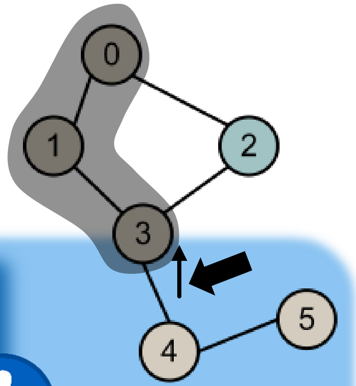

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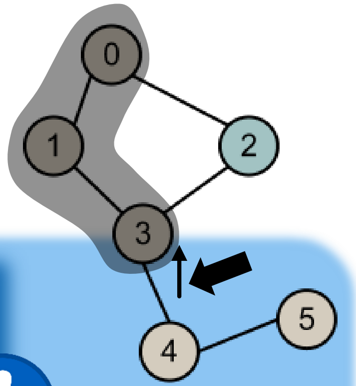

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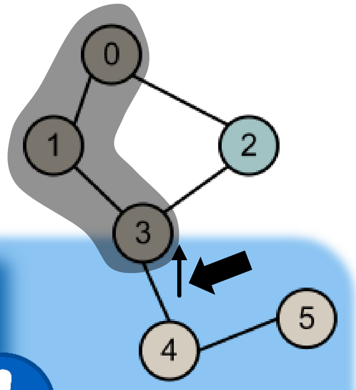

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$\log^2 n$ | $\log^2 n$ | $\log^2 n$ | ...

n bits: 10101010010 | 10001010101 | 11110000001 | 100001...

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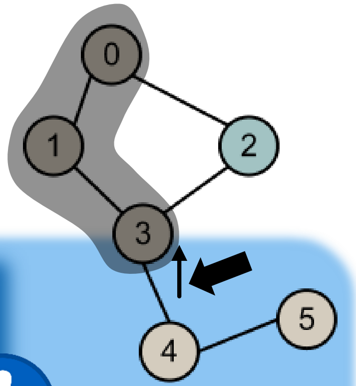

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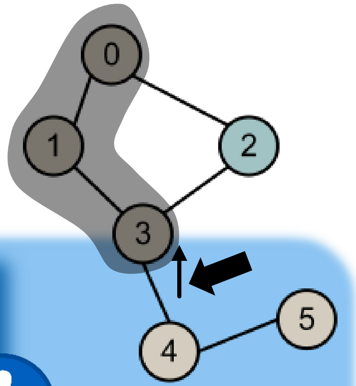

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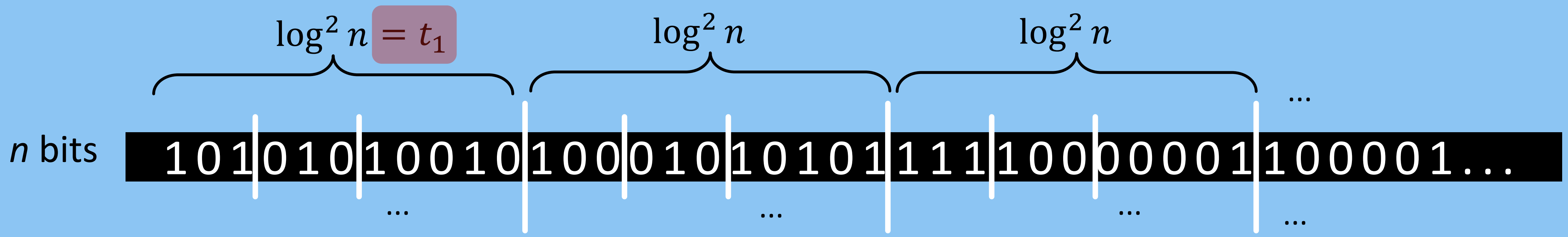

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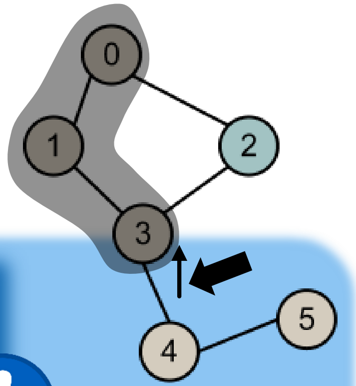

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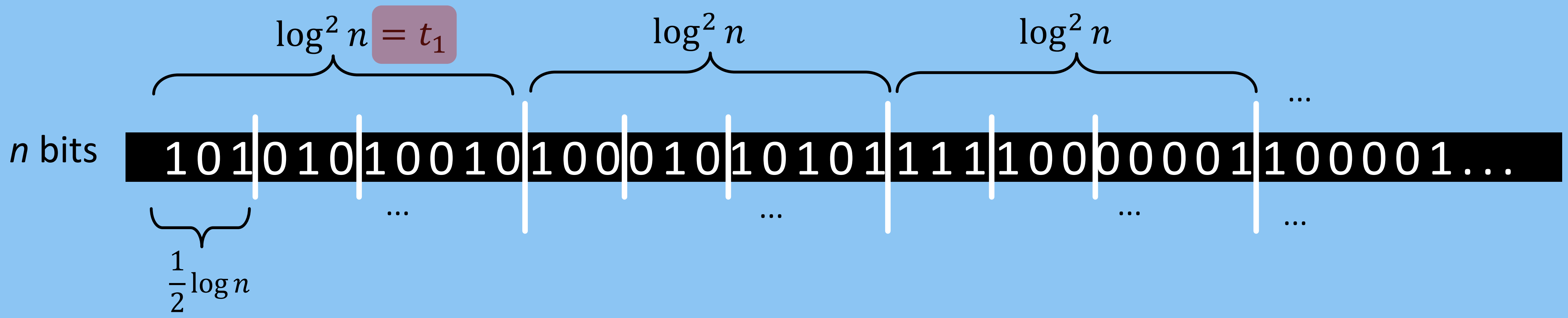

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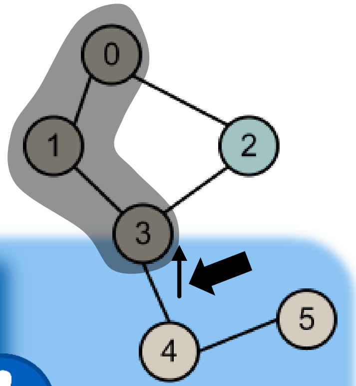

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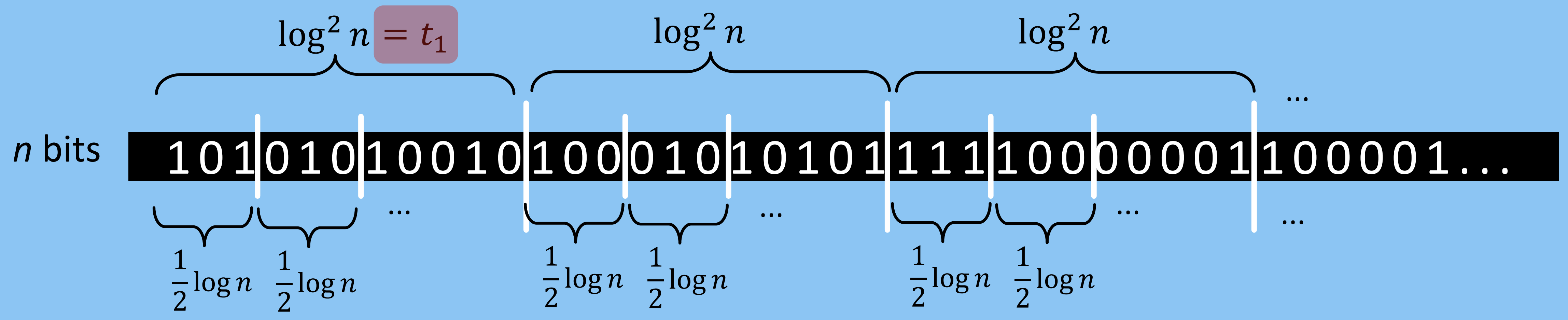

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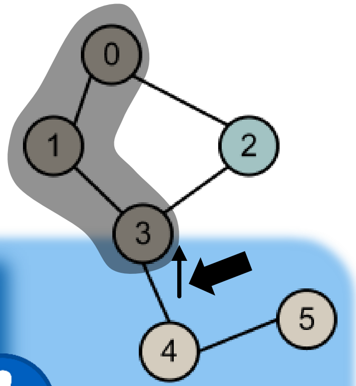

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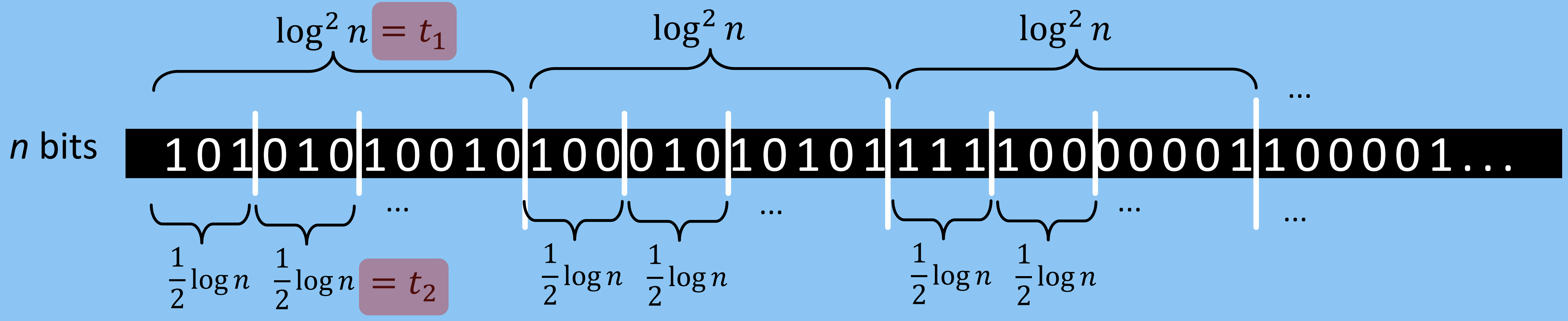

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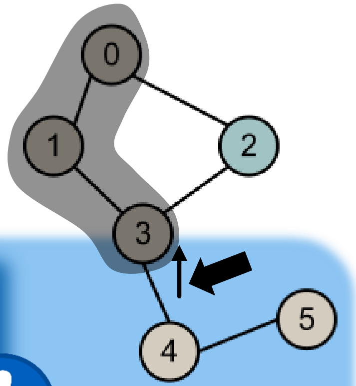

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Compute & store the number of 1s

$\log^2 n = t_1$ $\quad \log^2 n \quad \log^2 n \quad$ ...

n bits: 1 0 1 | 0 1 0 | 1 0 0 1 0 | 1 0 0 | 0 1 0 | 1 0 1 0 1 | 1 1 1 | 1 0 0 | 0 0 0 0 1 | 1 0 0 0 0 1 . . .

$\frac{1}{2}\log n \quad \frac{1}{2}\log n = t_2$ ... $\quad \frac{1}{2}\log n \quad \frac{1}{2}\log n$ ... $\quad \frac{1}{2}\log n \quad \frac{1}{2}\log n$ ... ...

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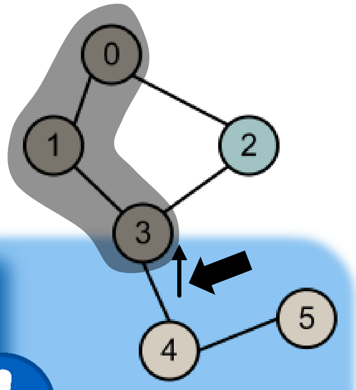

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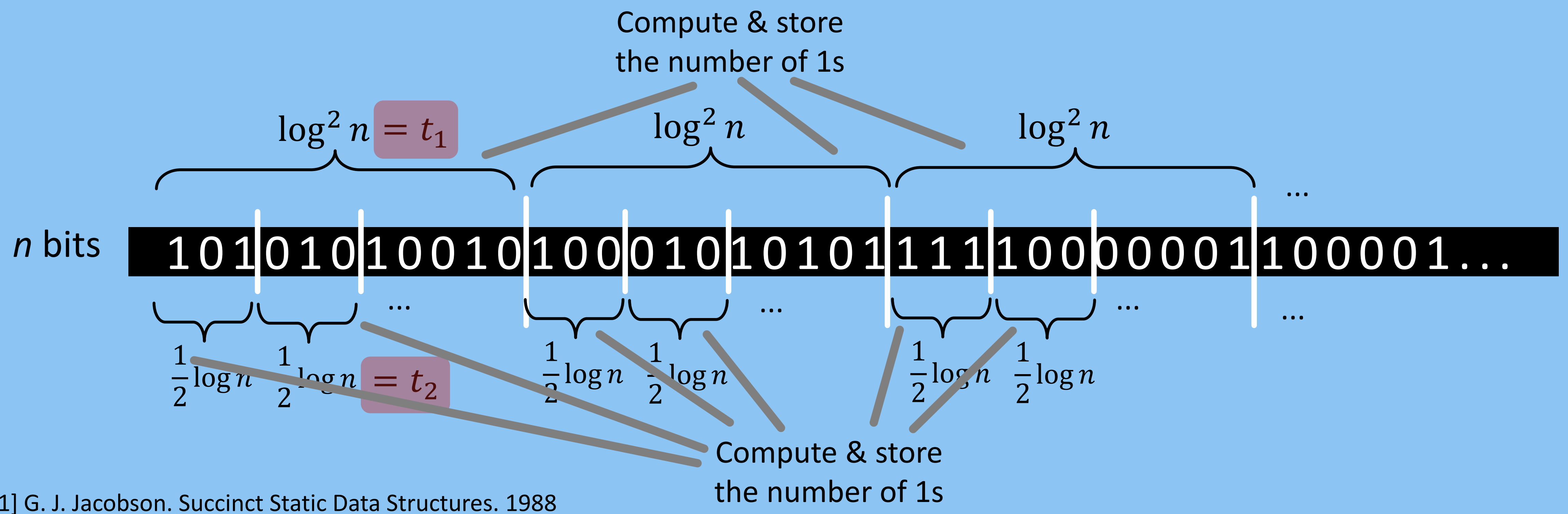

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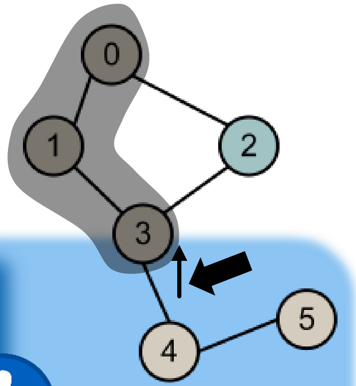

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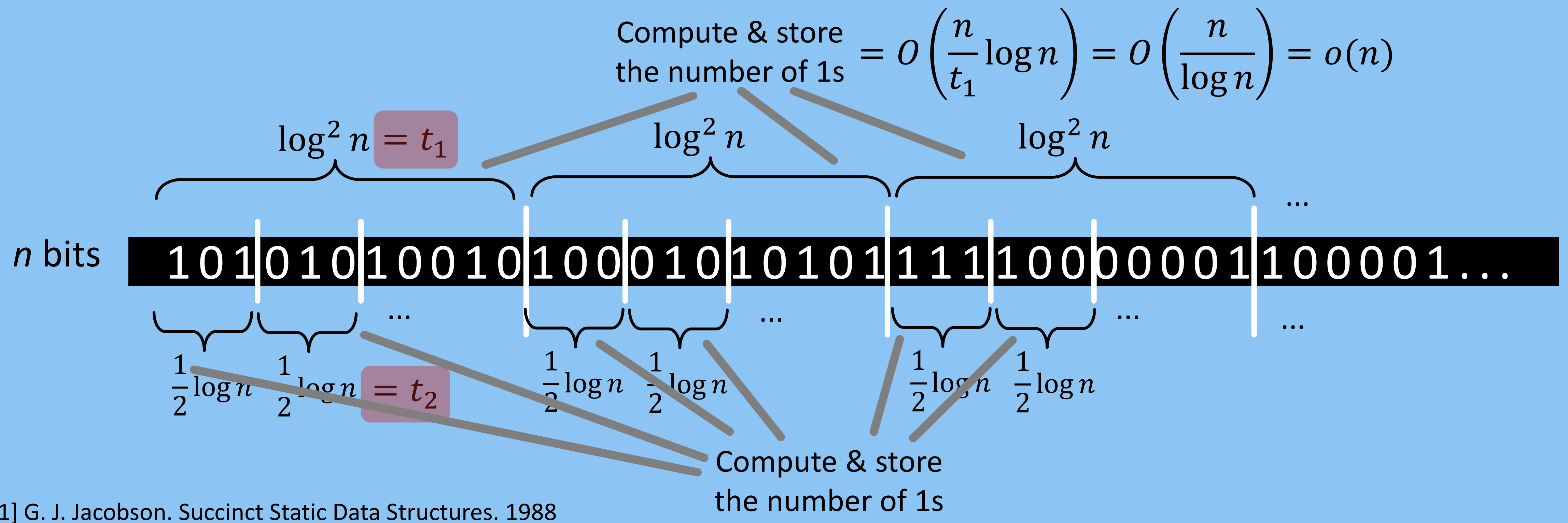

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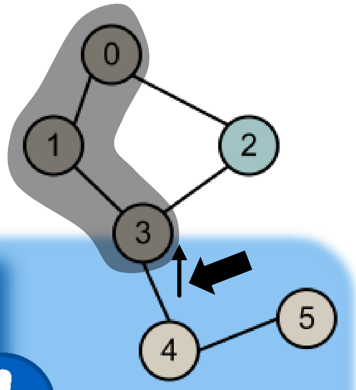

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Compute & store the number of 1s $= O\left(\frac{n}{t_1}\log n\right) = O\left(\frac{n}{\log n}\right) = o(n)$

$\log^2 n = t_1$ $\quad \log^2 n \quad \log^2 n \quad \ldots$

n bits: 1 0 1|0 1 0|1 0 0 1 0|1 0 0|0 1 0|1 0 1 0 1|1 1 1|1 0 0|0 0 0 1|1 0 0 0 0 1 . . .

$\frac{1}{2}\log n \quad \frac{1}{2}\log n = t_2 \quad \ldots \quad \frac{1}{2}\log n \quad \frac{1}{2}\log n \quad \ldots \quad \frac{1}{2}\log n \quad \frac{1}{2}\log n \quad \ldots \quad \ldots$

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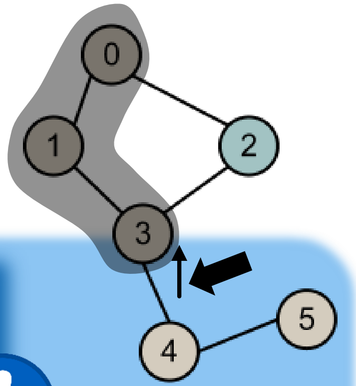

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Compute & store the number of 1s $= O\left(\frac{n}{t_1}\log n\right) = O\left(\frac{n}{\log n}\right) = o(n)$

$\log^2 n = t_1$ $\log^2 n$ $\log^2 n$ ...

n bits: 1 0 1 | 0 1 0 | 1 0 0 1 0 | 1 0 0 | 0 1 0 | 1 0 1 0 1 | 1 1 1 | 1 0 0 | 0 0 0 0 1 | 1 0 0 0 0 1 . . .

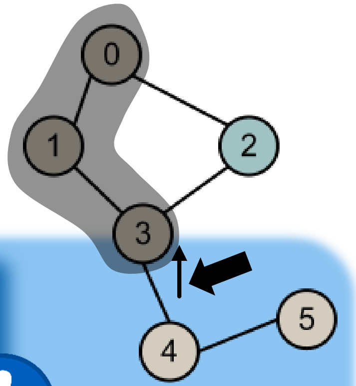
$\frac{1}{2}\log n$ $\frac{1}{2}\log n = t_2$ ... $\frac{1}{2}\log n$ $\frac{1}{2}\log n$ ... $\frac{1}{2}\log n$ $\frac{1}{2}\log n$ ... ...

Compute & store the number of 1s $= O\left(\frac{n}{t_2}\log t_1\right) = O\left(\frac{n\log\log n}{\log n}\right) = o(n)$

[1] G. J. Jacobson. Succinct Static Data Structures. 1988

2 Log (Offset structure)

...Encode the resulting bit vectors as **succinct bit vectors [1]** !



Succinct bit vectors

They use **[Q] + *o*(Q) bits** ([Q] - lower bound), they answer various queries in ***o*(Q) time**.

= small + fast (hopefully) !

Total storage:

$$n + o(n) + o(n) + \cdots = n + o(n)$$!

Compute & store the number of 1s $= O\left(\frac{n}{t_1}\log n\right) = O\left(\frac{n}{\log n}\right) = o(n)$

$\log^2 n = t_1$ $\log^2 n$ $\log^2 n$ ...

n bits: 1 0 1 | 0 1 0 | 1 0 0 1 0 | 1 0 0 | 0 1 0 | 1 0 1 0 1 | 1 1 1 | 1 0 0 | 0 0 0 0 1 | 1 0 0 0 0 1 . . .

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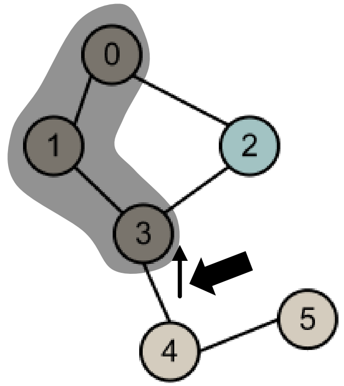

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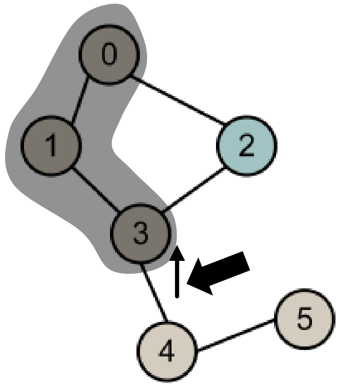

Formal analyses

2 Log (Offset structure)

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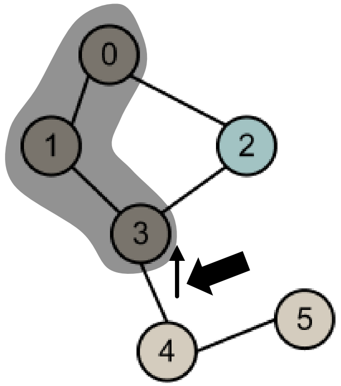

Formal analyses

$\mathcal{O}$	**ID**	**Asymptotic size** [bits]	**Exact size** [bits]	*select* or $\mathcal{O}[v]$
Pointer array	ptr$W$	$O(Wn)$	$W(n+1)$	$O(1)$
Plain [44]	bvPL	$O\left(\frac{Wm}{B}\right)$	$\frac{2Wm}{B}$	$O(1)$
Interleaved [44]	bvIL	$O\left(\frac{Wm}{B}+\frac{Wm}{L}\right)$	$2Wm\left(\frac{1}{B}+\frac{64}{L}\right)$	$O\left(\log\frac{Wm}{B}\right)$
Entropy based [31, 78]	bvEN	$O\left(\frac{Wm}{B}\log\frac{Wm}{B}\right)$	$\approx\log\binom{\frac{2Wm}{B}}{n}$	$O\left(\log\frac{Wm}{B}\right)$
Sparse [76]	bvSD	$O\left(n+n\log\frac{Wm}{Bn}\right)$	$\approx n\left(2+\log\frac{2Wm}{Bn}\right)$	$O(1)$
B-tree based [1]	bvBT	$O\left(\frac{Wm}{B}\right)$	$\approx 1.1\cdot\frac{2Wm}{B}$	$O(\log n)$
Gap-compressed [1]	bvGC	$O\left(\frac{Wm}{B}\log\frac{Wm}{Bn}\right)$	$\approx 1.3\cdot\frac{2Wm}{B}\log\frac{2Wm}{Bn}$	$O(\log n)$

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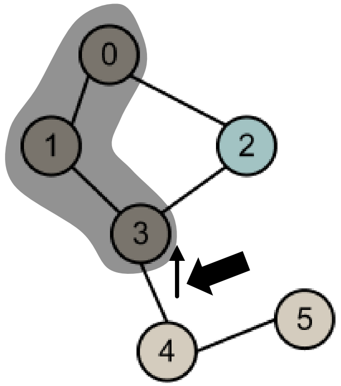

Formal analyses

Check the paper for details ☺

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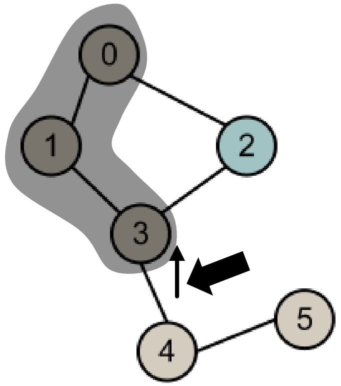

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Interleaved [44]	[illegible]	[illegible]	[illegible]	[illegible]
Entropy based [3[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Sparse [76]	[illegible]	[illegible]	[illegible]	[illegible]
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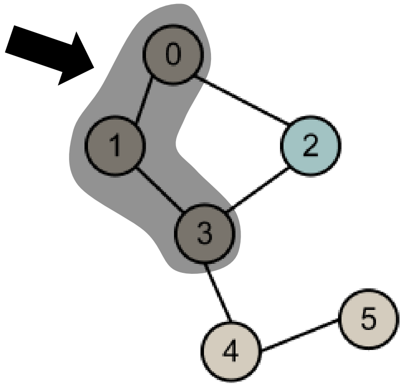
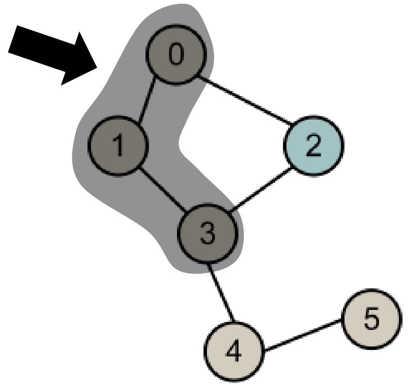
! **We will show that some are in practice both small and fast!**

3 **Log (** Adjacency structure **)**



3 **Log (** Adjacency structure **)**

Use different relabelings !



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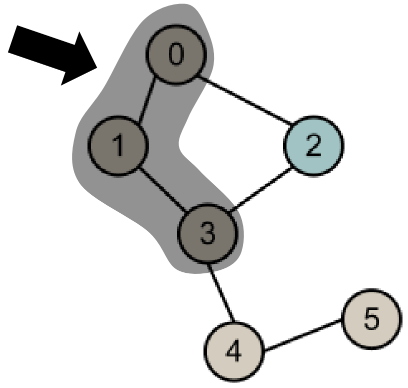
Use different relabelings !



Degree-Minimizing: Targeting general graphs (no assumptions on graph structure)

3 **Log (** Adjacency structure **)**

Use different relabelings

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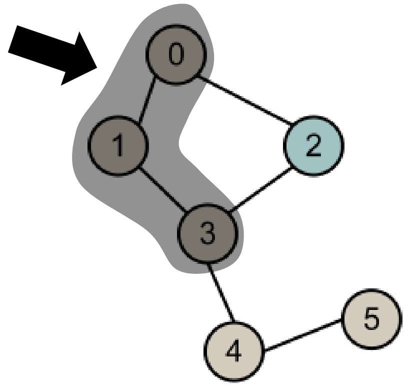


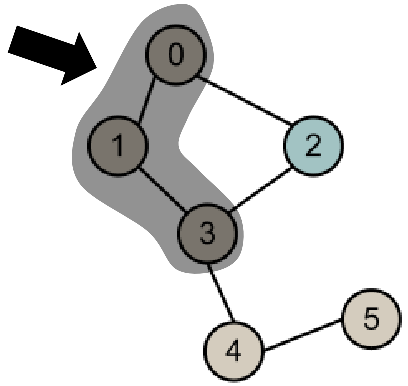
Degree-Minimizing: Targeting general graphs (no assumptions on graph structure)

More schemes that assume specific classes of graphs

...

3 **Log (** Adjacency structure **)**

Use different relabelings **!**



Degree-Minimizing: Targeting general graphs
(no assumptions on graph structure)

Permute(2 | 3 | 4 | 5 | 1M) = v | w | x | y | z

(simultaneously for all other neighborhoods)

More schemes that assume specific classes of graphs

...

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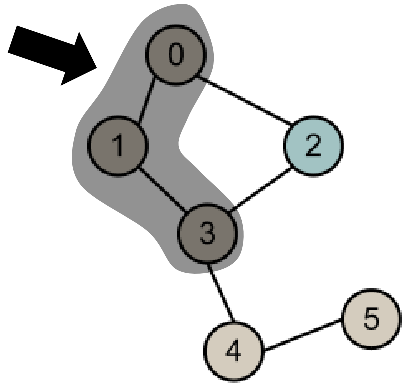
Use different relabelings !



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(simultaneously for all other neighborhoods)

(1) The more often a label occurs (i.e., the higher vertex degree), the smaller permuted value it receives

More schemes that assume specific classes of graphs

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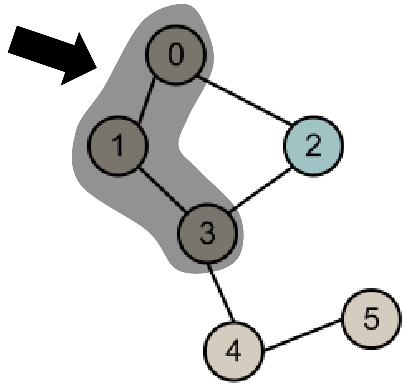
Use different relabelings !



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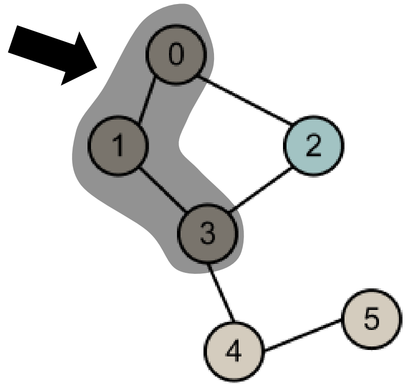
Gap-encode(v | w | x | y | z) = v | w-v | x-w | y-x | z-y

More schemes that assume specific classes of graphs

...

3 **Log (** Adjacency structure **)**

Use different relabelings **!**



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(simultaneously for all other neighborhoods)

(1) The more often a label occurs (i.e., the higher vertex degree), the smaller permuted value it receives

Gap-encode(v | w | x | y | z) = v | w-v | x-w | y-x | z-y

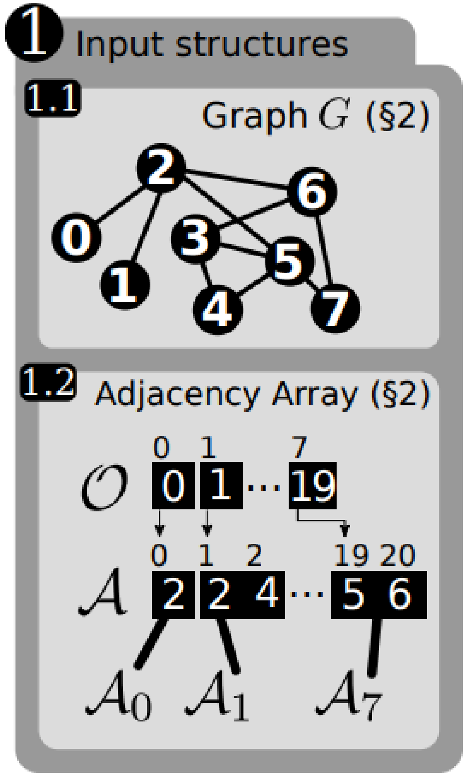
(2) Encode new labels with gap encoding (differences between consecutive labels instead of full labels)

More schemes that assume specific classes of graphs

...

Overview of Full Log(Graph) Design

Overview of Full Log(Graph) Design



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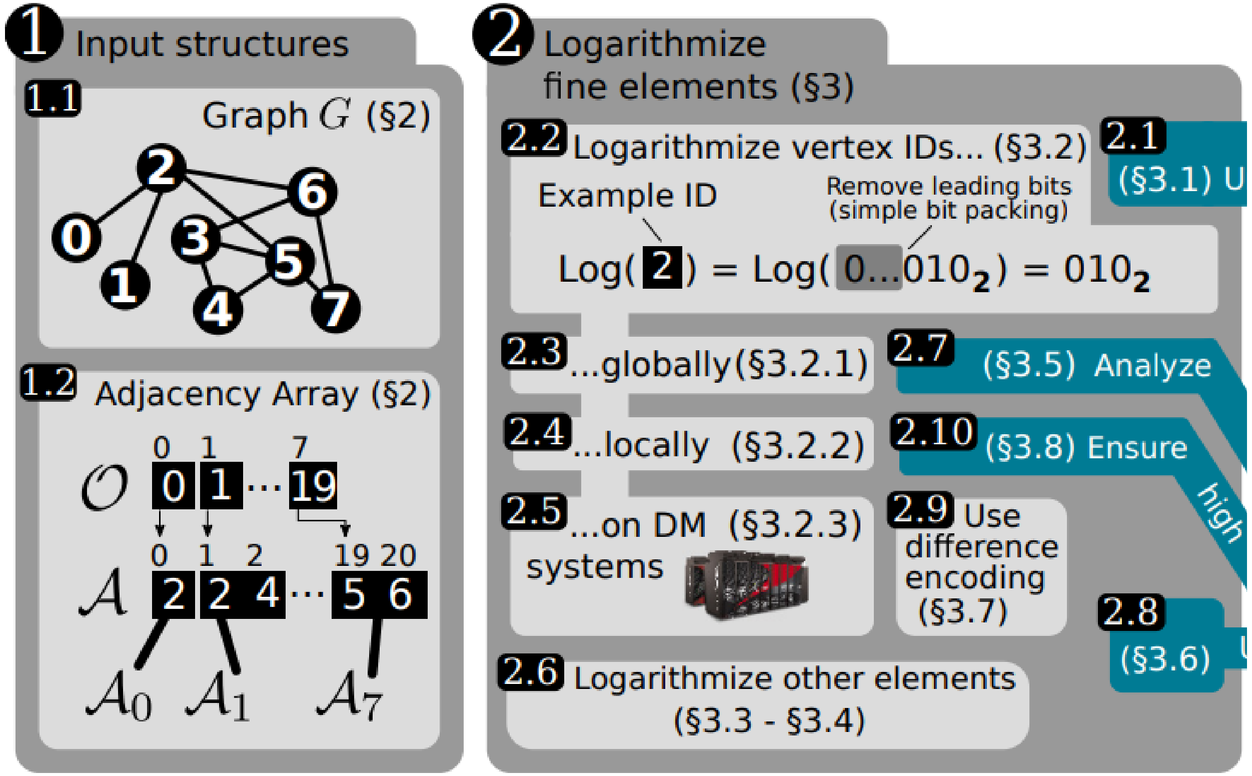

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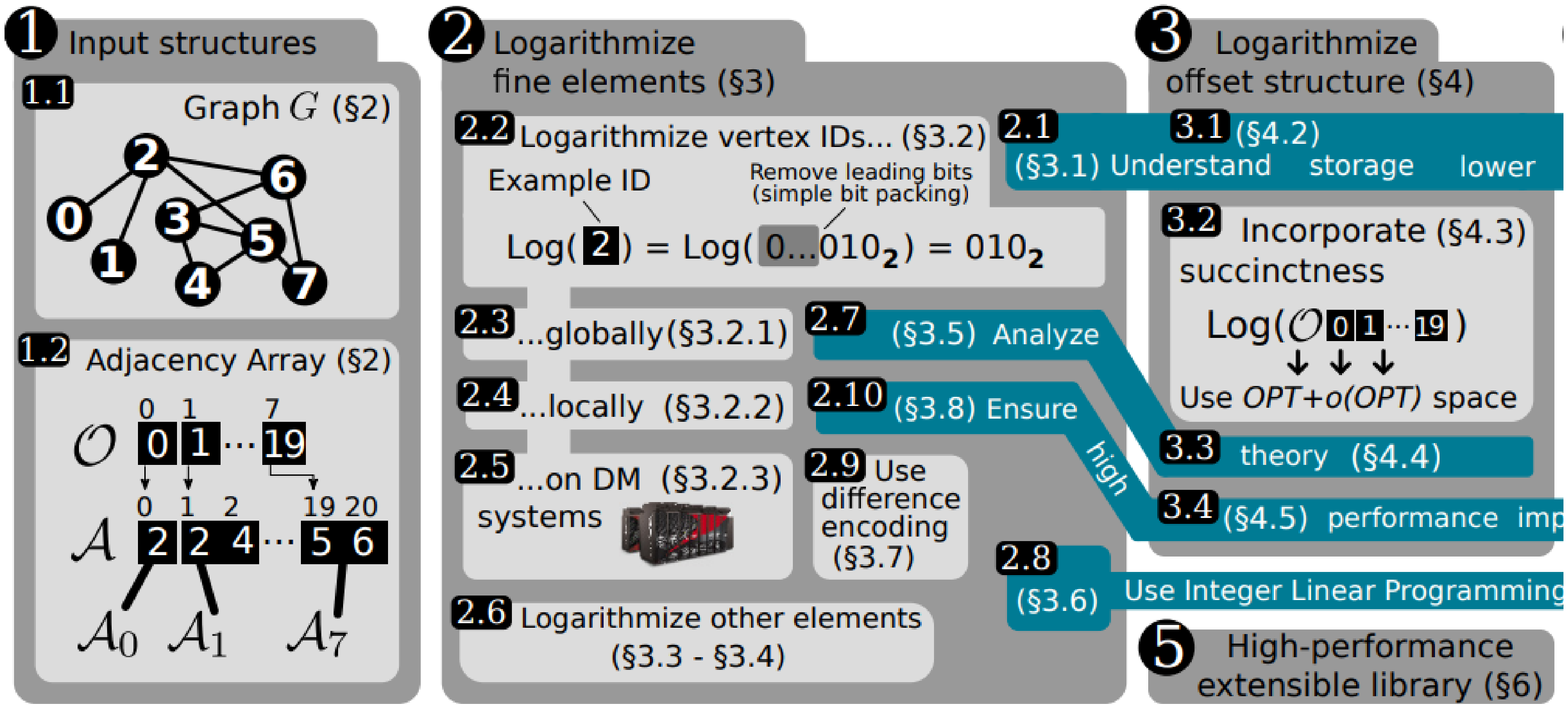

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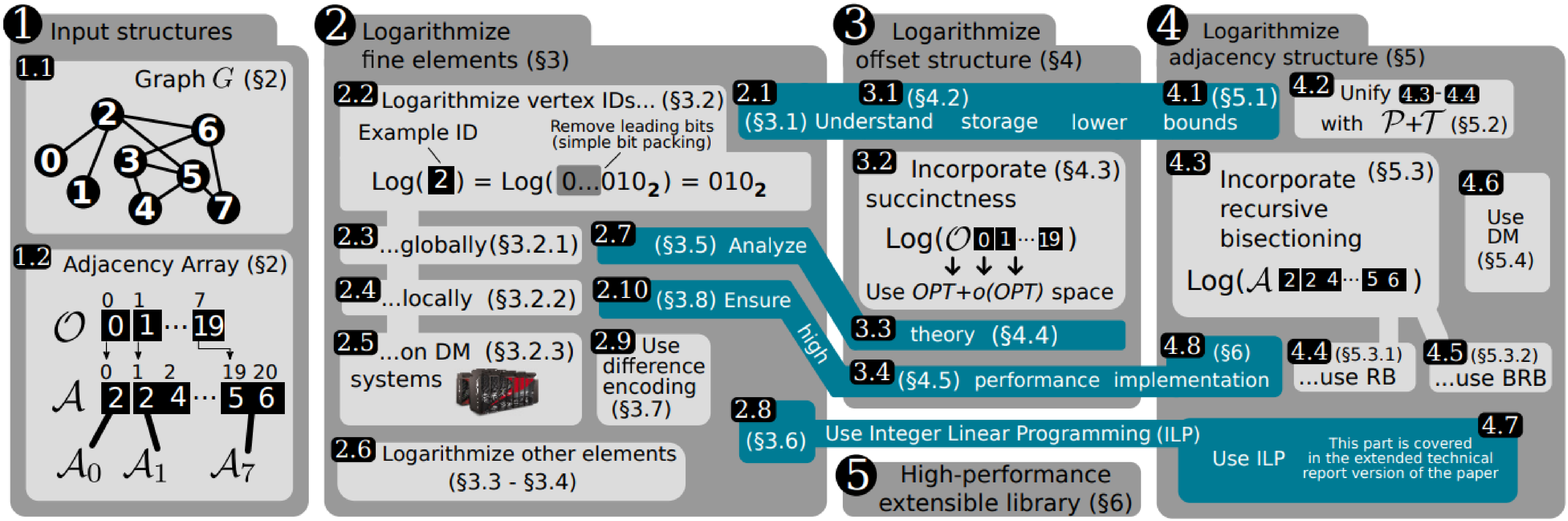

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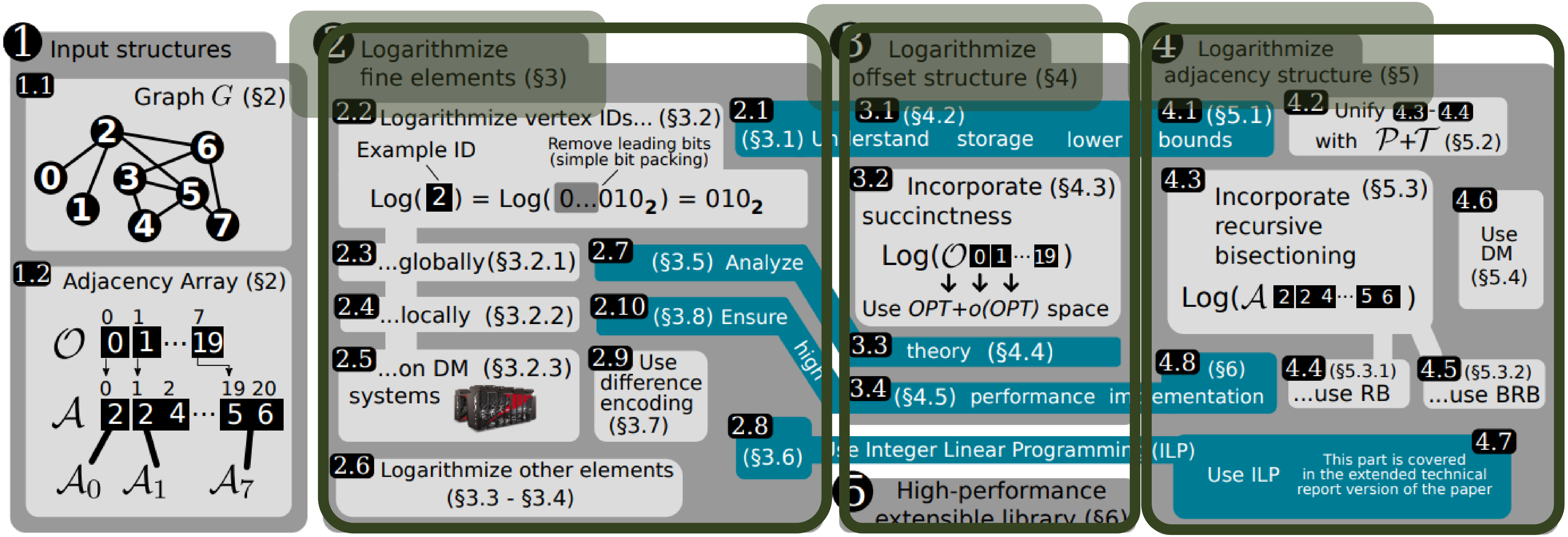

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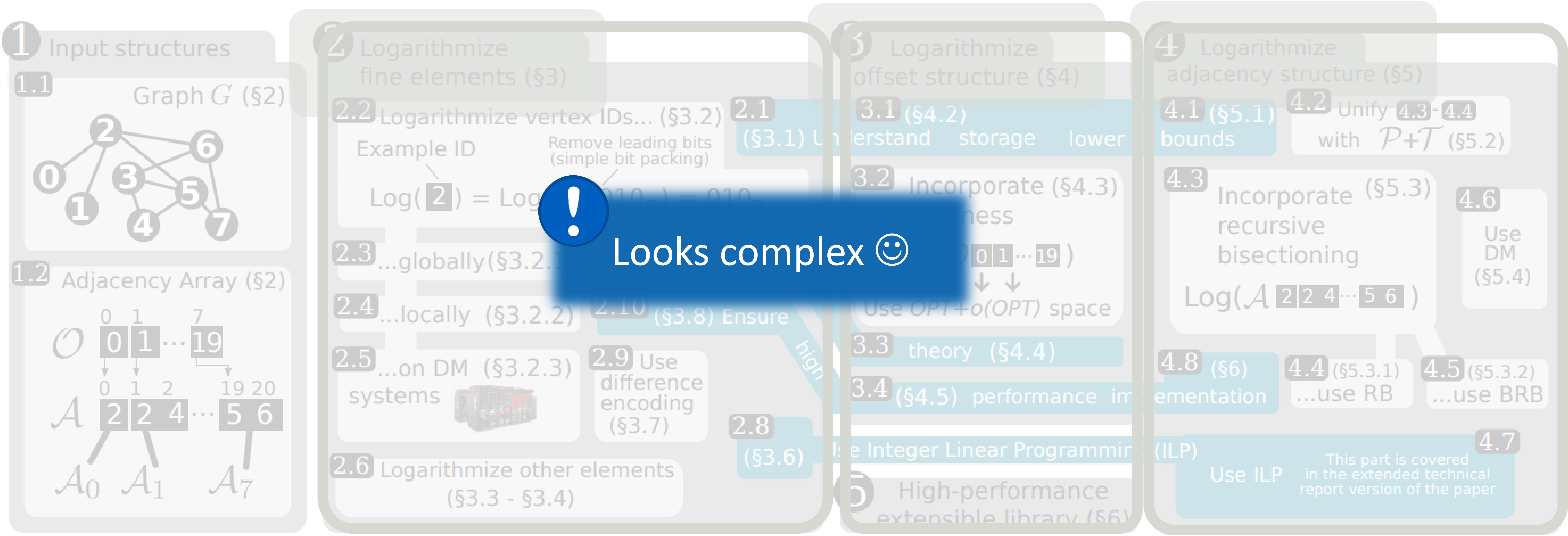
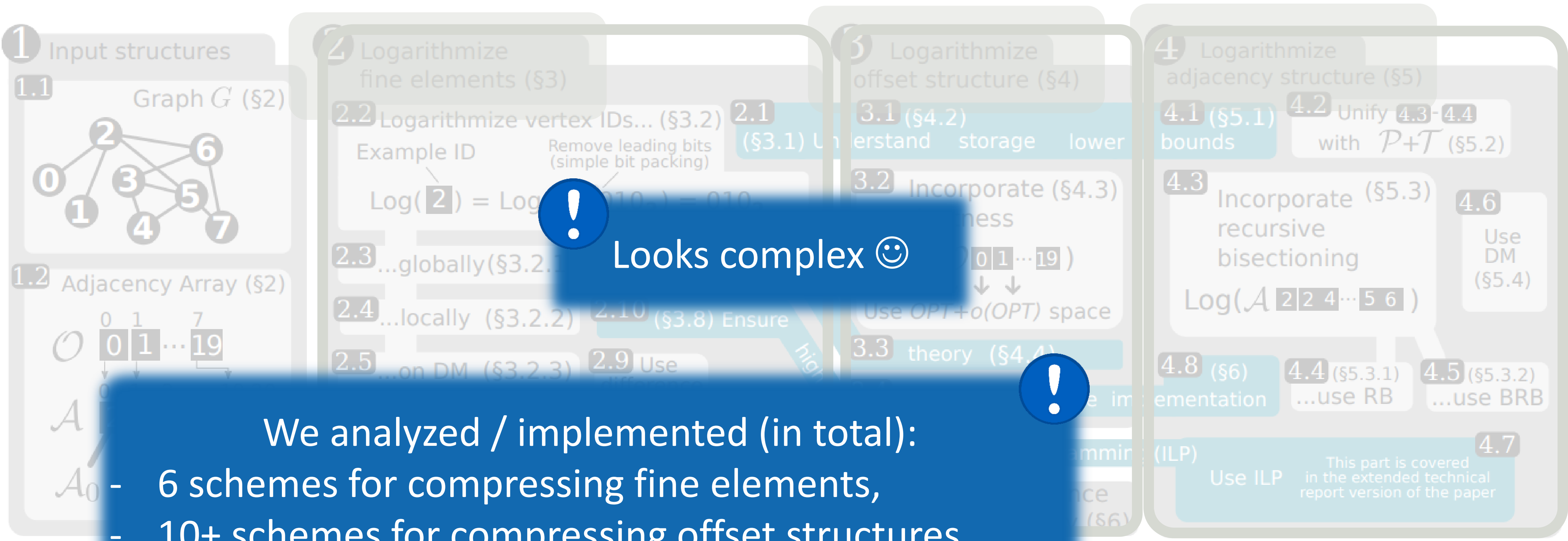


Looks complex ☺

Overview of Full Log(Graph) Design

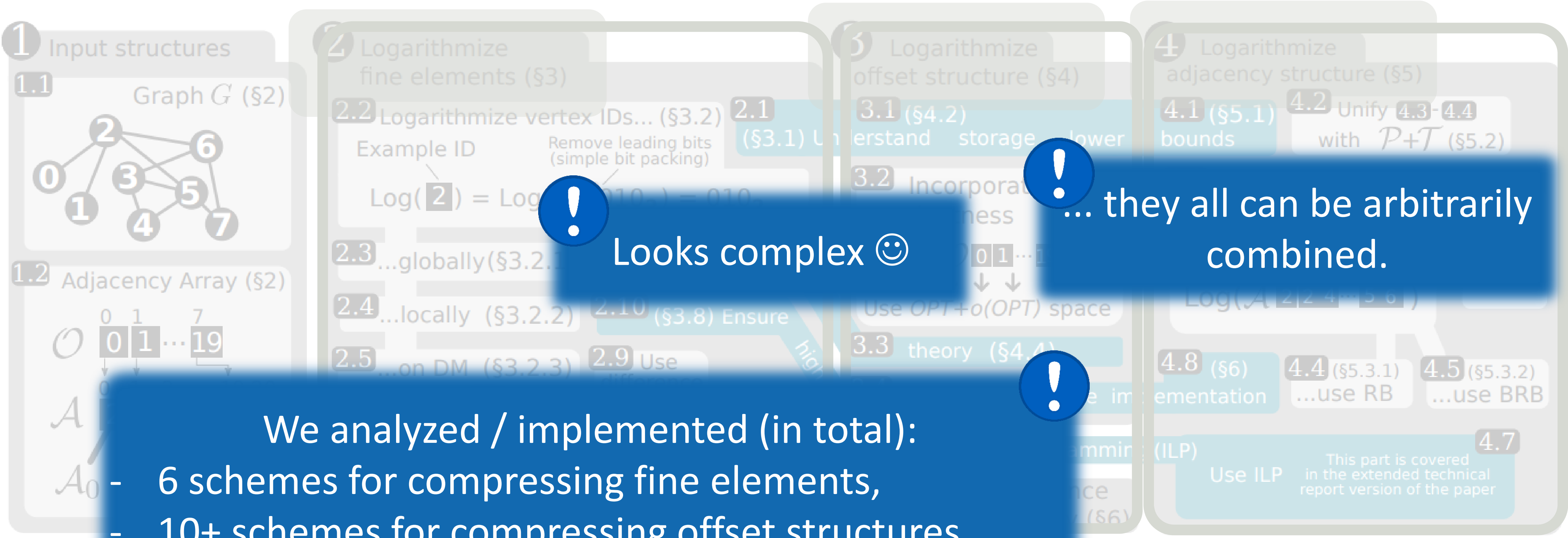


Looks complex ☺

We analyzed / implemented (in total):

- 6 schemes for compressing fine elements,
- 10+ schemes for compressing offset structures,
- 4+ schemes for compressing adjacency structures

Overview of Full Log(Graph) Design



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... they all can be arbitrarily combined.

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Overview of Full Log(Graph) Design

How to ensure fast, manageable, and extensible implementation of all these schemes?

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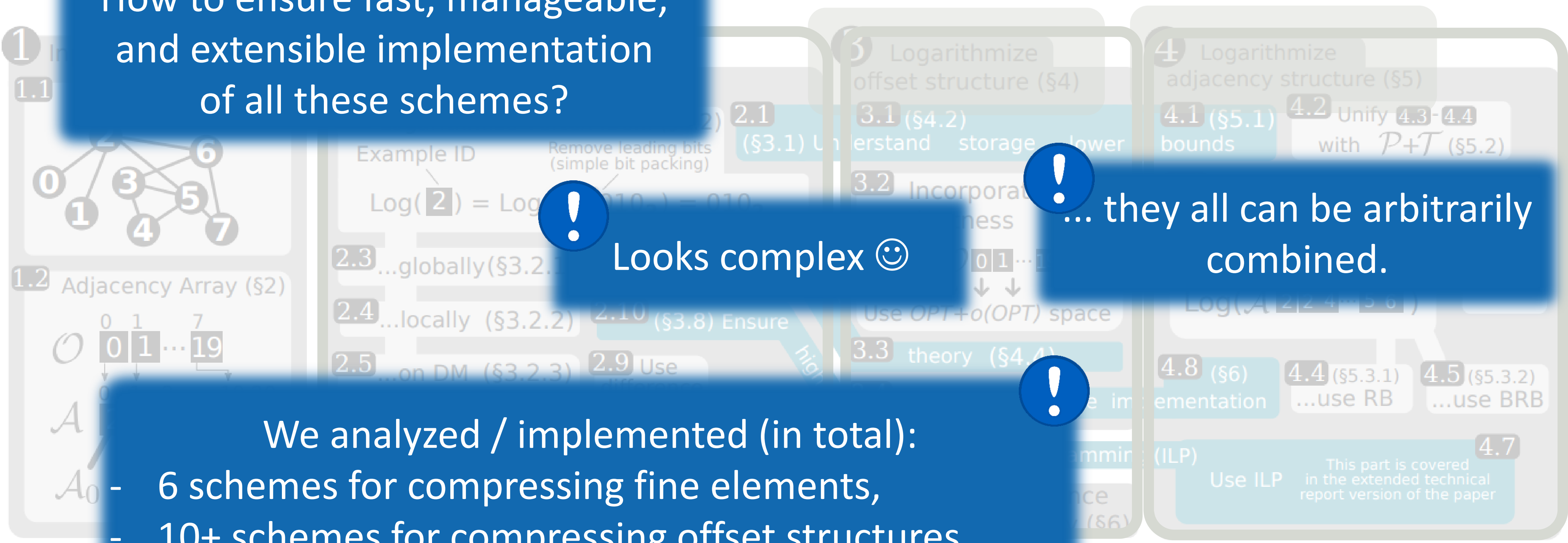

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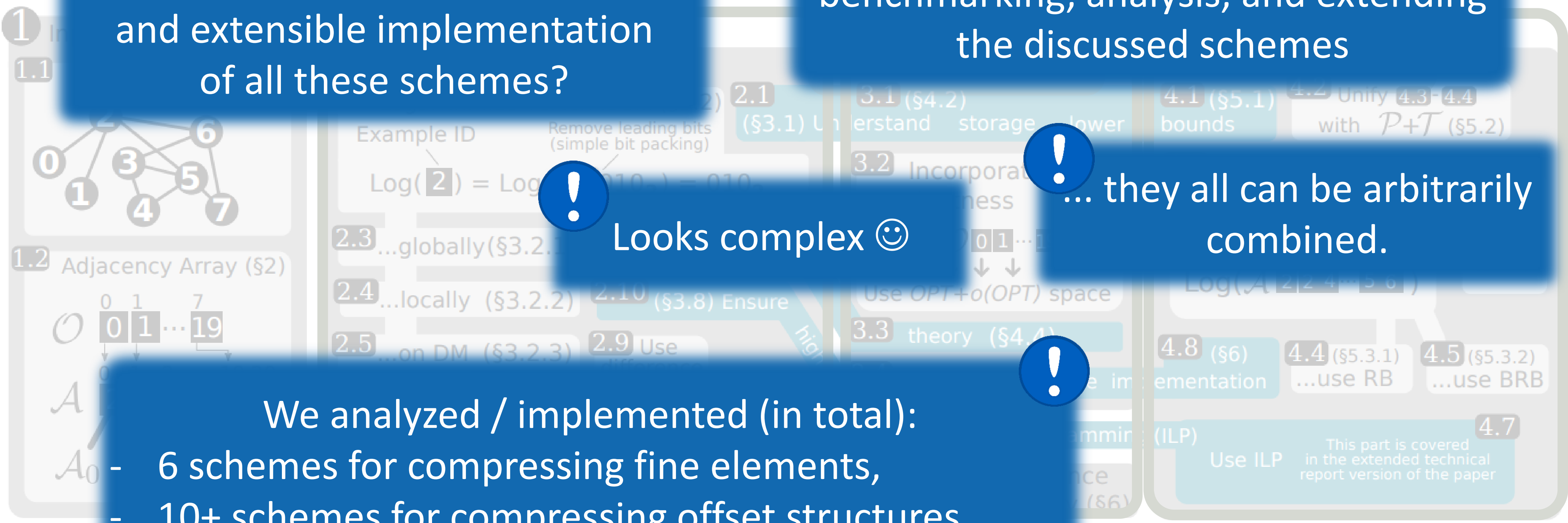
Overview of Full Log(Graph) Design

How to ensure fast, manageable, and extensible implementation of all these schemes?

We use C++ templates to develop a library that facilitates implementation, benchmarking, analysis, and extending the discussed schemes

Looks complex ☺

... they all can be arbitrarily combined.



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Performance Analysis

Types of machines

CSCS Cray Piz Daint

Performance Analysis

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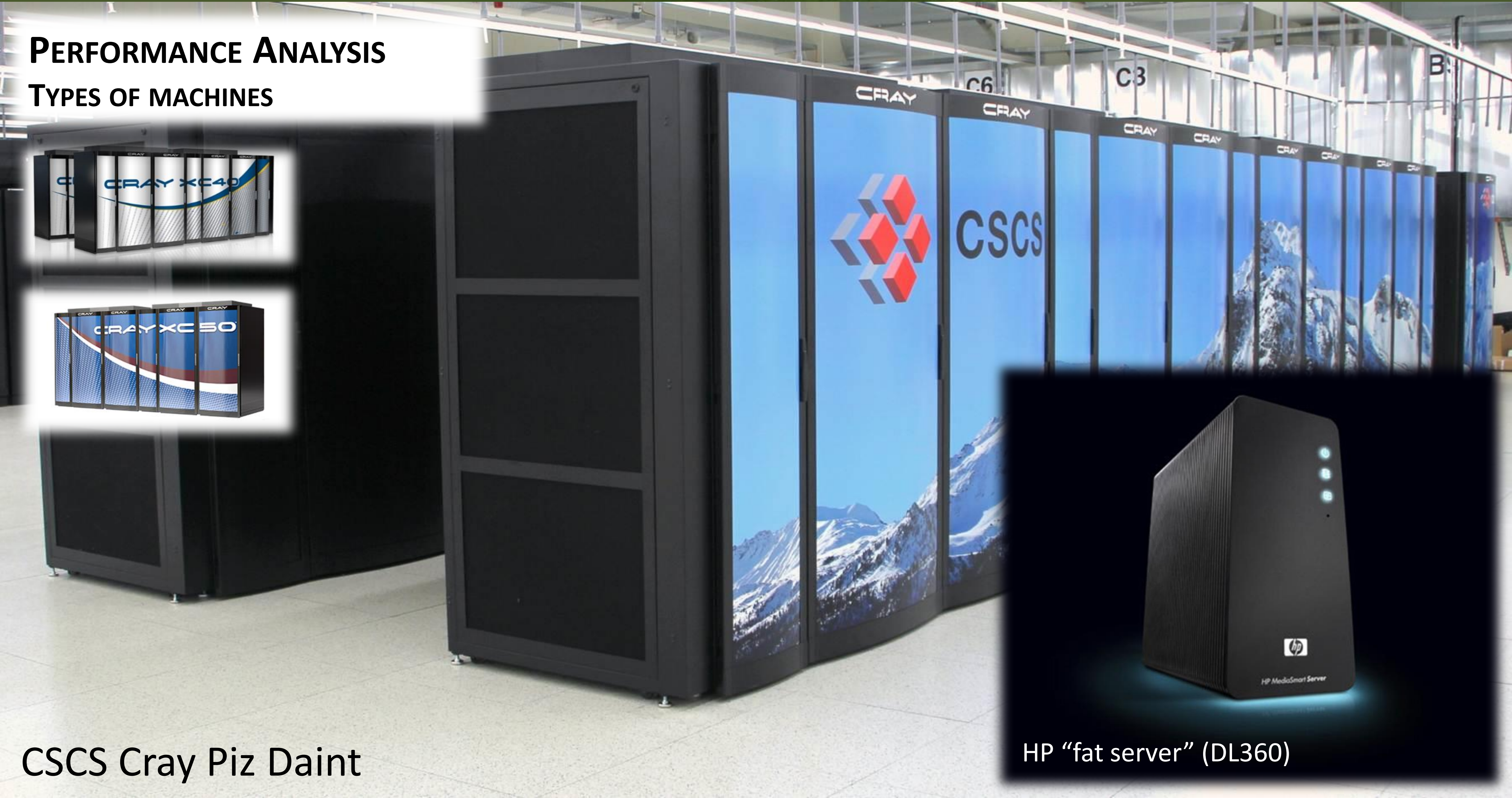


CSCS Cray Piz Daint

HP “fat server” (DL360)

Performance Analysis

Types of graphs

Performance Analysis

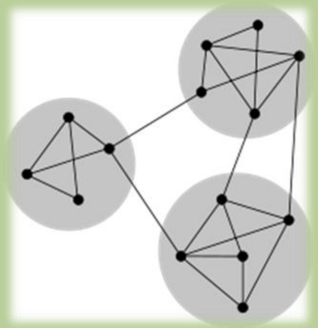
Types of graphs

Synthetic graphs

Performance Analysis

Types of graphs

Synthetic graphs



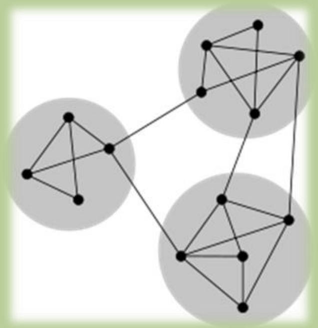
Kronecker [1]

[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010.

Performance Analysis

Types of graphs

Synthetic graphs



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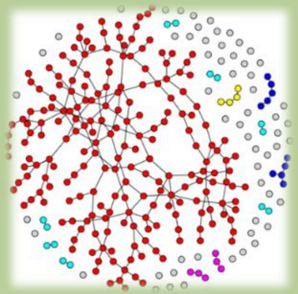

Erdös-Rényi [2]

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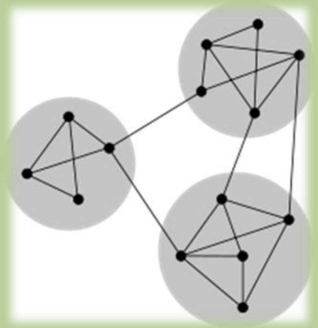

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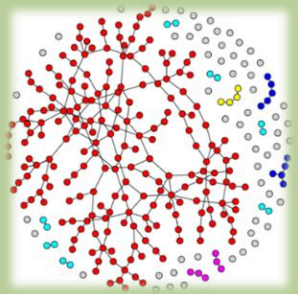

Erdös-Rényi [2]

Real-world graphs (SNAP [3], KONECT [4], Webgraph [5], DIMACS [6])

[3] SNAP. https://snap.stanford.edu

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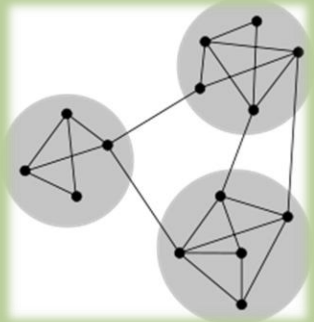

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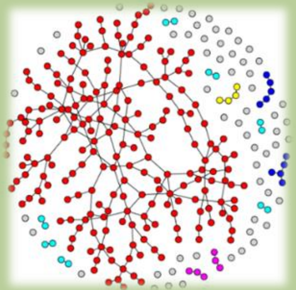

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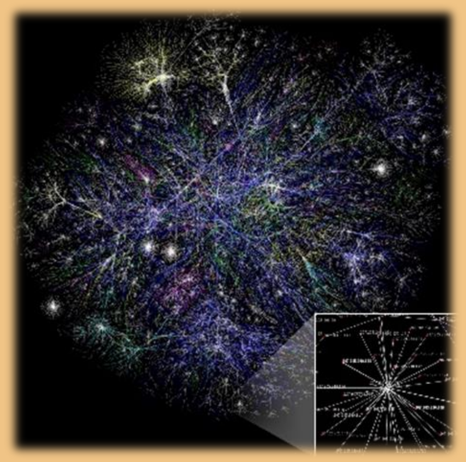
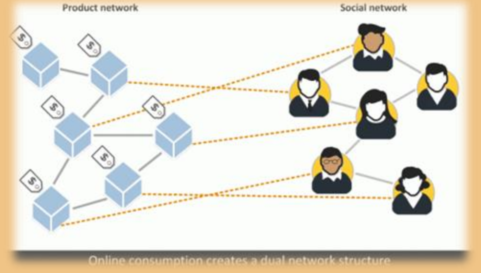


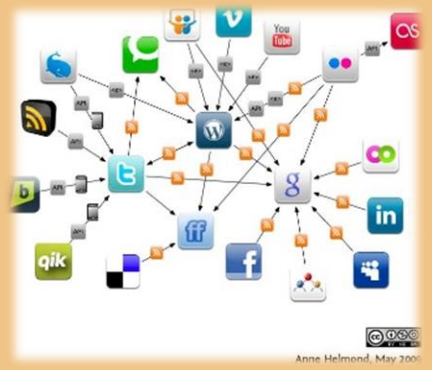
Web graphs

Road networks

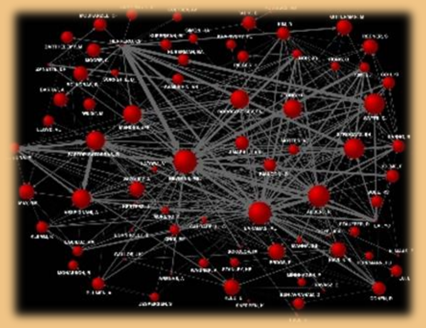
Social networks



Purchase networks



Communication graphs



Citation graphs

[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010.
[2] P. Erdos and A. Renyi. On the evolution of random graphs. Pub. Math. Inst. Hun. A. Science. 1960.

[3] SNAP. https://snap.stanford.edu
[4] KONECT. https://konect.cc
[5] DIMACS Challenge
[6] Webgraphs. https://law.di.unimi.it/datasets.php

Performance Analysis

Algorithms

Performance Analysis

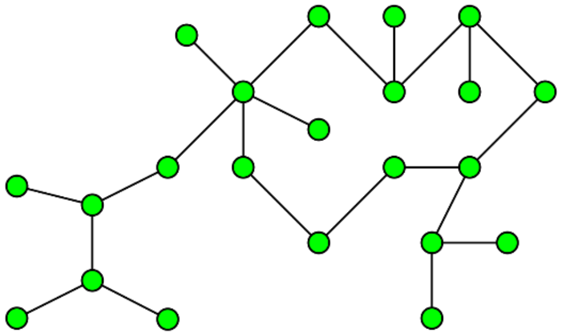
Algorithms

Connected Components
(Shiloach-Vishkin [1])

Performance Analysis

Algorithms

Connected Components
(Shiloach-Vishkin [1])

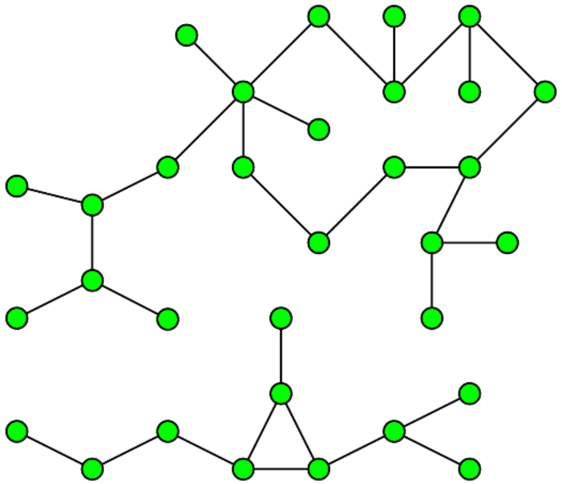


[1] Y. Shiloach, U. Vishkin. An O (log n) parallel connectivity algorithm. 1980.

PERFORMANCE ANALYSIS

ALGORITHMS

Connected Components
(Shiloach-Vishkin [1])



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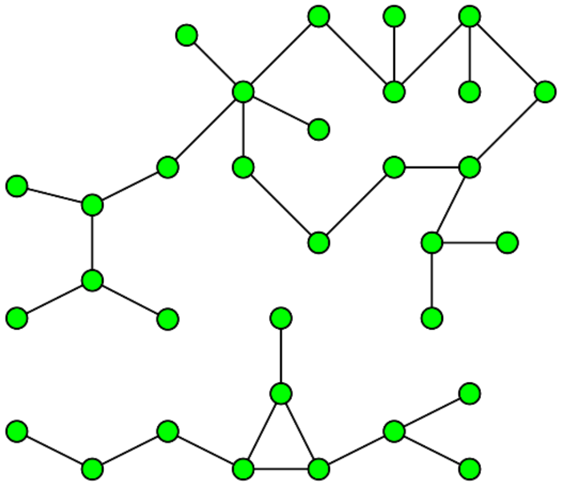
Performance Analysis

Algorithms

Connected Components
(Shiloach-Vishkin [1])

BFS (direction optimization [2])

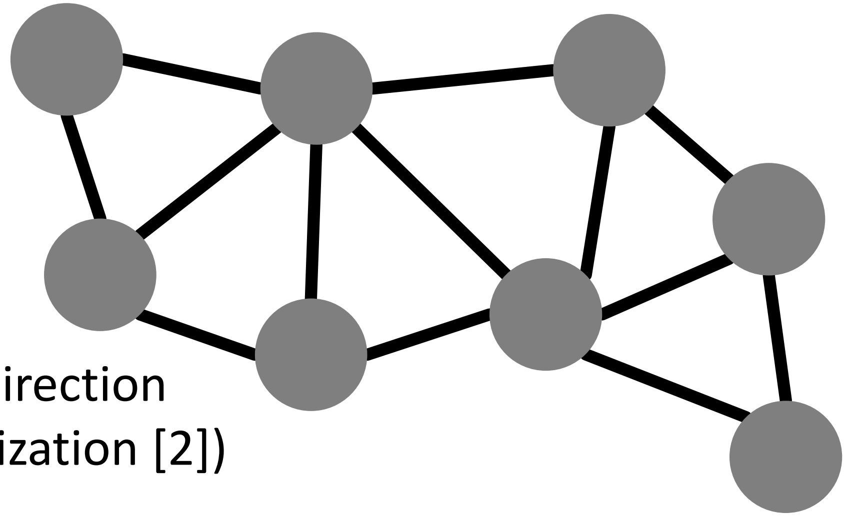


[1] Y. Shiloach, U. Vishkin. An O (log n) parallel connectivity algorithm. 1980.

[2] S Beamer et al. Direction-Optimizing Breadth-First Search. 2013.

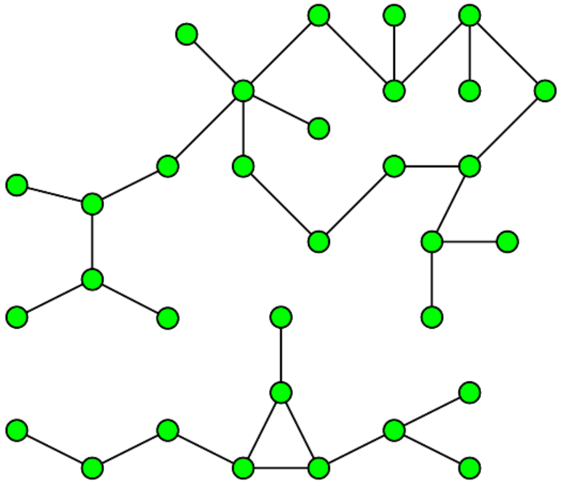
Performance Analysis

Algorithms



BFS (direction optimization [2])

Connected Components
(Shiloach-Vishkin [1])



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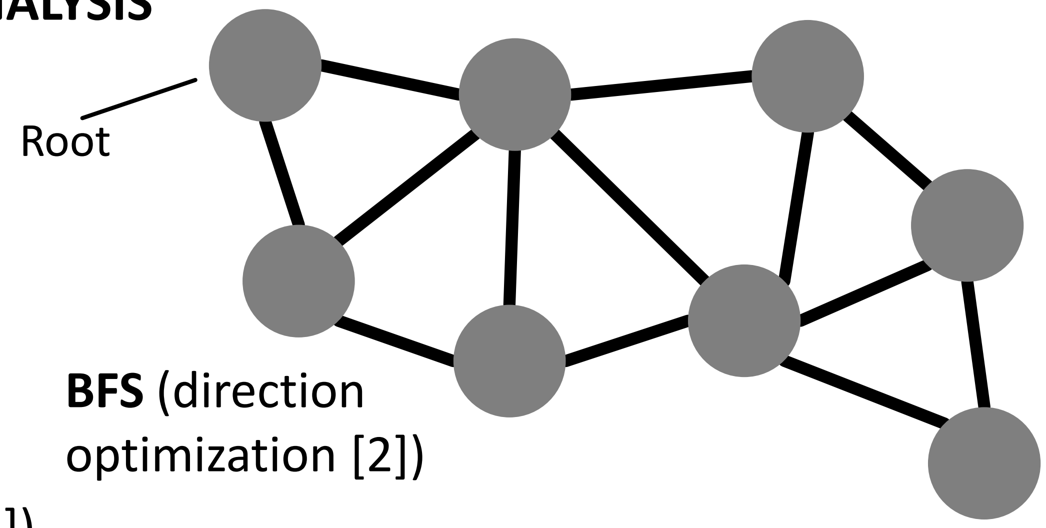
Performance Analysis

Algorithms



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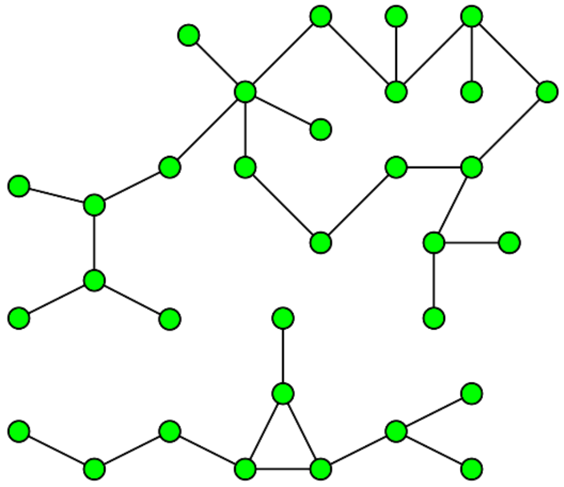

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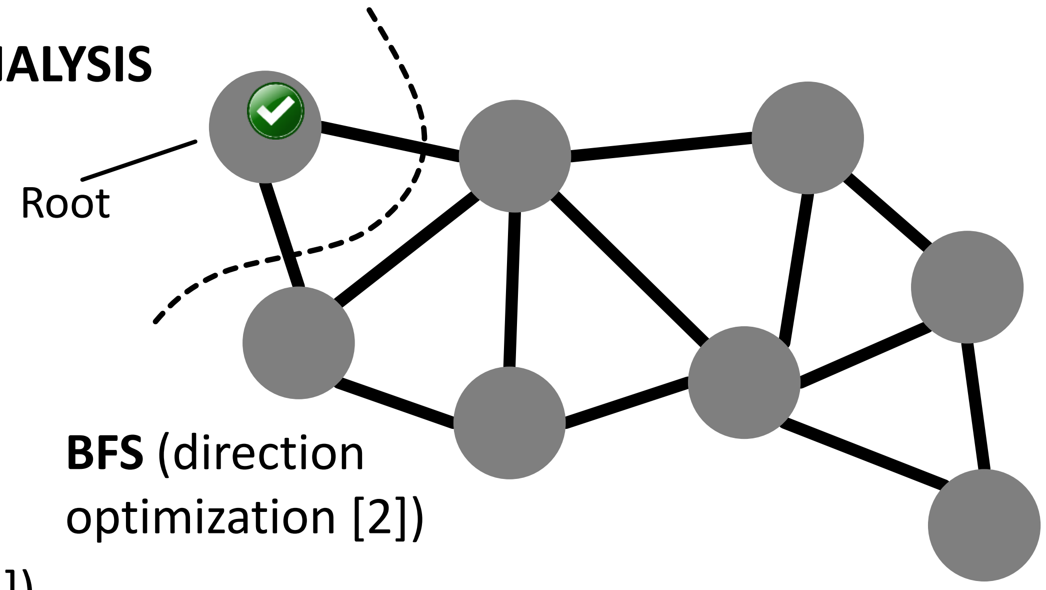

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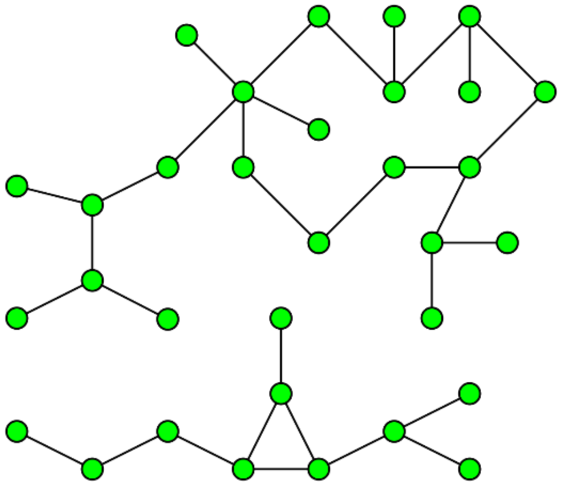

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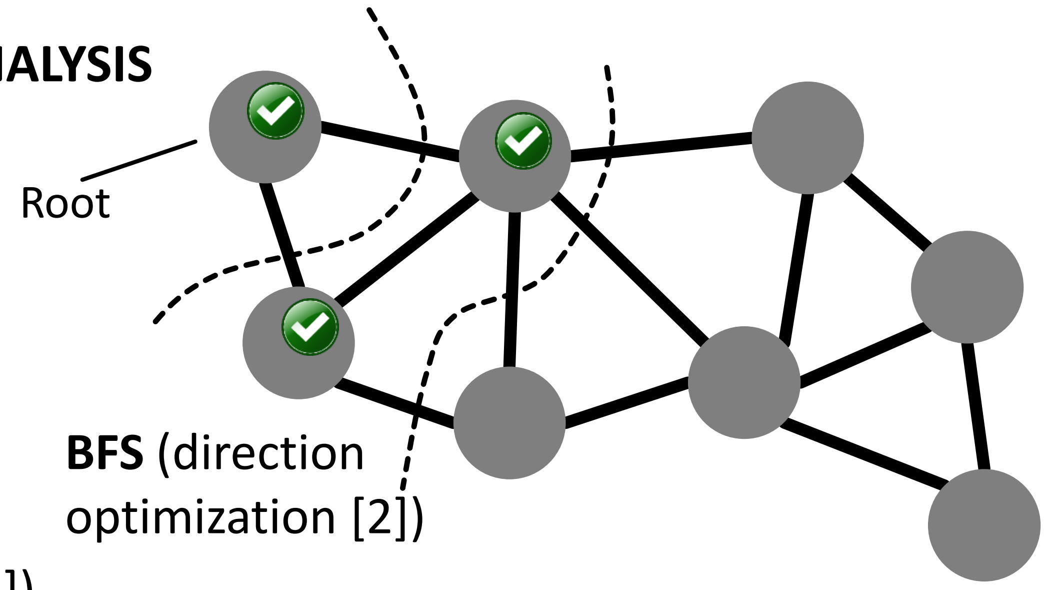

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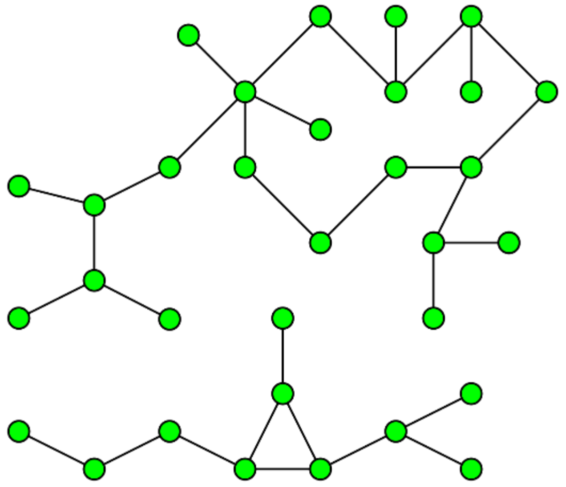

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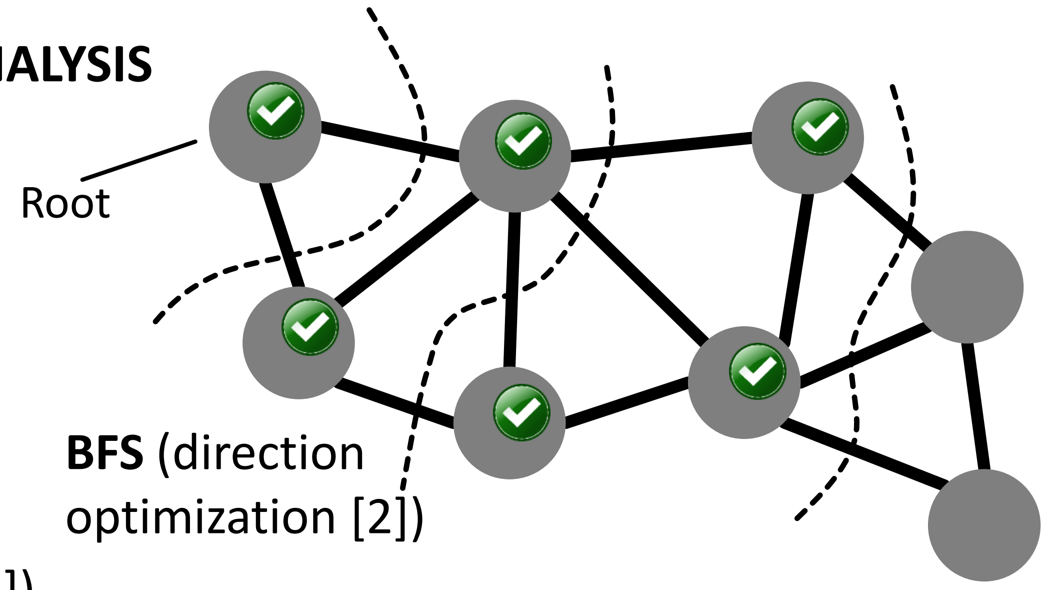

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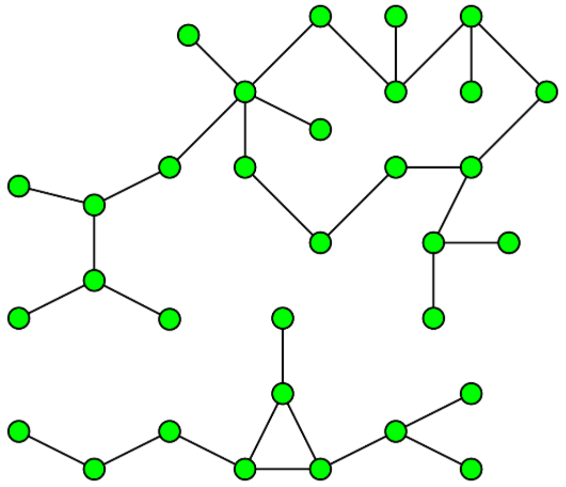

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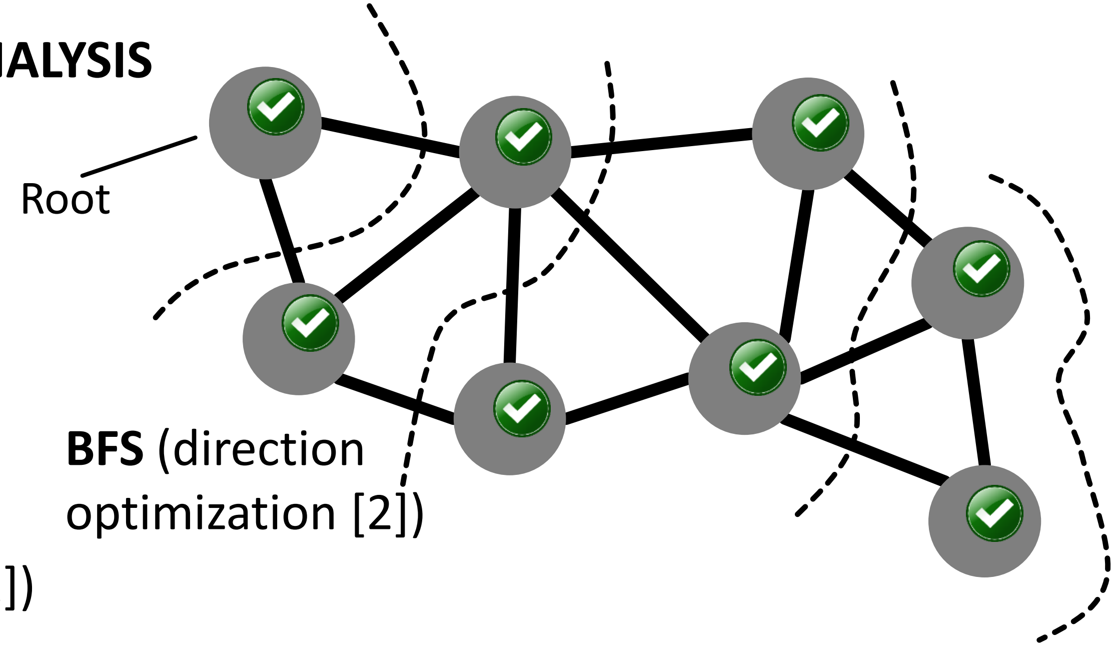

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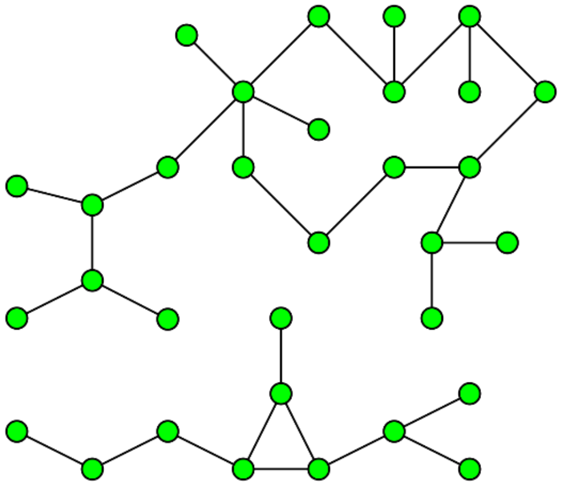

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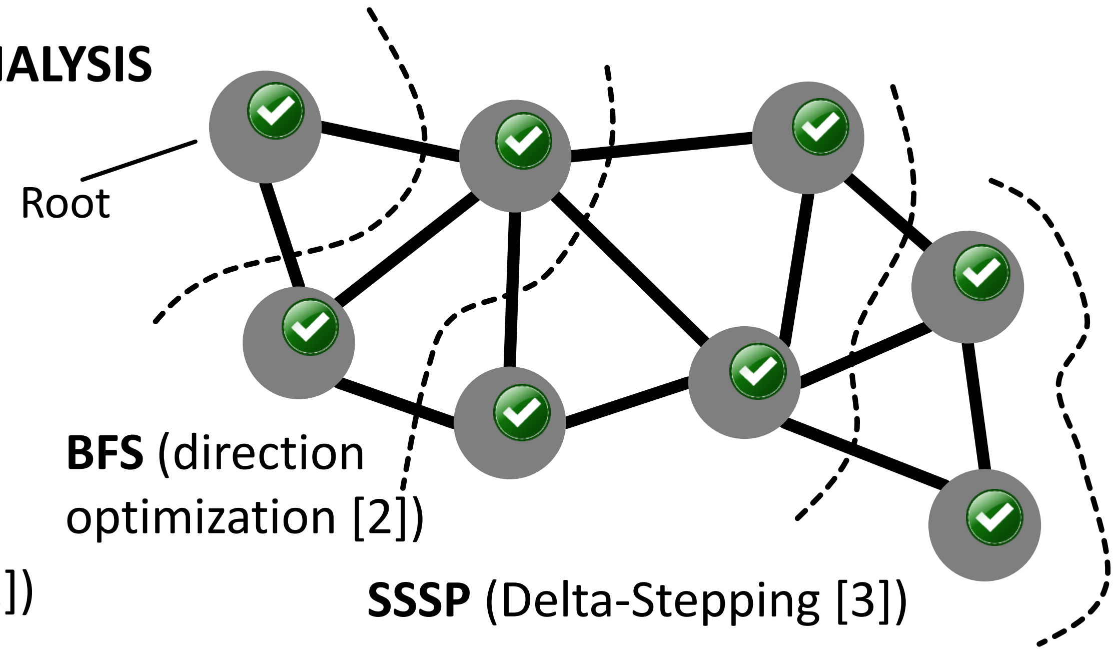

BFS (direction optimization [2])

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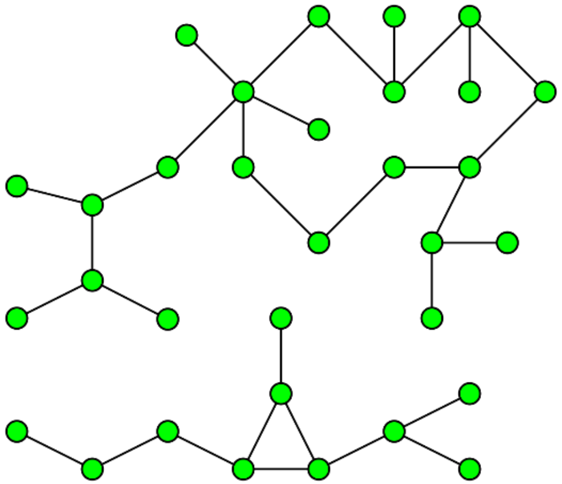

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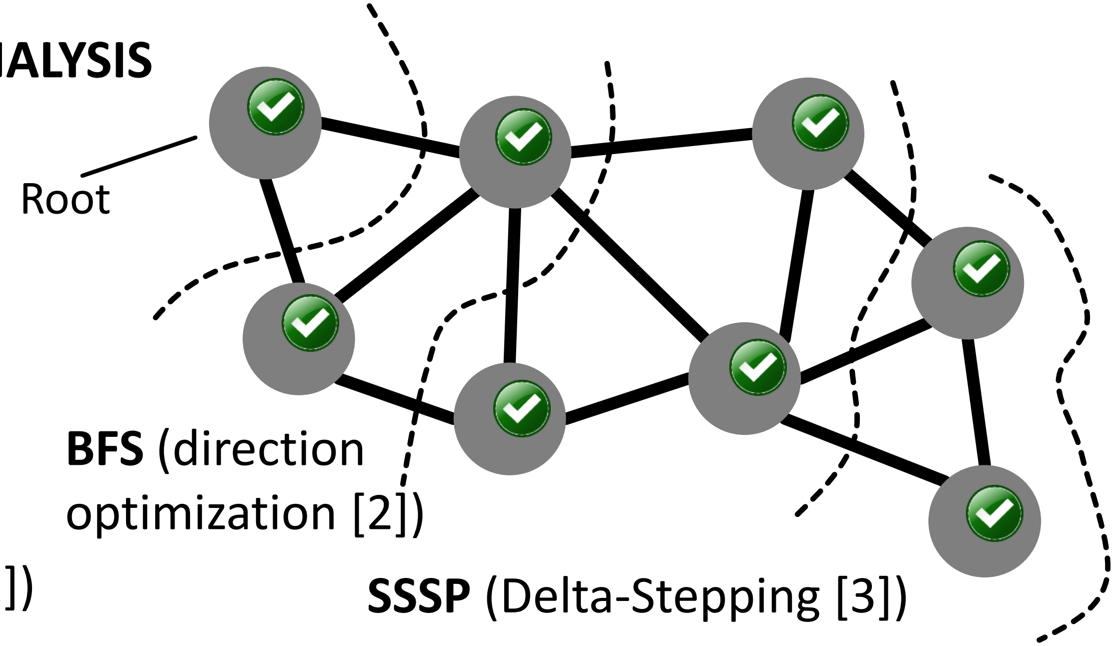

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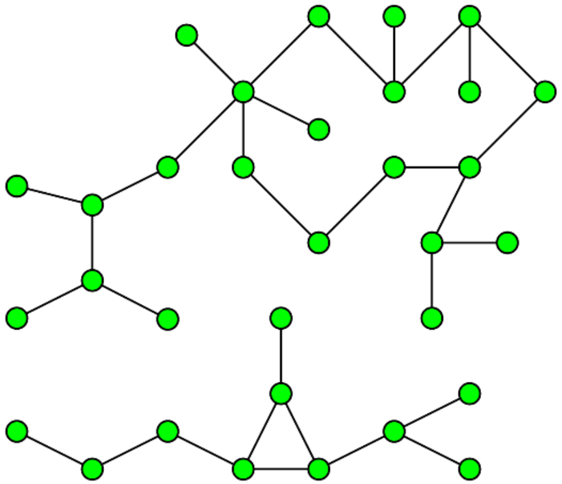

Triangle Counting

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Performance Analysis

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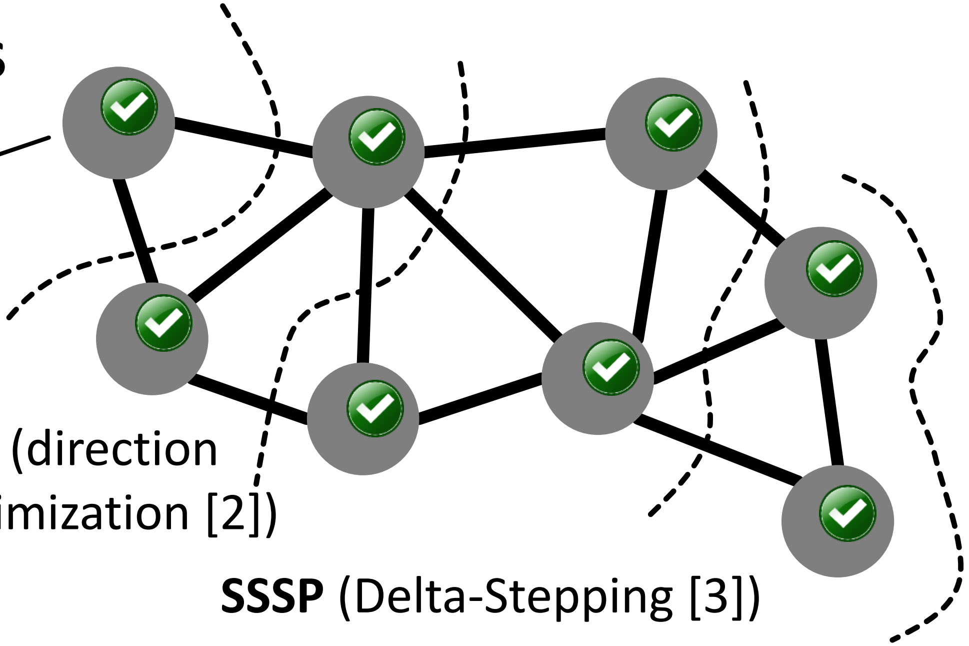

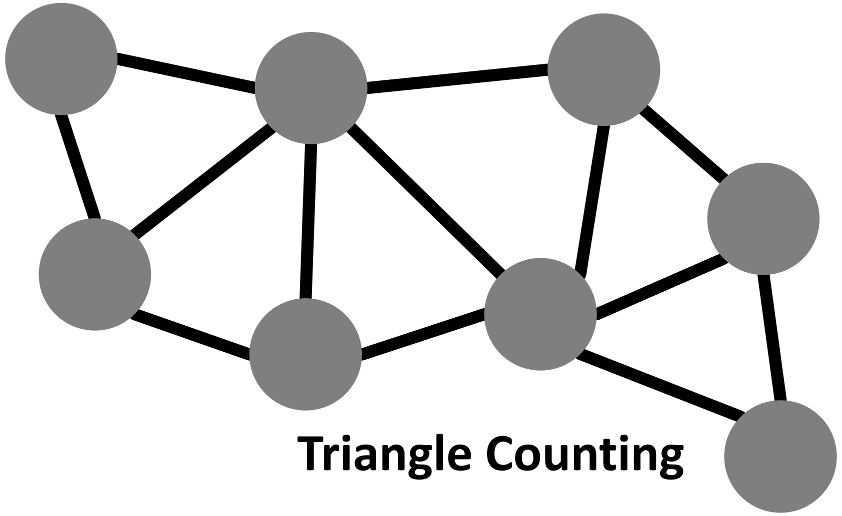

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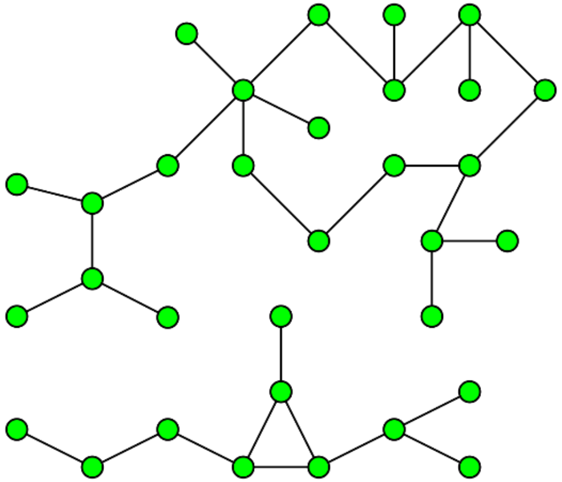

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Performance Analysis

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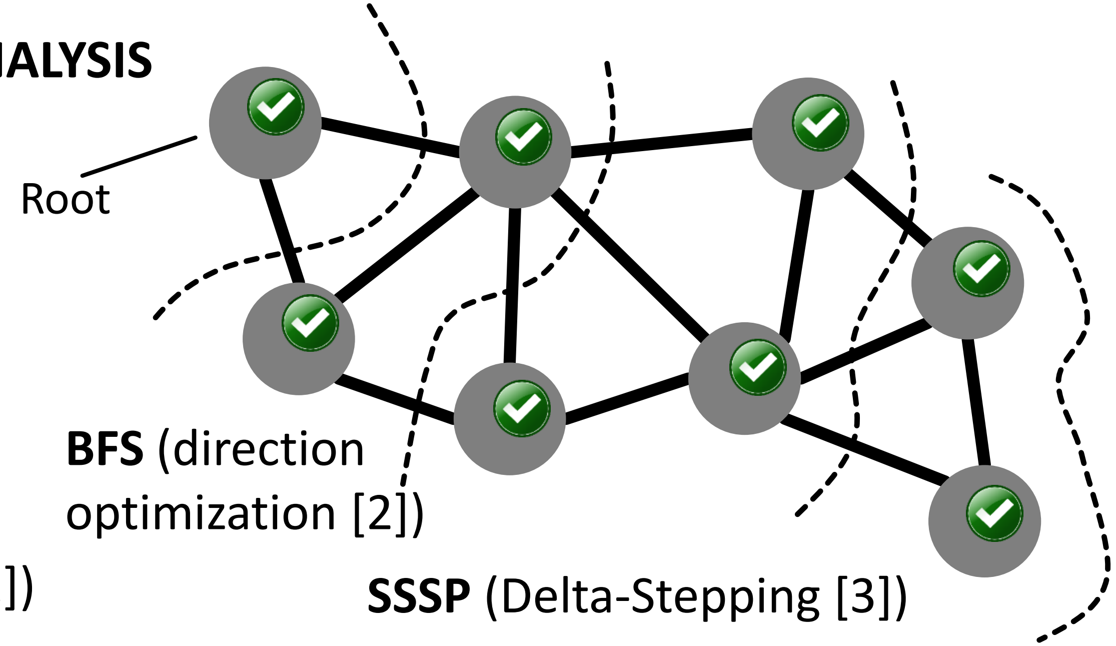

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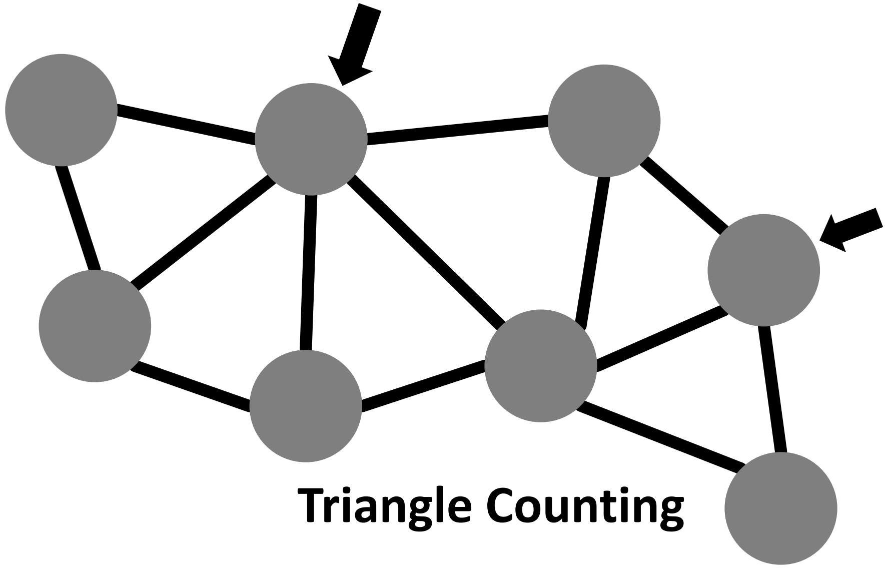

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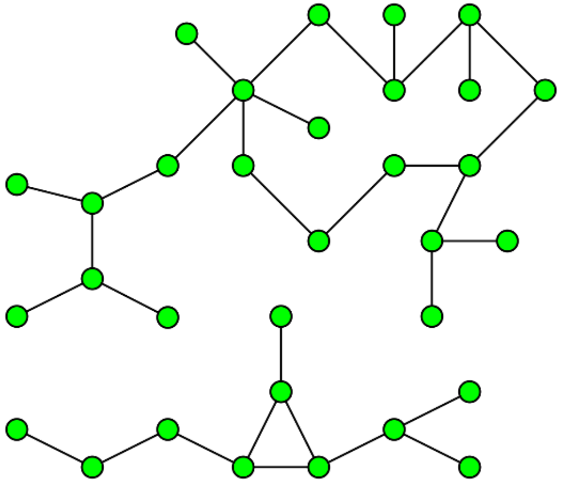

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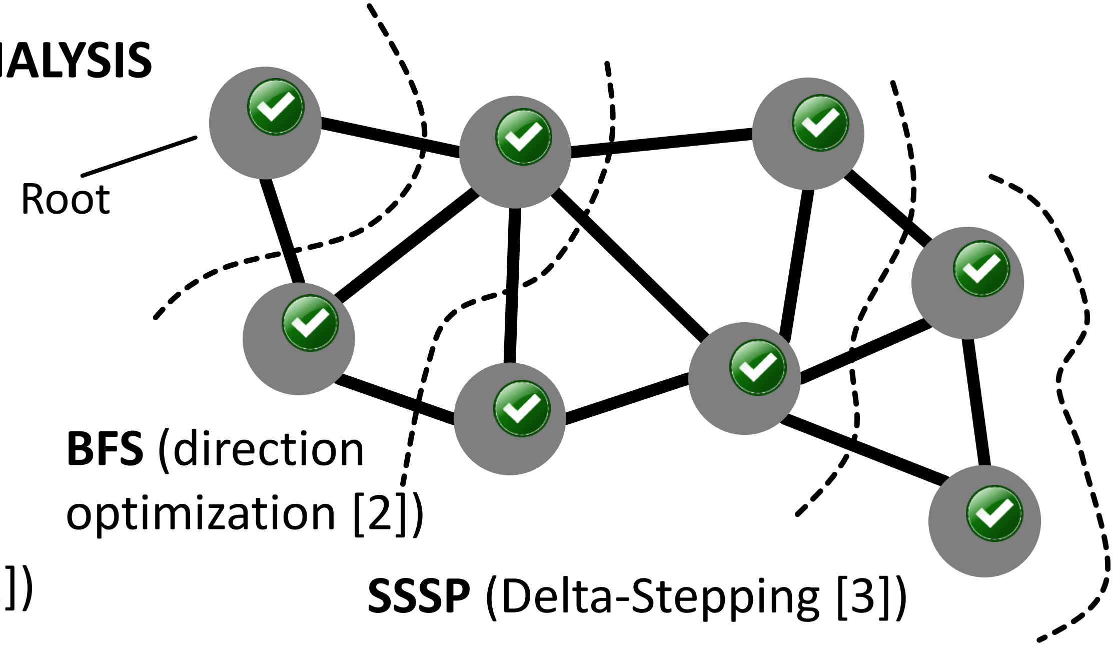

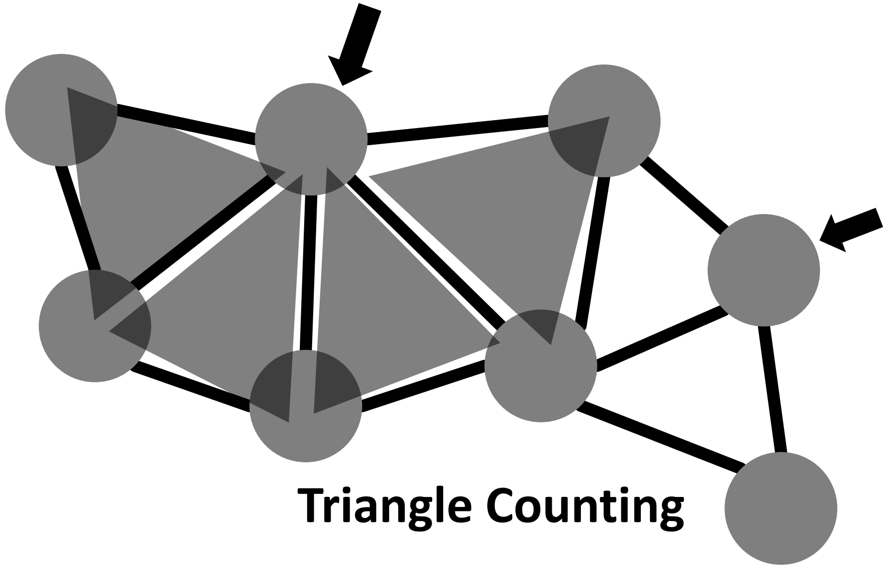

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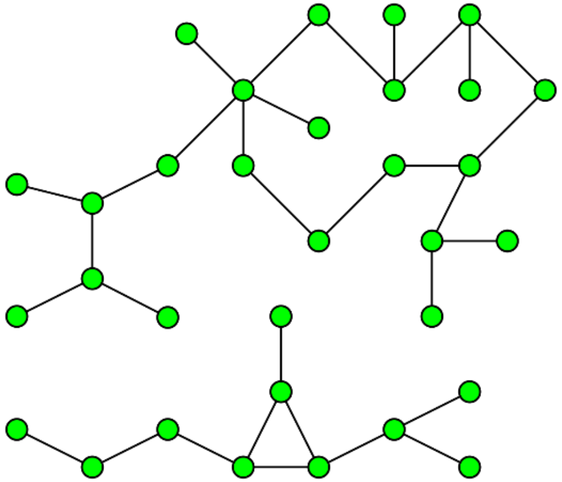

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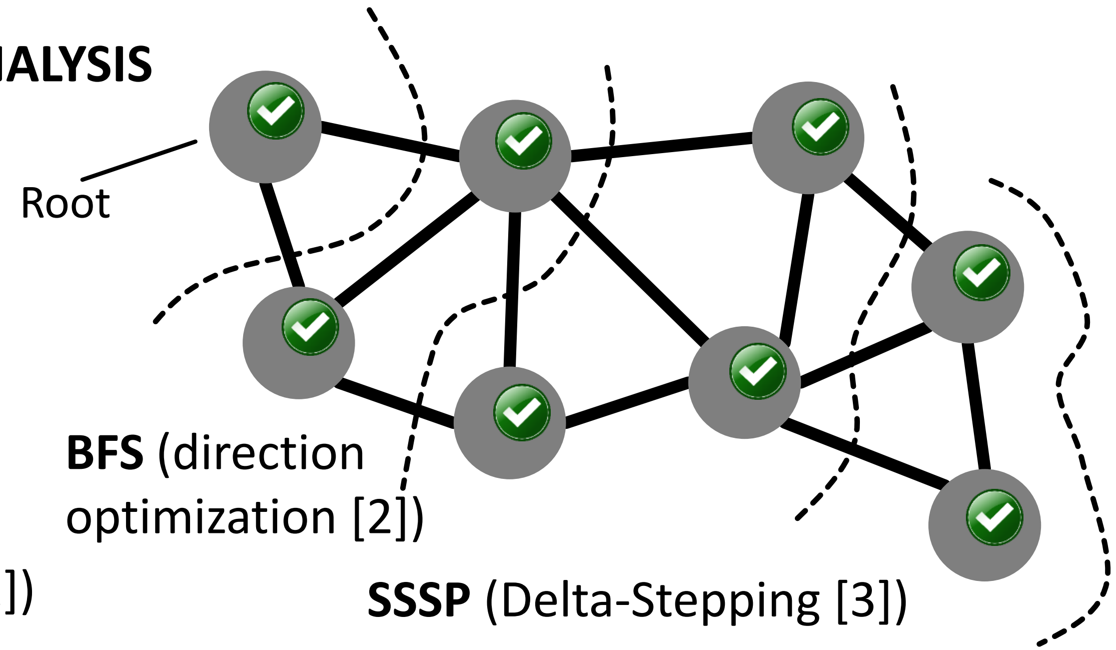

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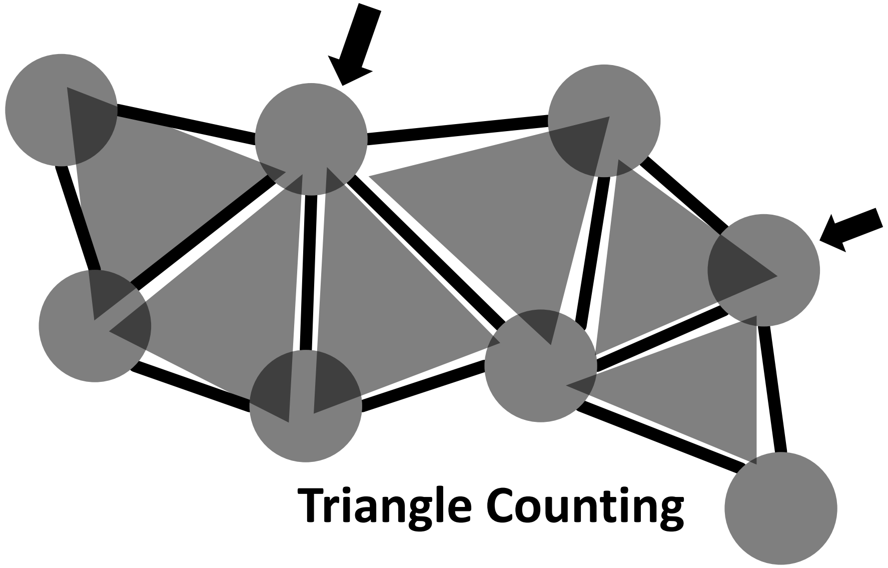

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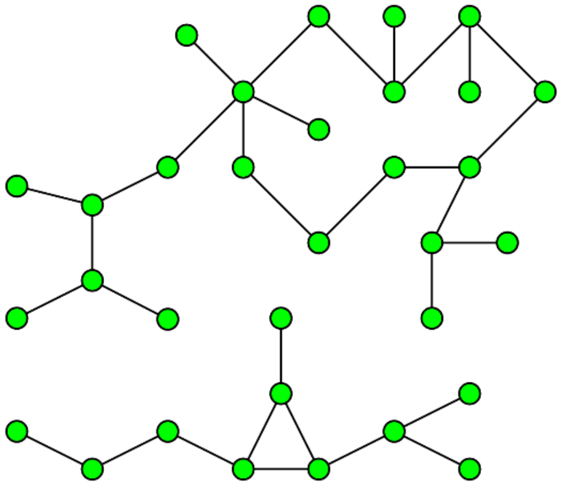

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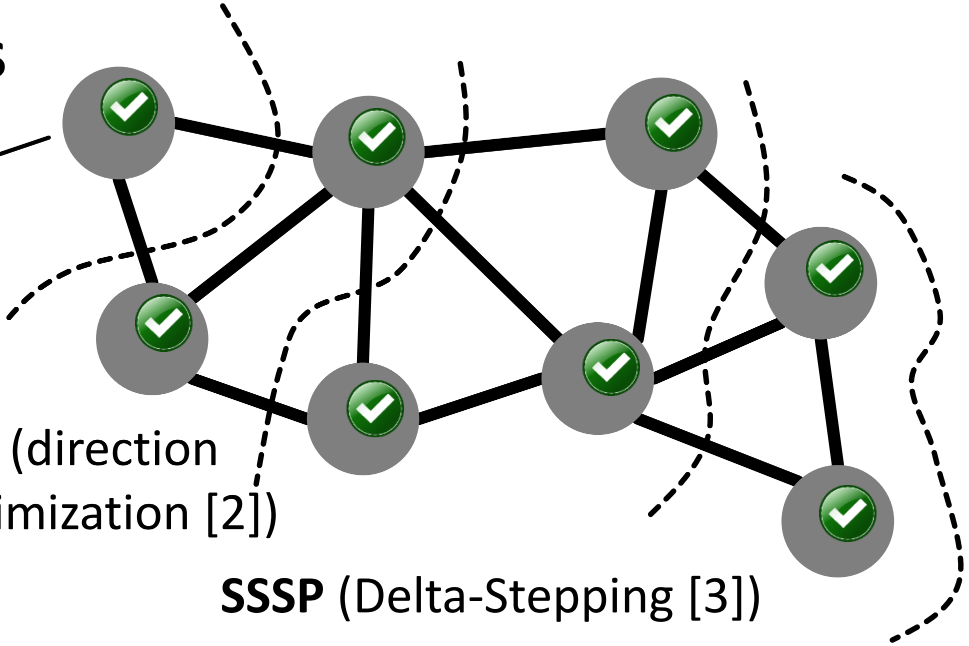
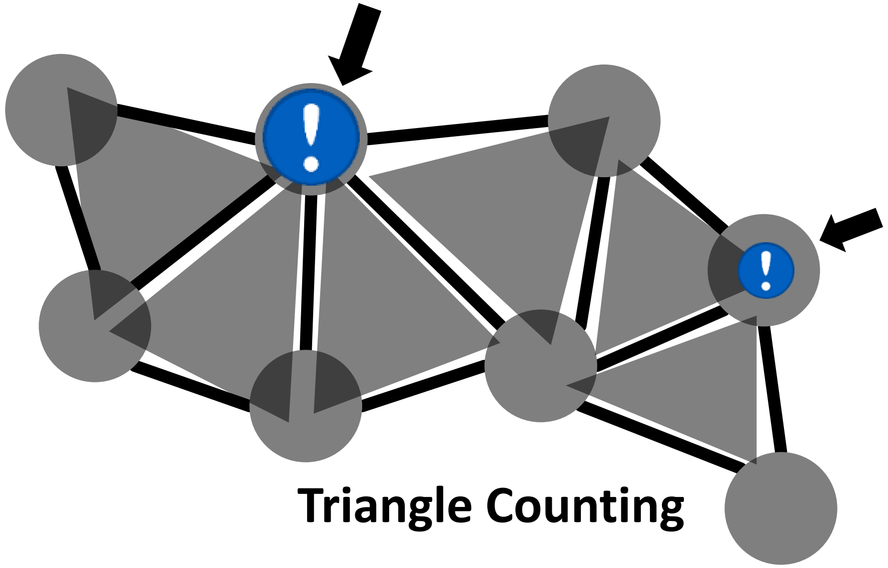
PERFORMANCE ANALYSIS
ALGORITHMS



Root

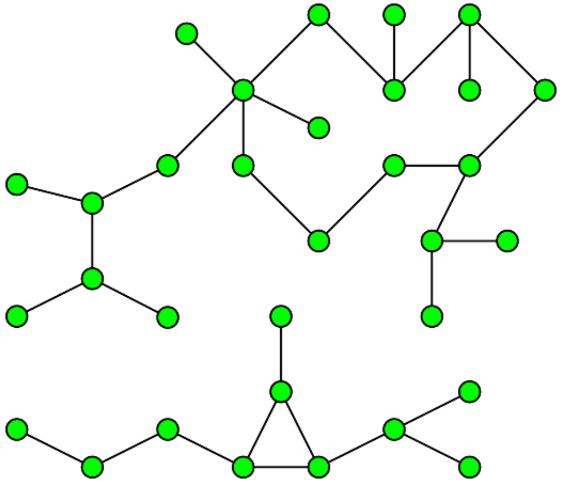
BFS (direction optimization [2])

SSSP (Delta-Stepping [3])



Triangle Counting

Connected Components
(Shiloach-Vishkin [1])



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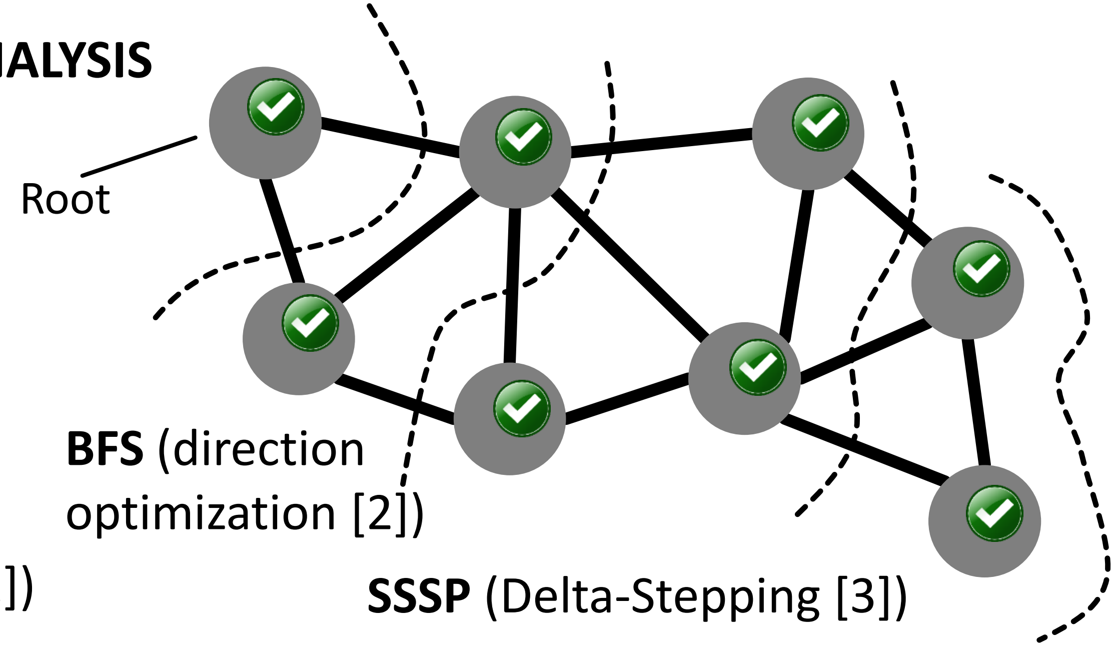

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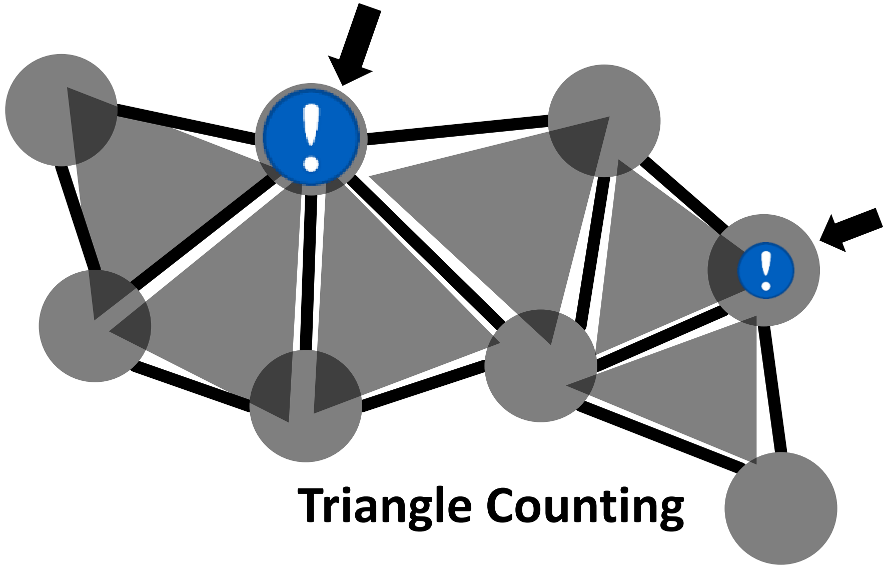

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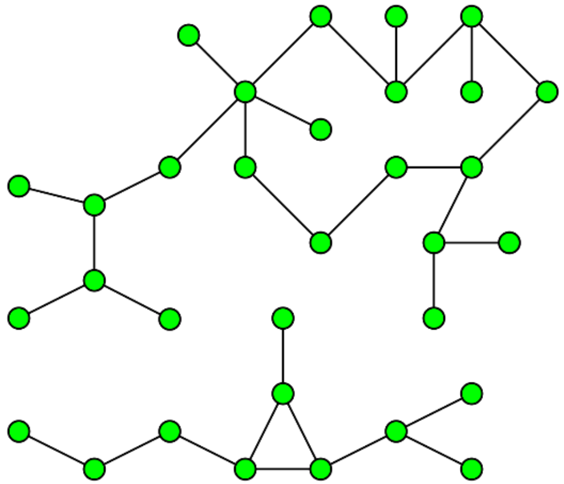

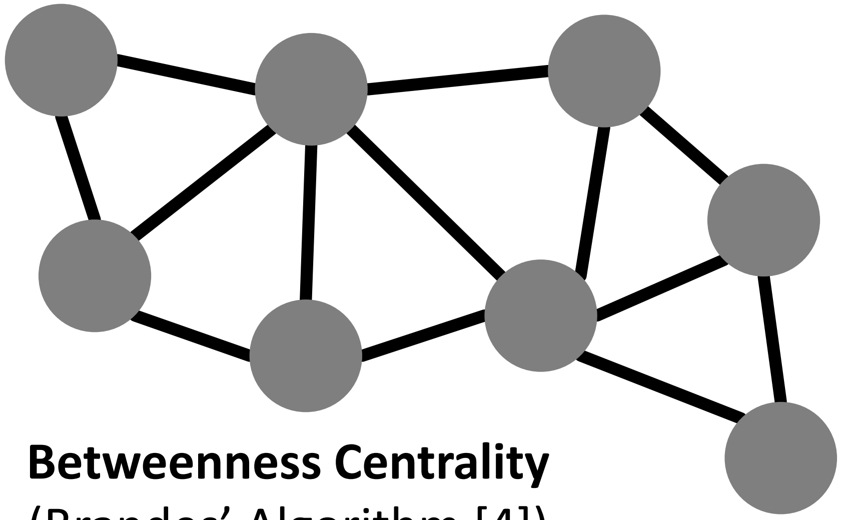

Betweenness Centrality (Brandes' Algorithm [4])

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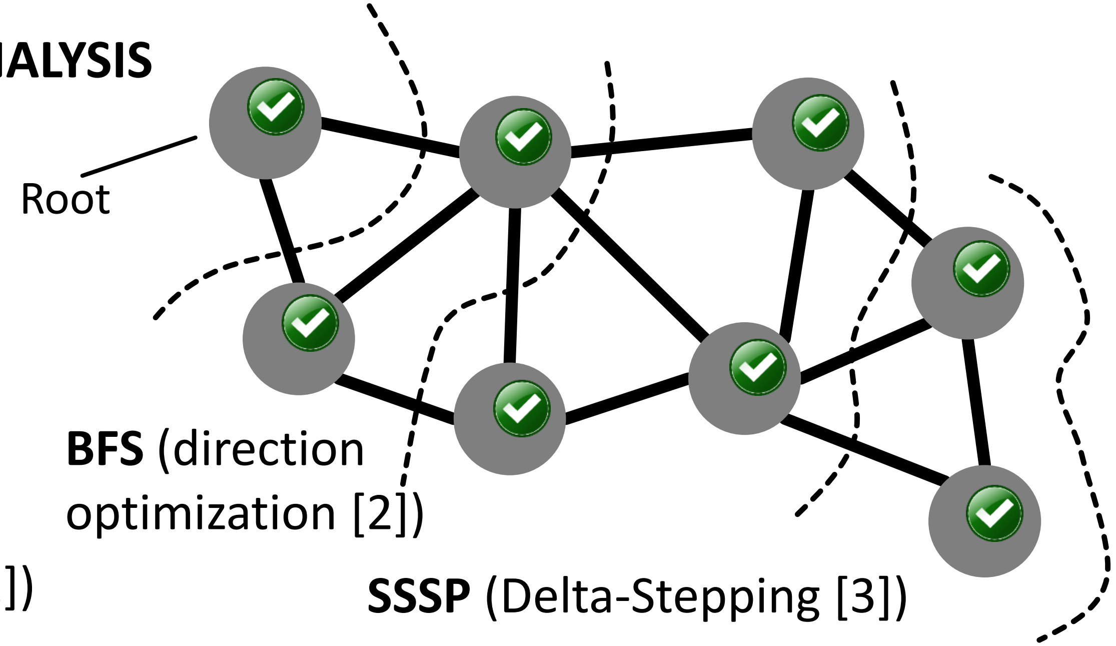
ALGORITHMS



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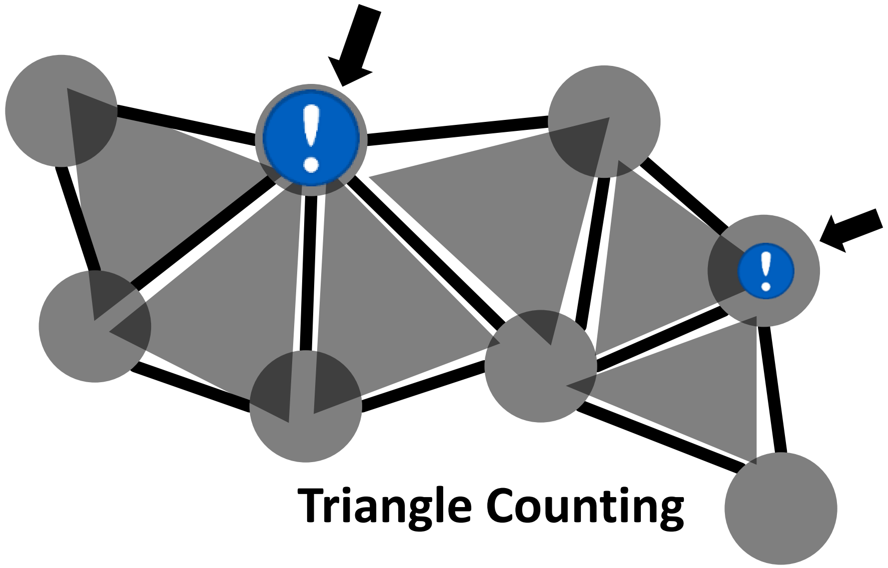

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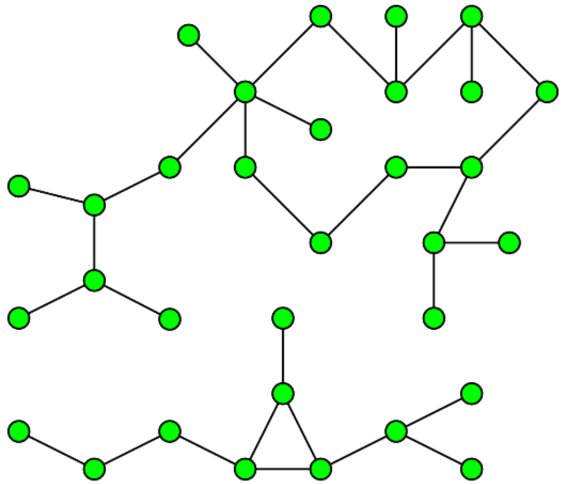

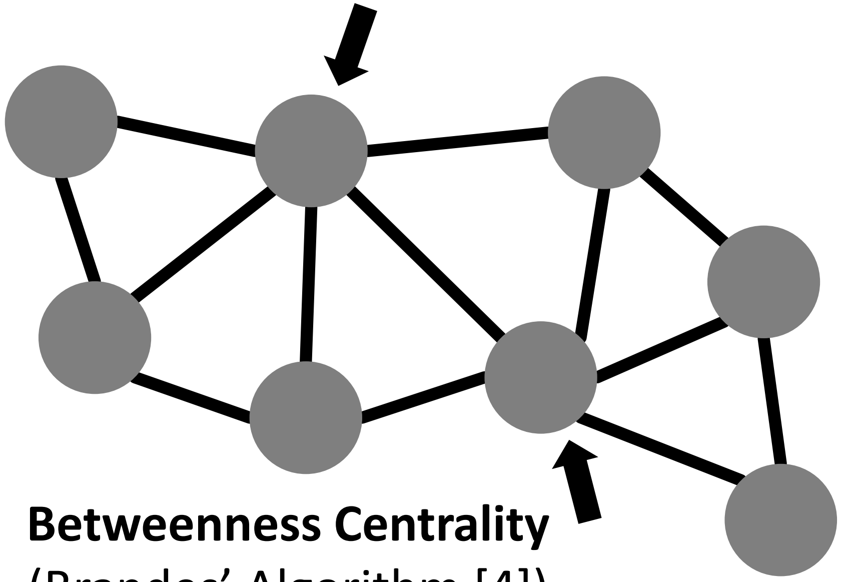

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Performance Analysis

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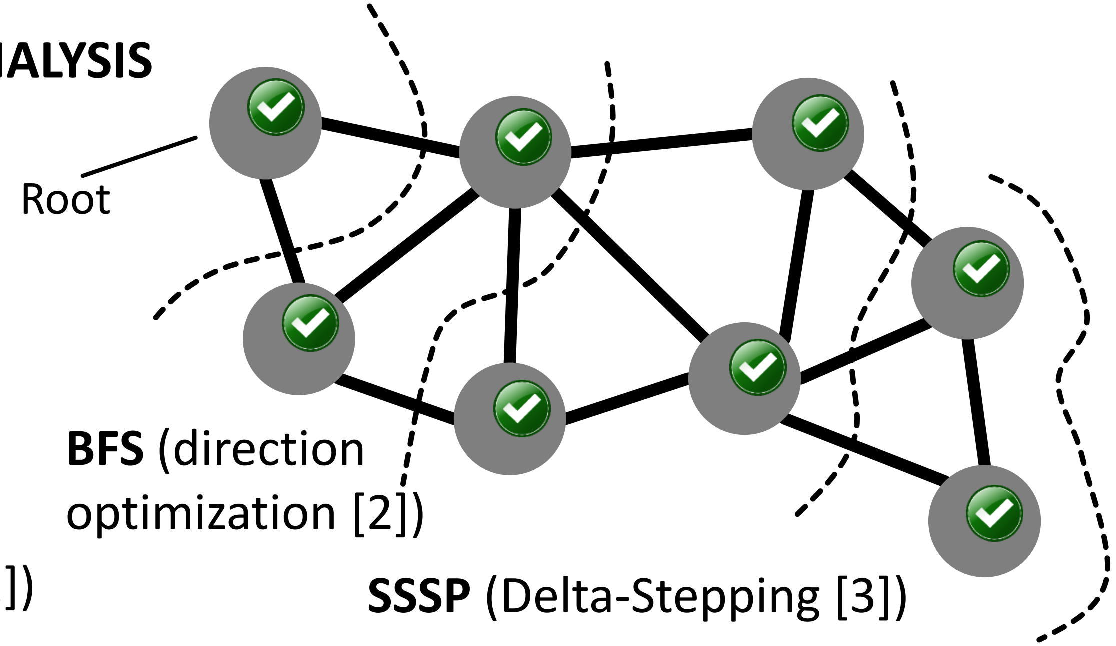

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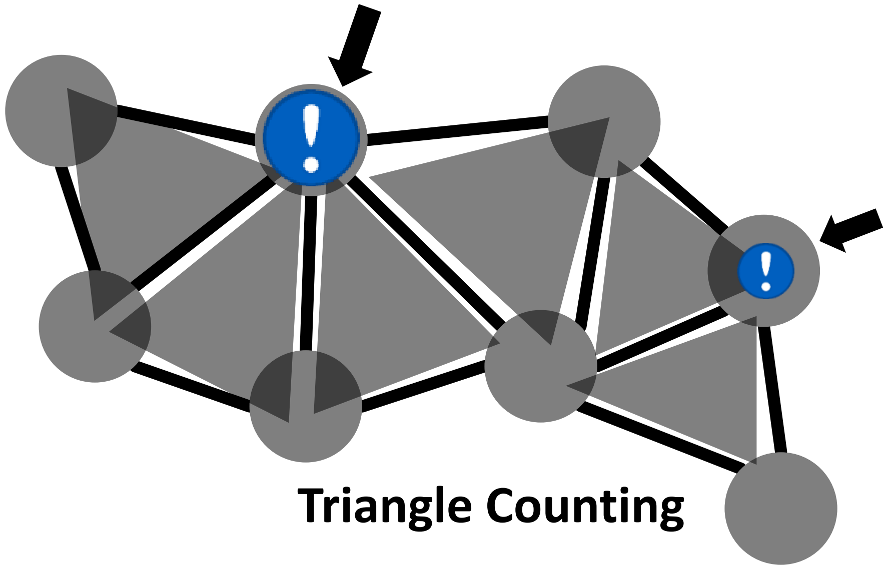

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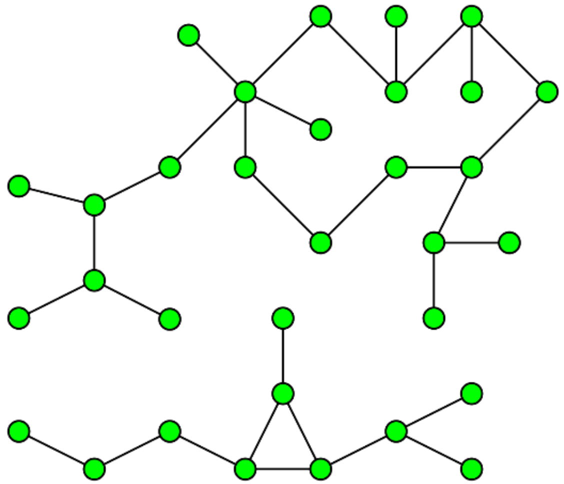

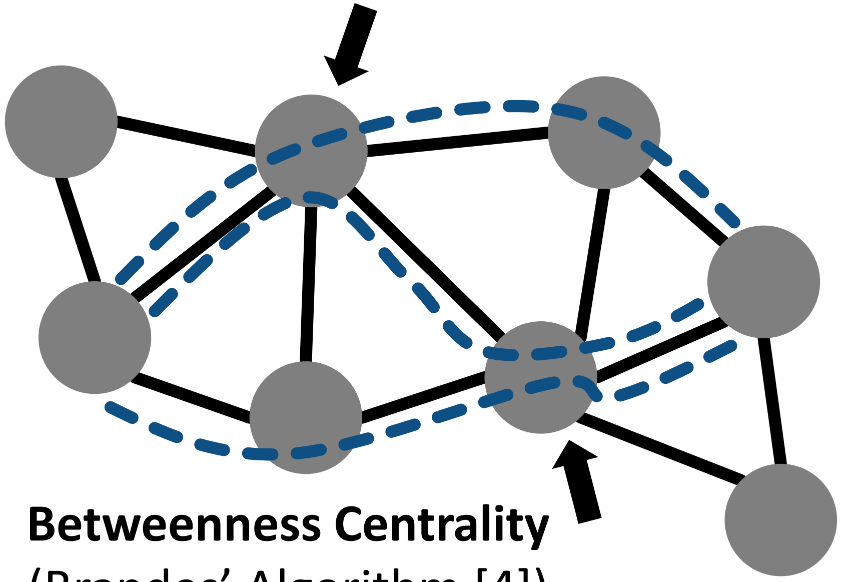

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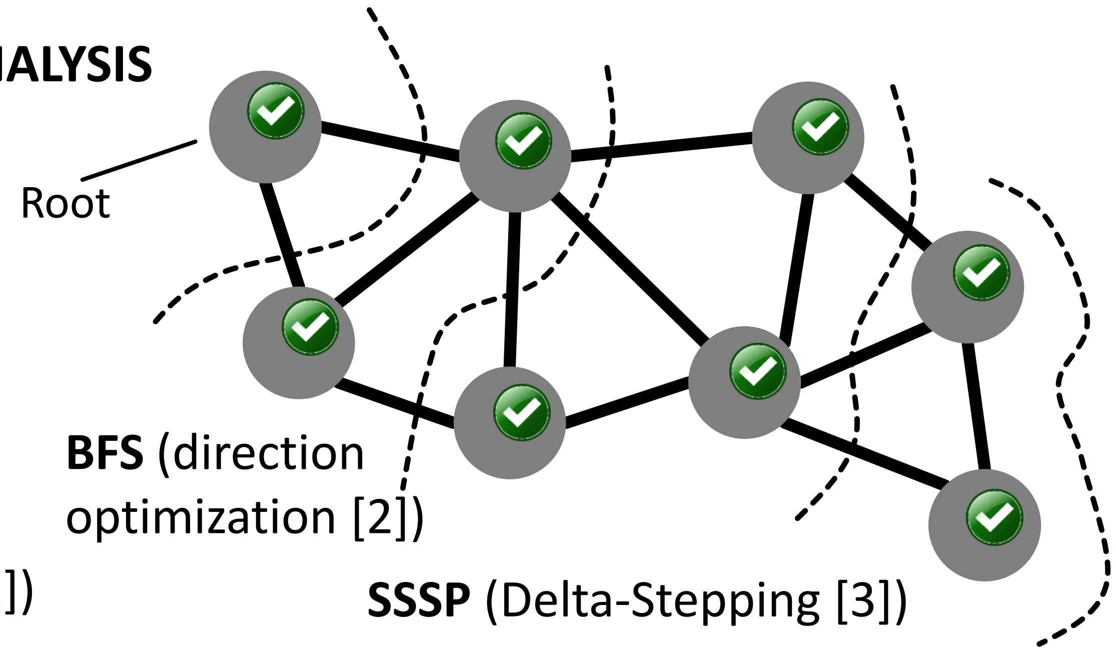

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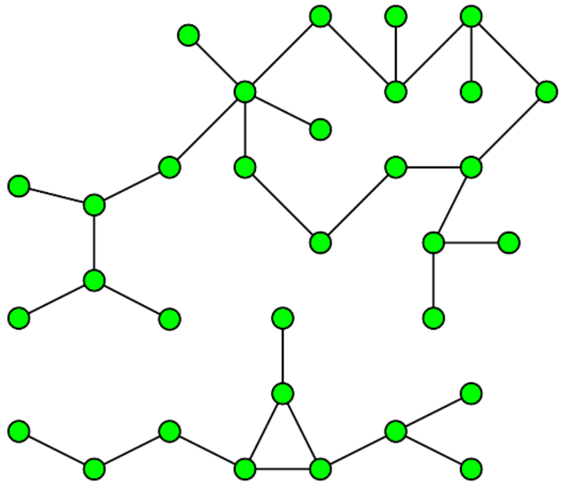

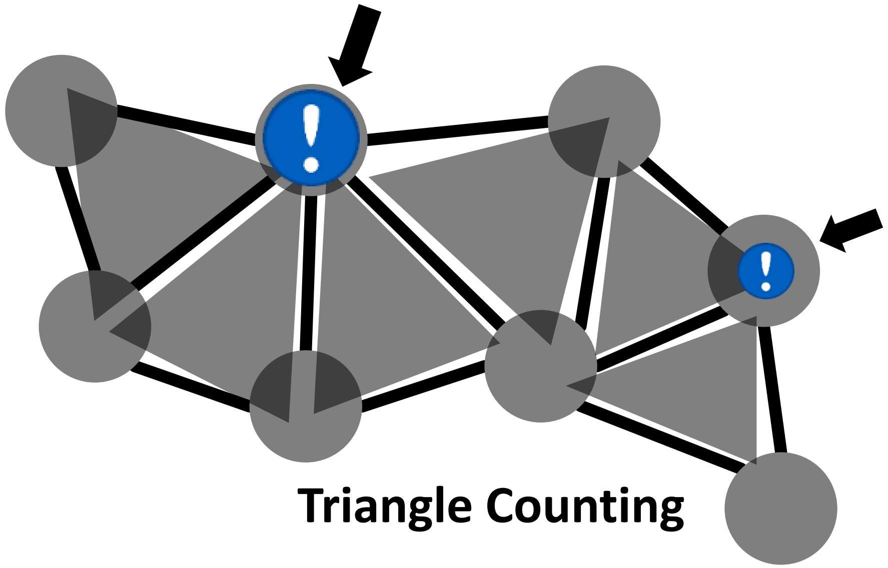

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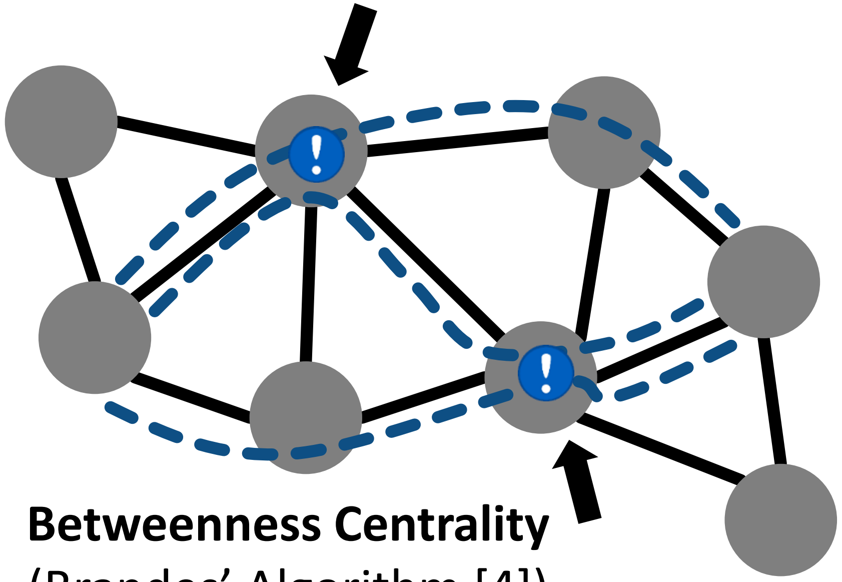

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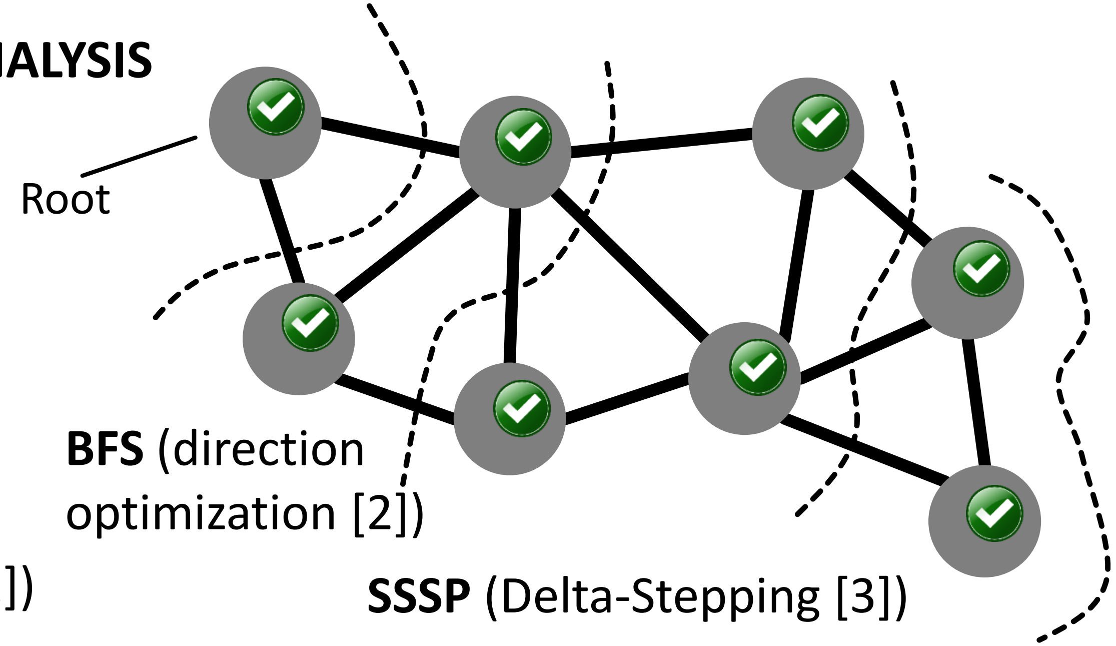
ALGORITHMS



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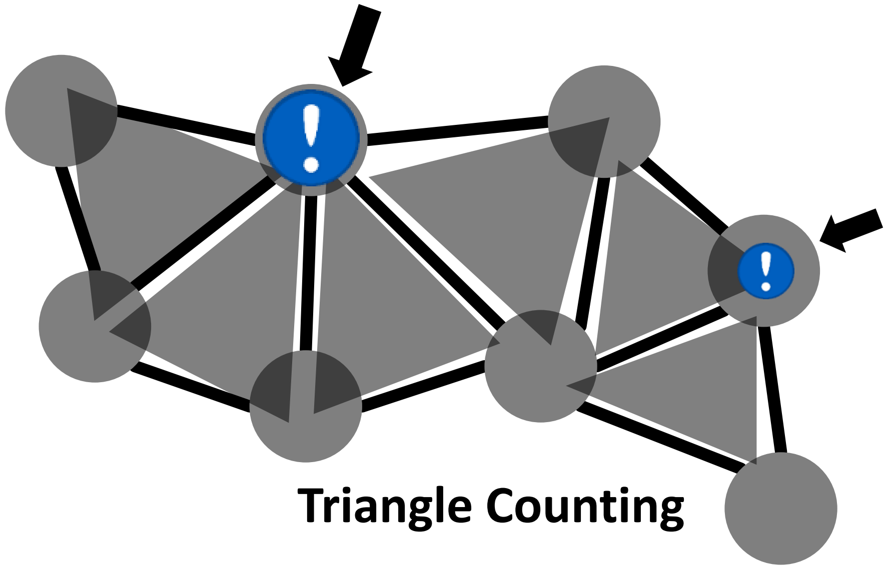

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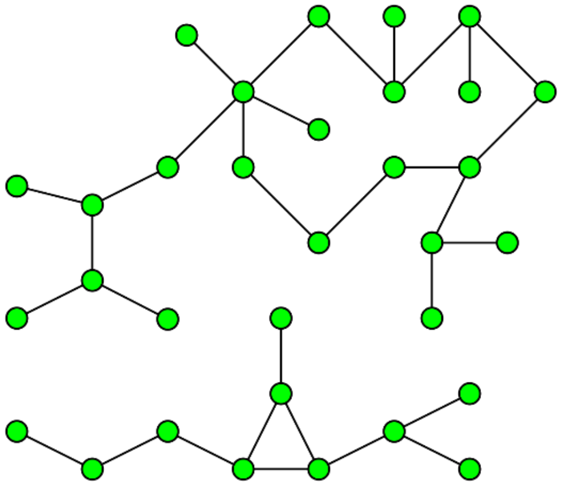
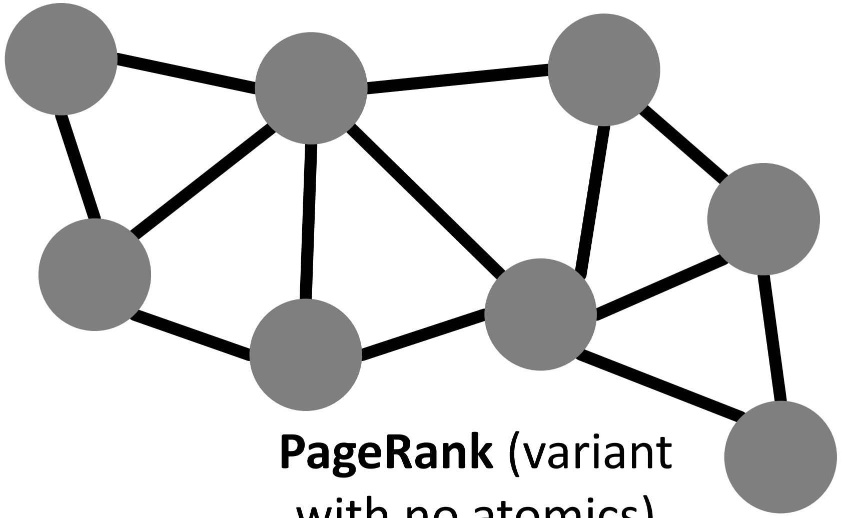
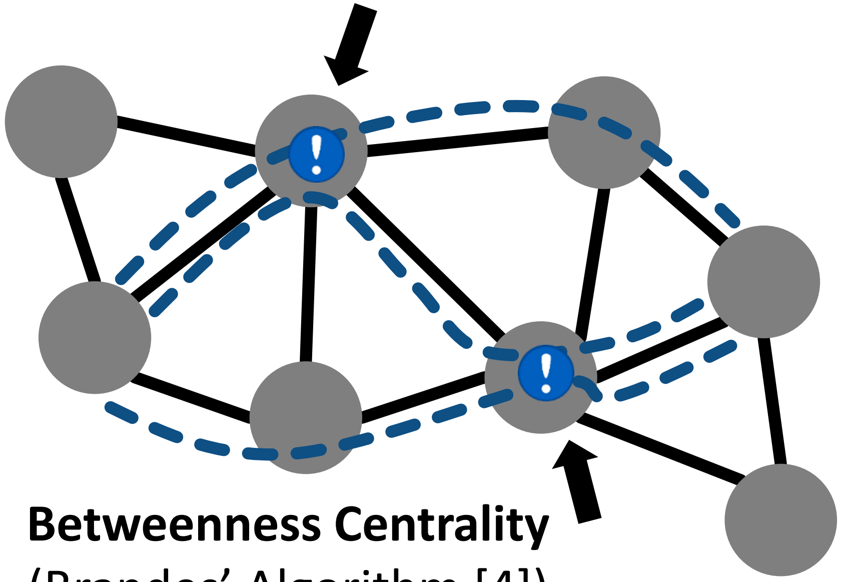




PageRank (variant with no atomics)



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Performance Analysis

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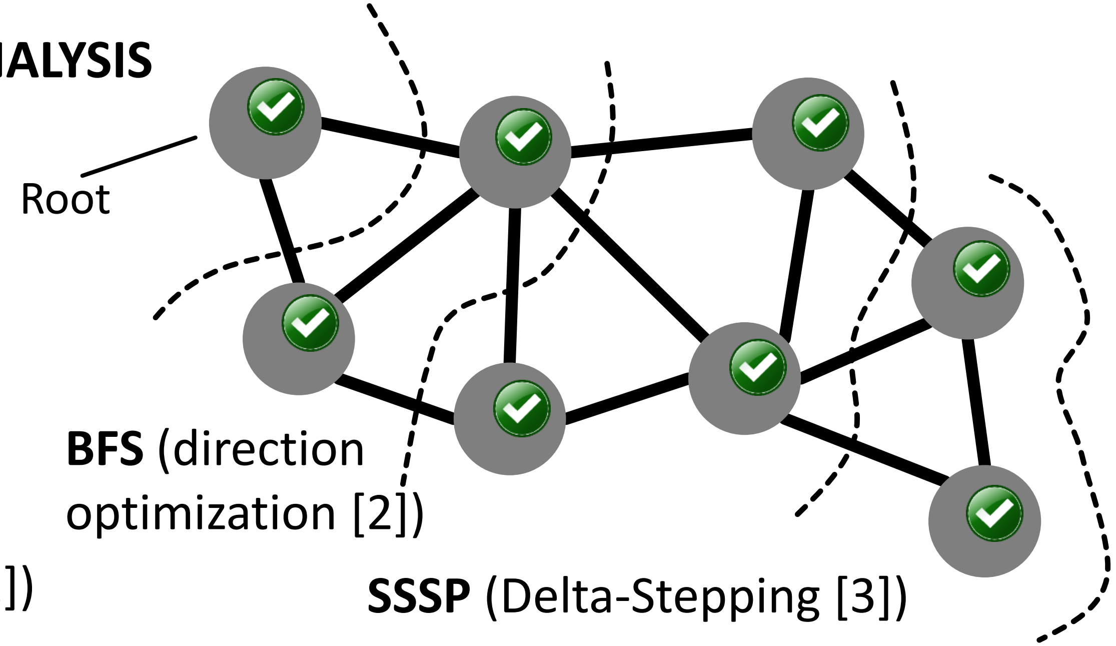

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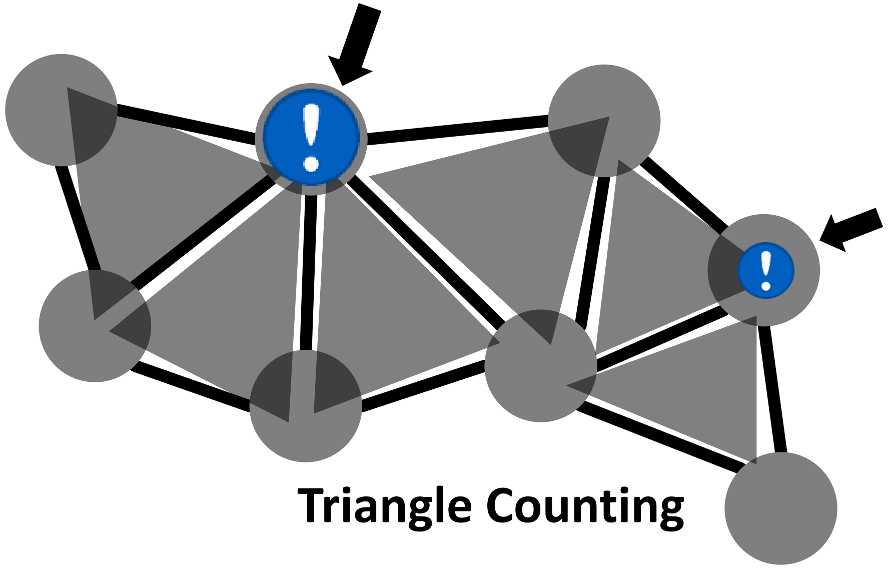

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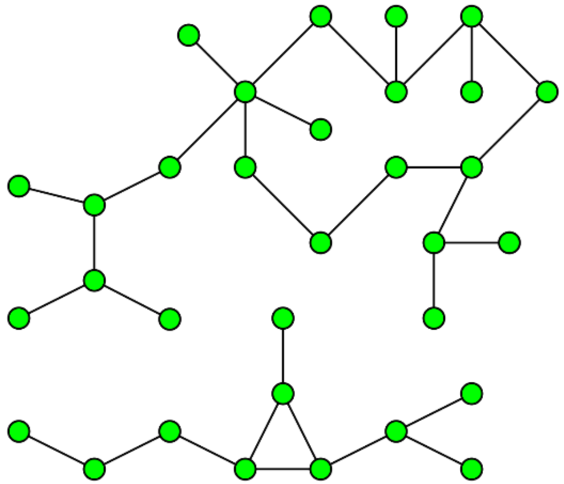
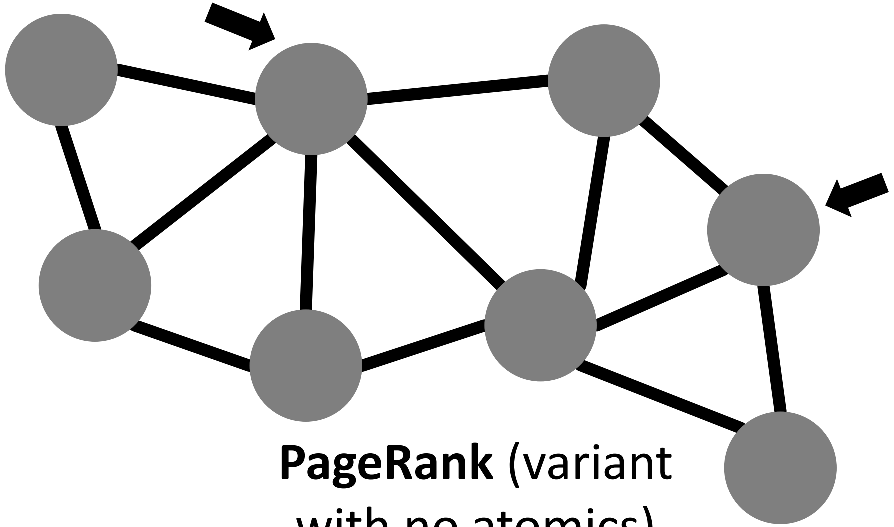
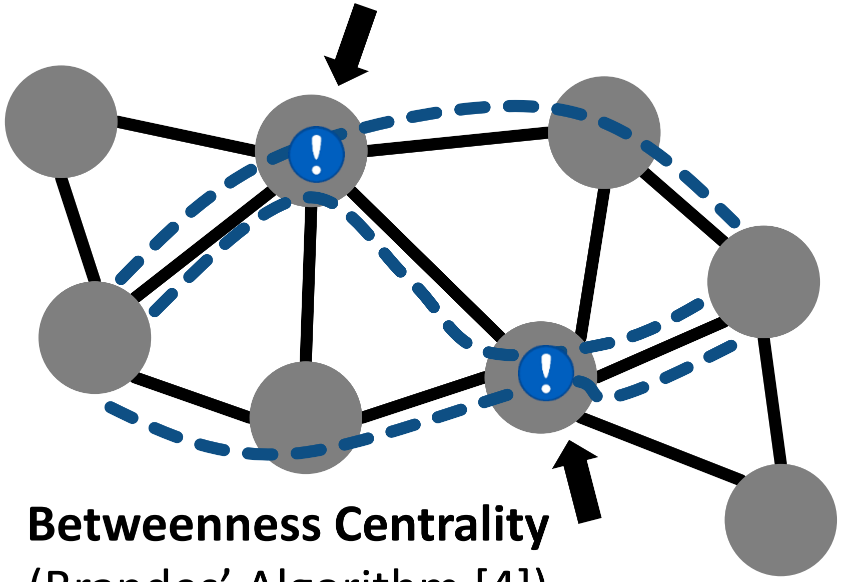




PageRank (variant with no atomics)



Betweenness Centrality (Brandes' Algorithm [4])

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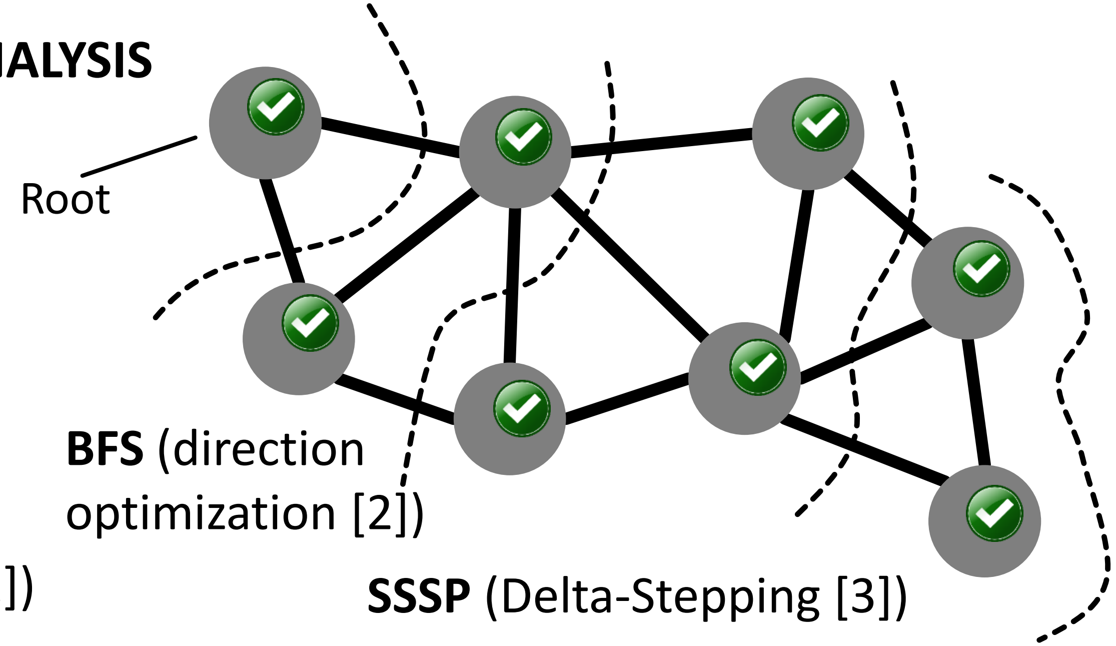
Performance Analysis
Algorithms



BFS (direction optimization [2])

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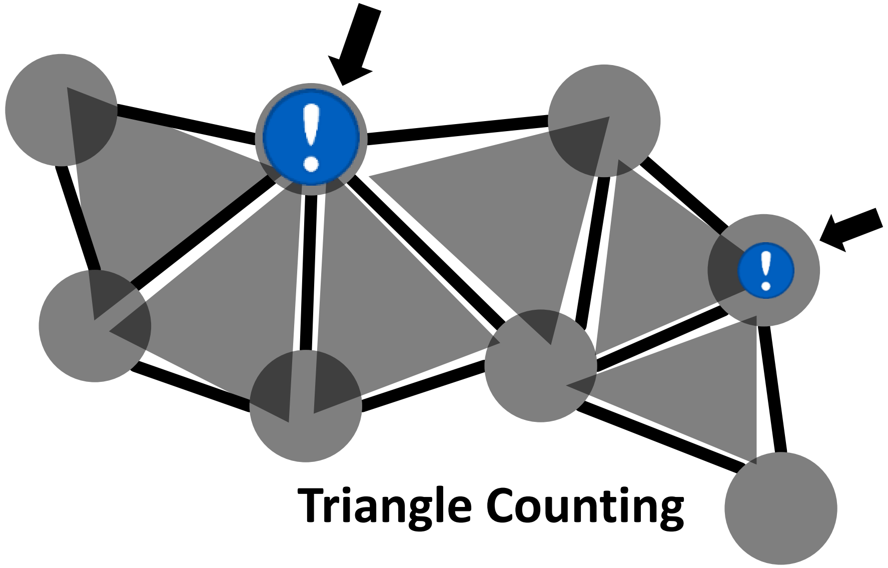
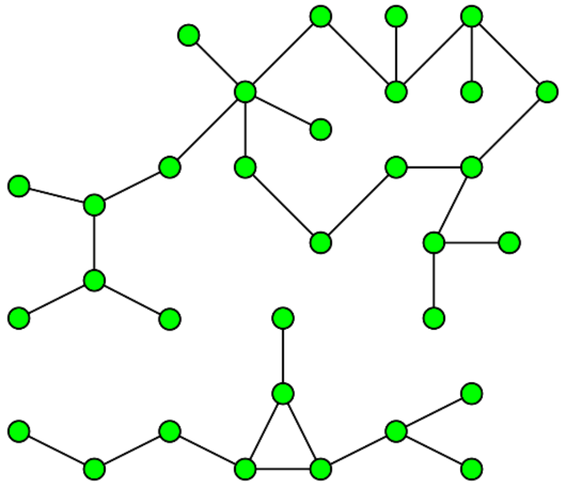
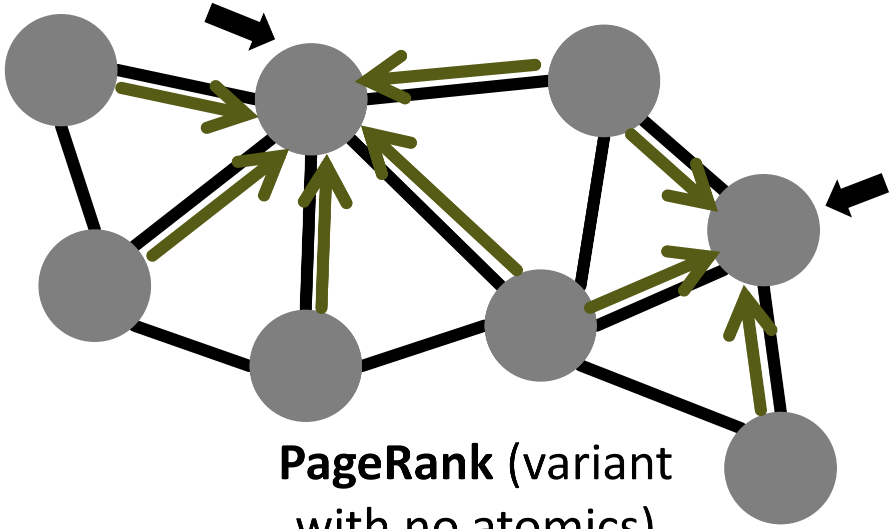


Triangle Counting

Connected Components (Shiloach-Vishkin [1])





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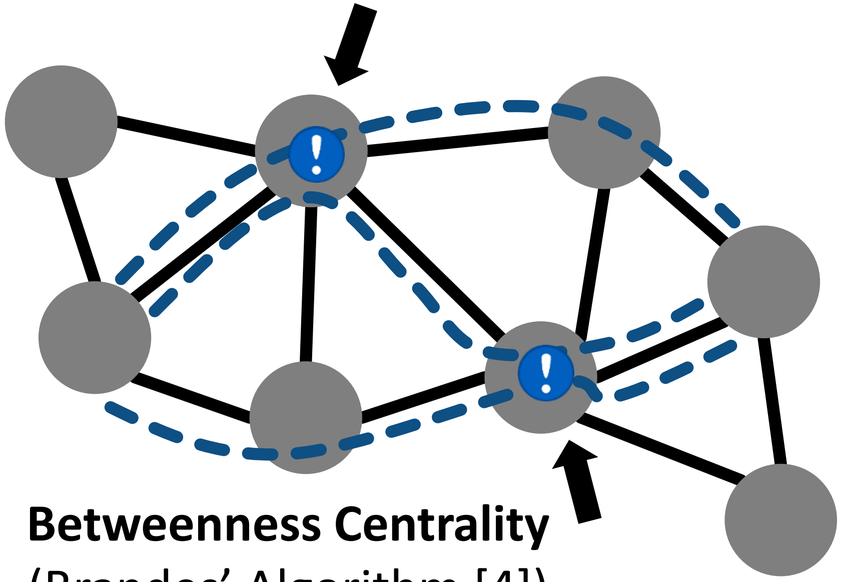

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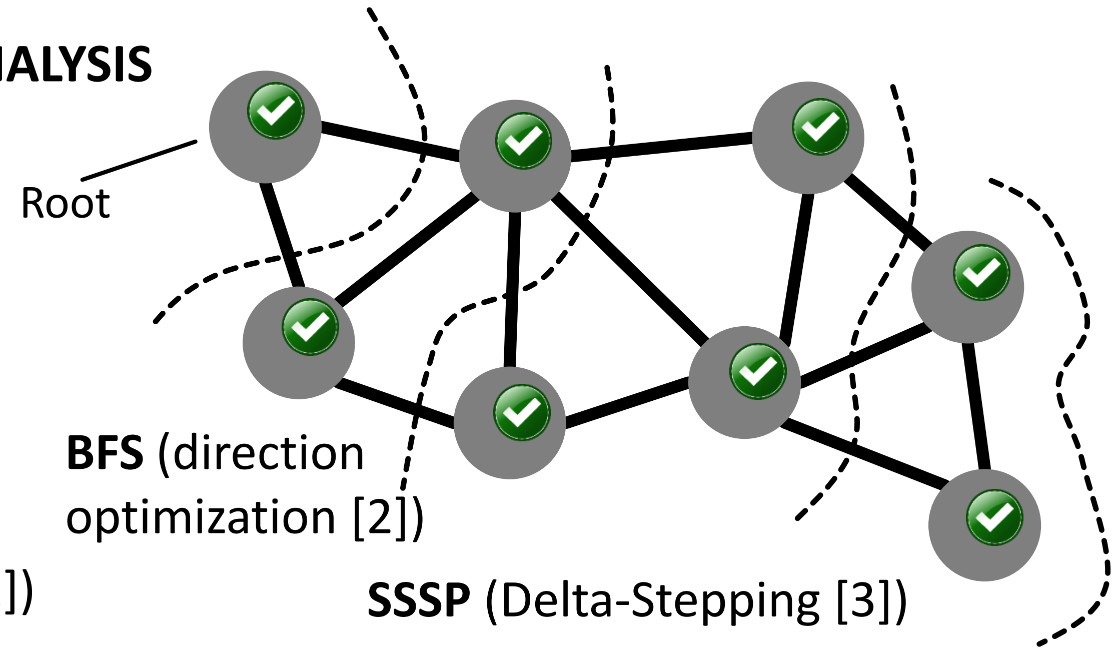

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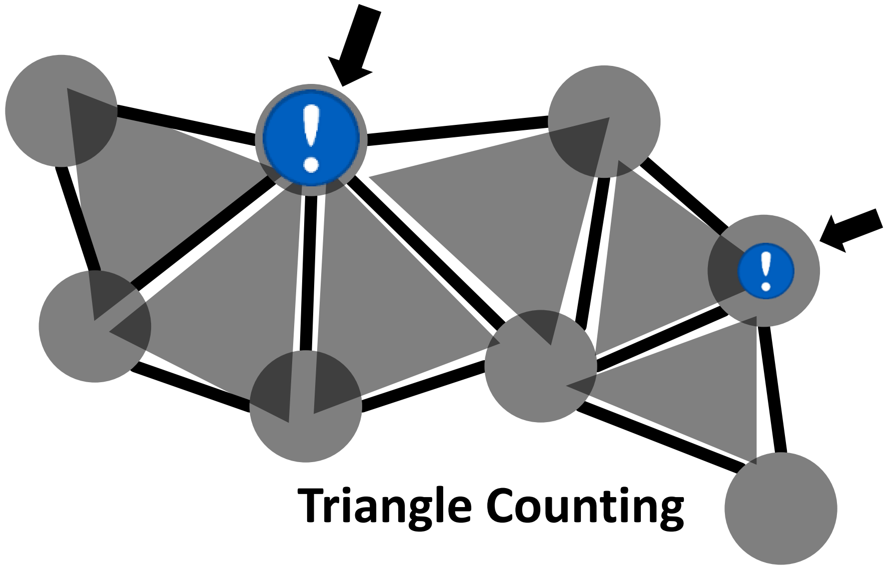

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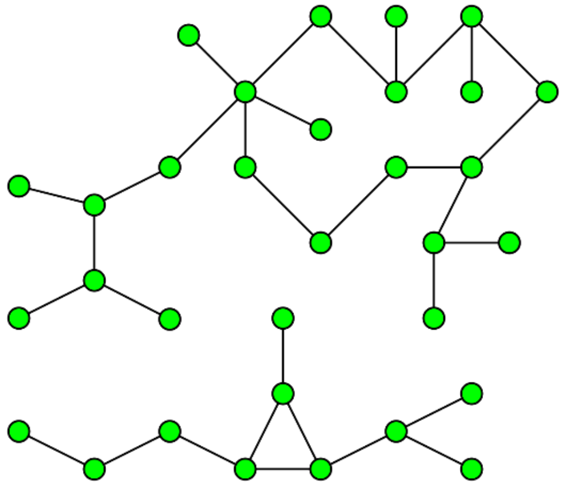

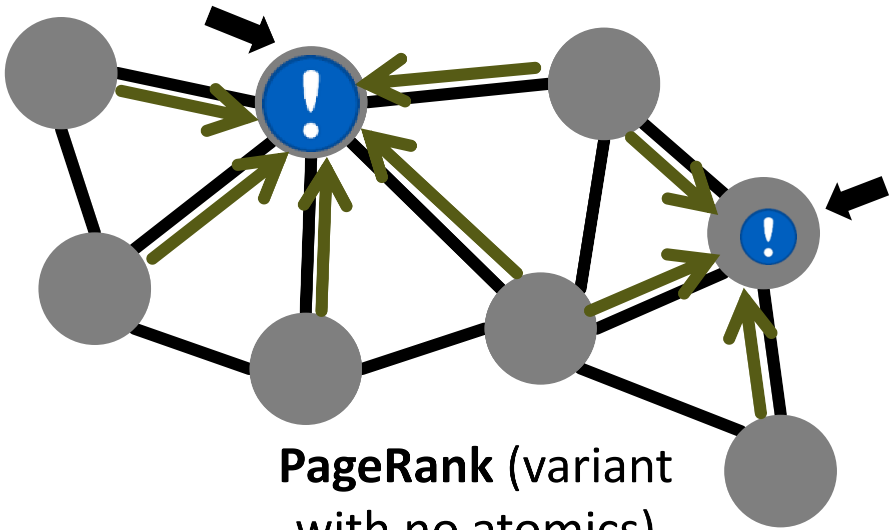

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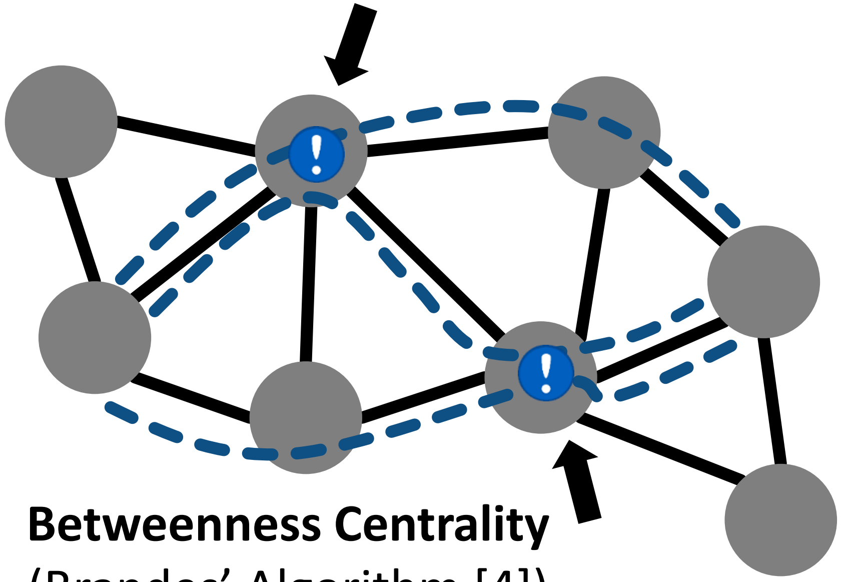

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Performance Analysis

Comparison Targets

Performance Analysis

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Performance Analysis

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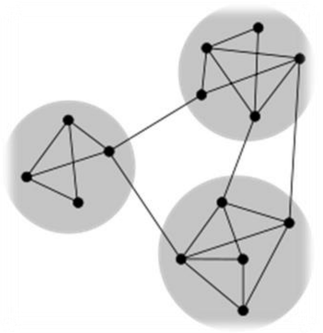
SSSP

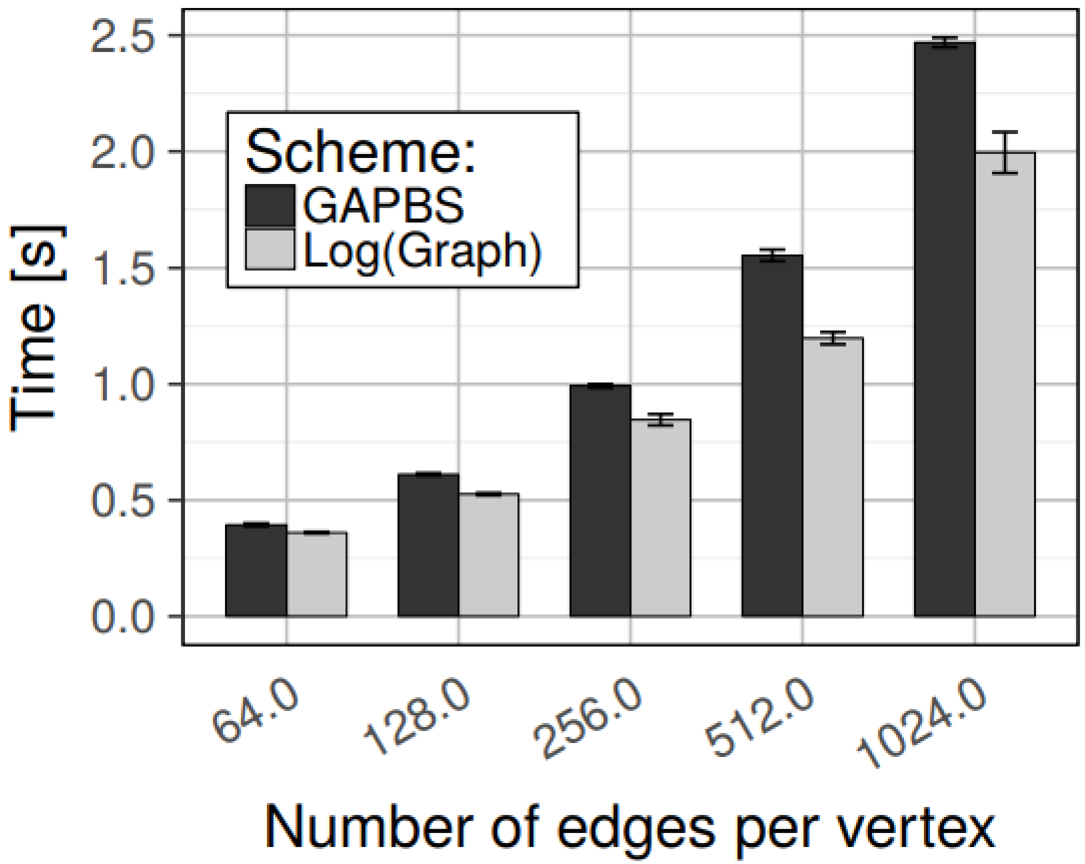
1 Log (Vertex labels), Log (Edge weights)   Storage, Performance

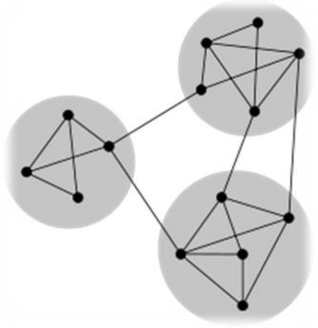


Kronecker graphs
Number of vertices: 4M

SSSP

1 **Log (Vertex labels), Log (Edge weights)** **Storage, Performance**





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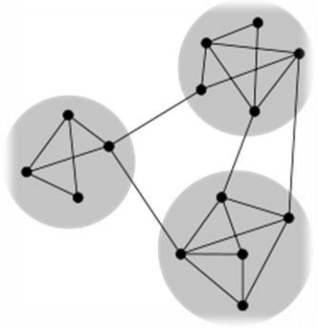
SSSP



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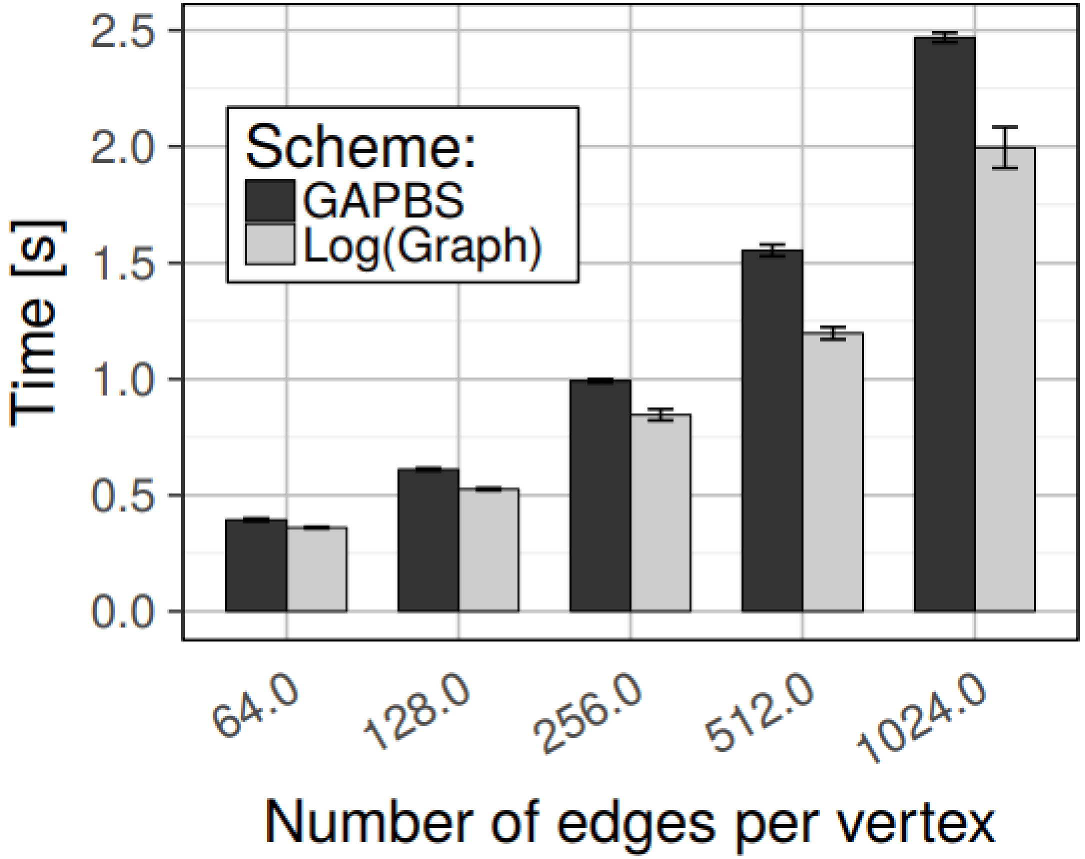

Log(Graph) consistently reduces storage overhead (by 20-35%)

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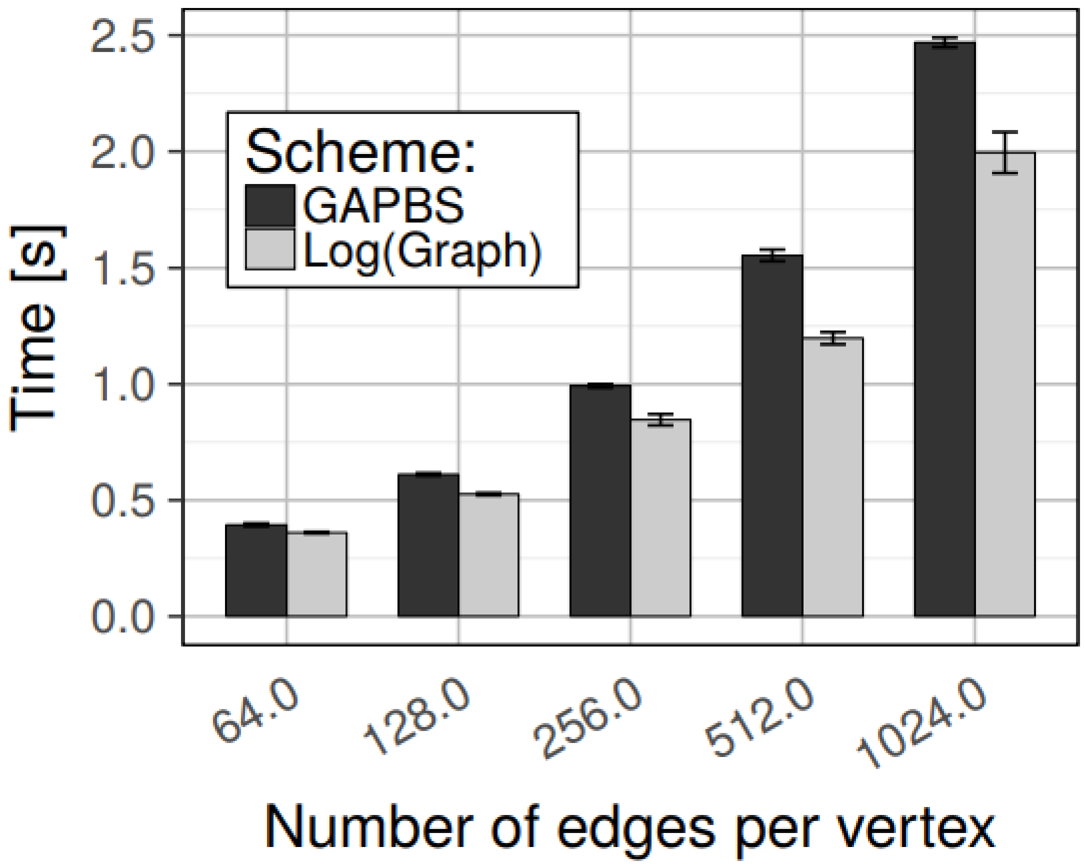
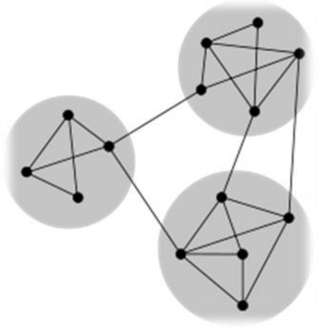


Log(Graph) **accelerates** GAPBS



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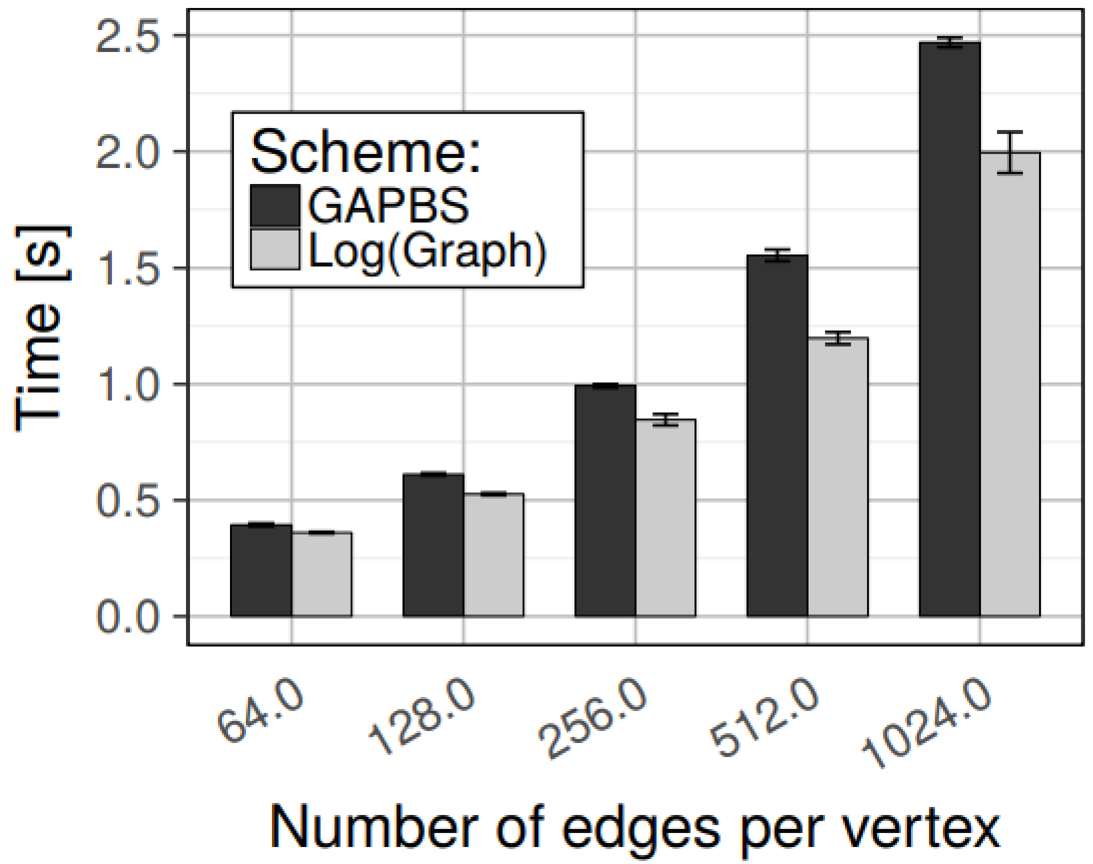

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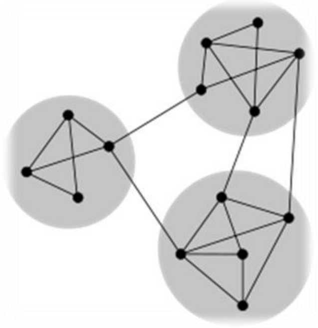

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Both storage and performance are improved **simultaneously**

Log(Graph) consistently reduces storage overhead (by 20-35%)

2 **Log (Offset structure) Storage**

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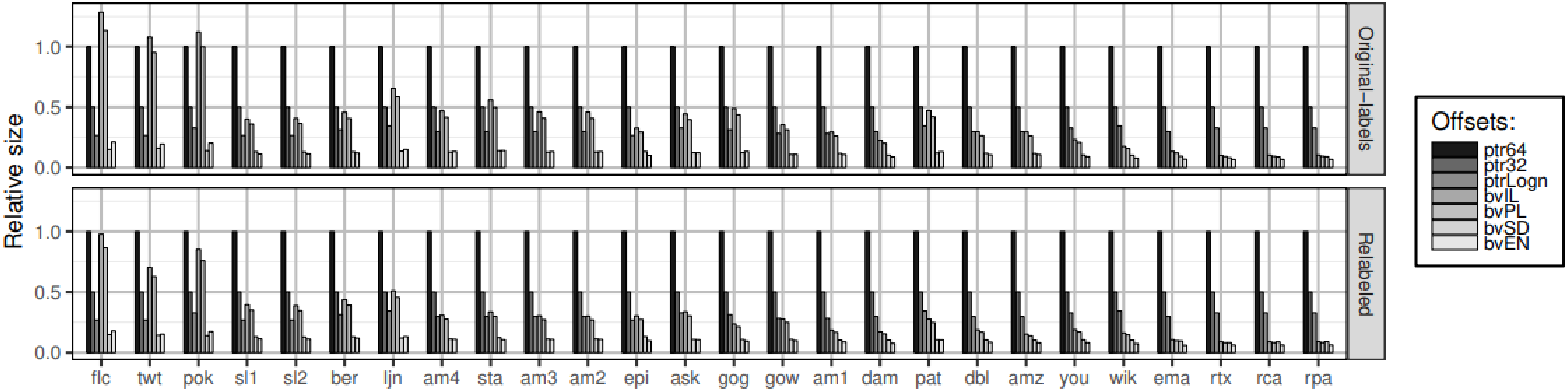
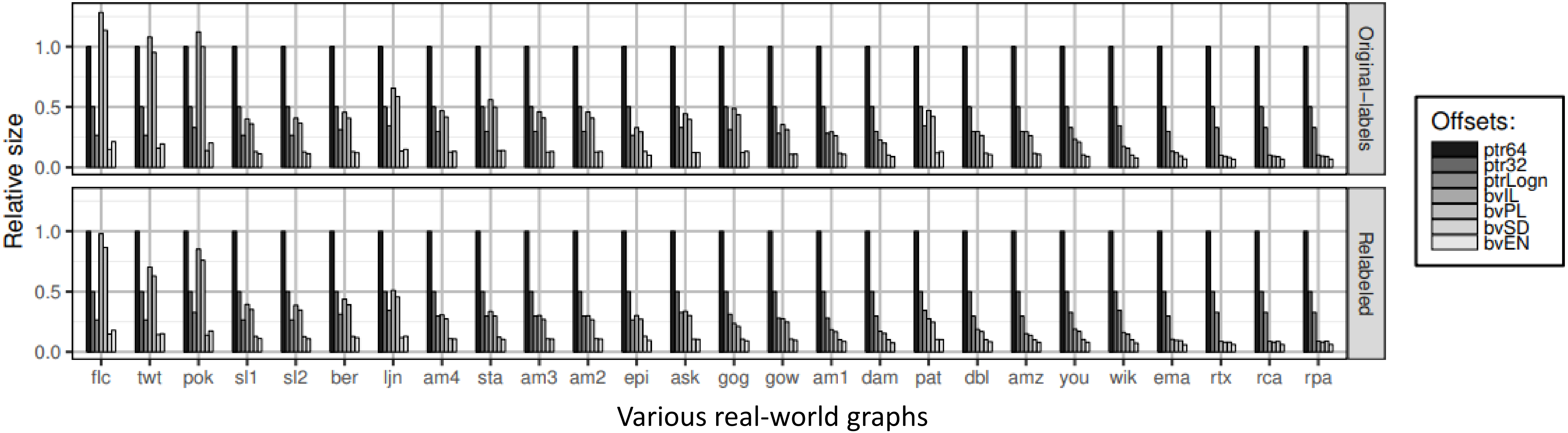


Various real-world graphs

2 Log (Offset structure) Storage



Lots of data ☺

Conclusions:

2 Log (Offset structure) Storage

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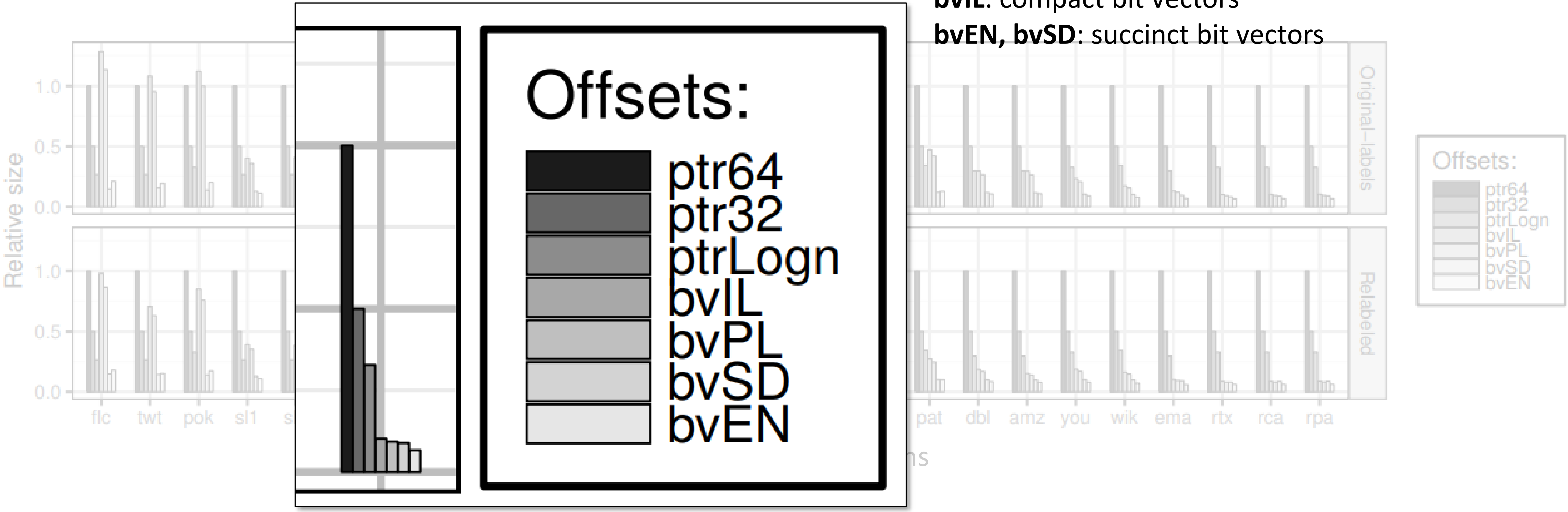
Conclusions:

2 **Log (** Offset structure **)** **Storage**

ptr64, ptr32: traditional array of offsets
ptrLogn: separate compression of each offset
bvPL: plain bit vectors
bvIL: compact bit vectors
bvEN, bvSD: succinct bit vectors



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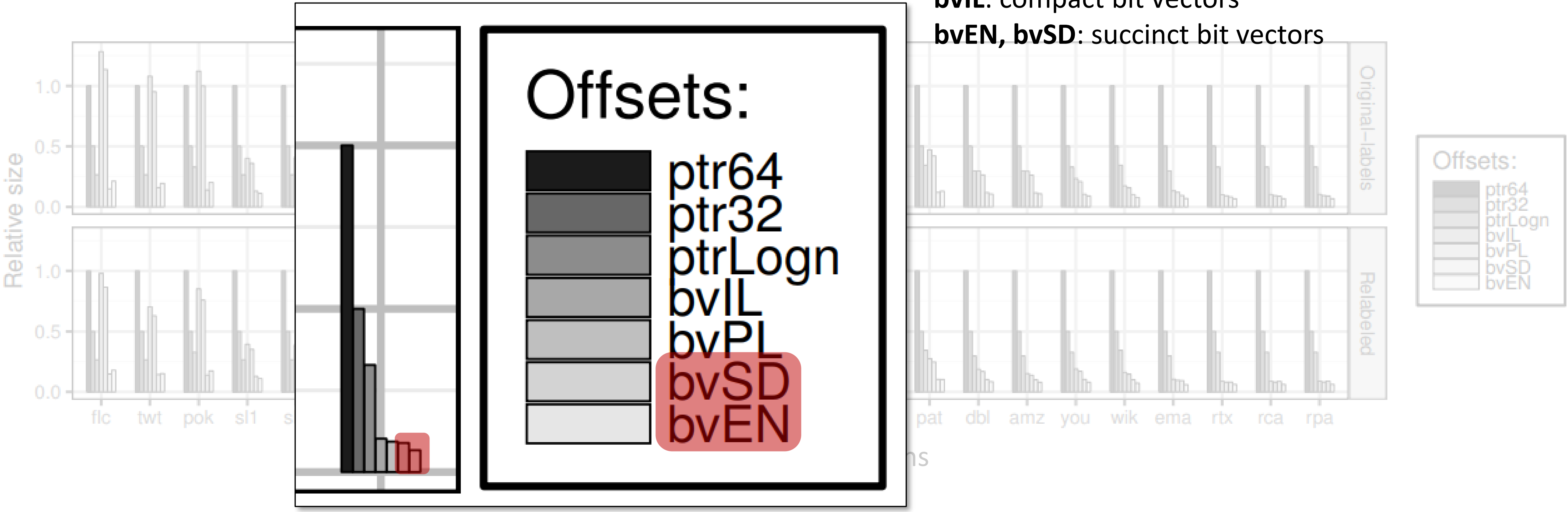

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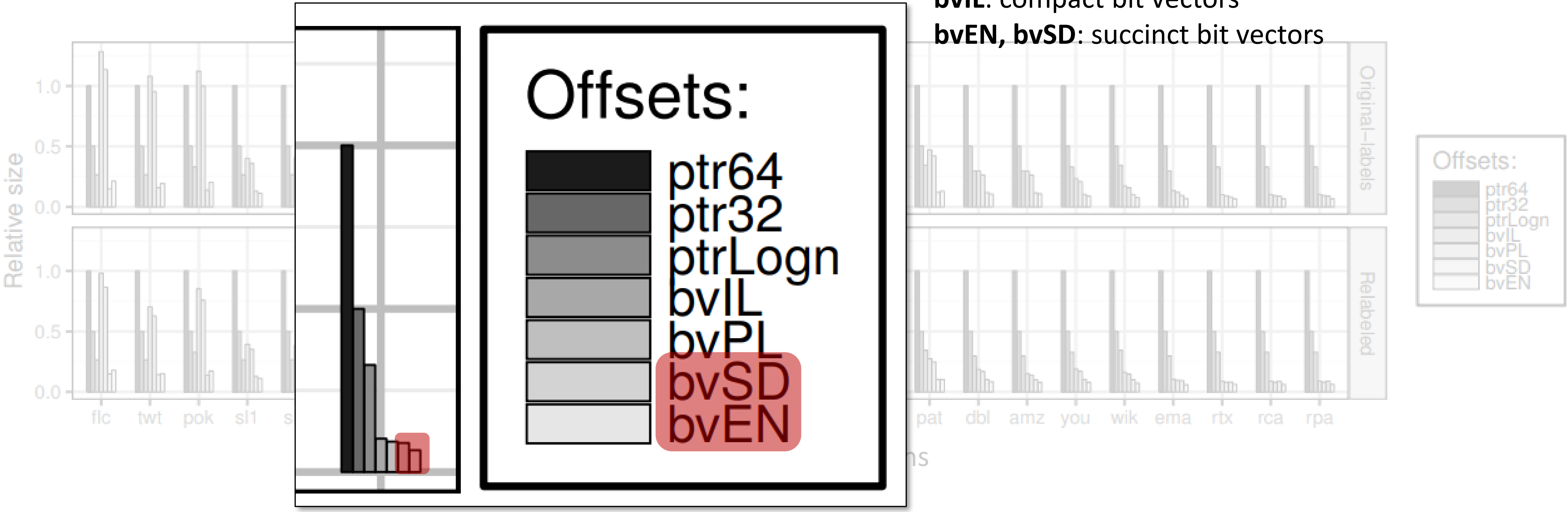

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Succinct bit vectors consistently ensure best storage reductions

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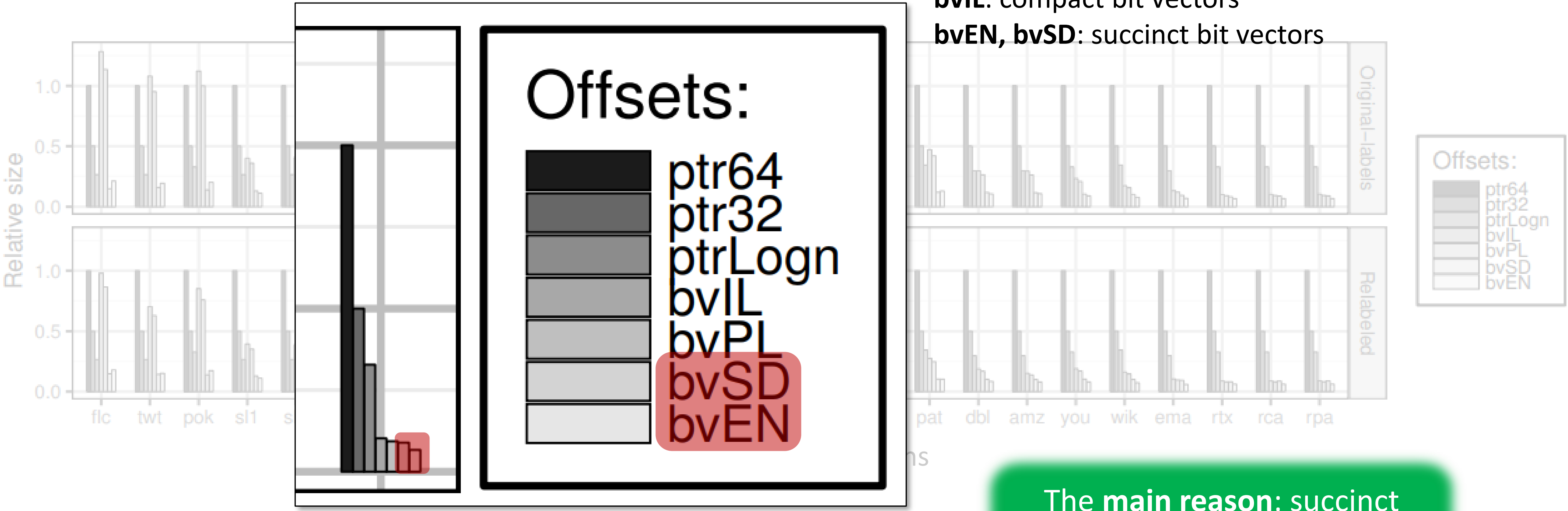

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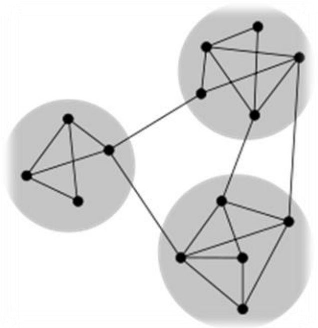
Conclusions:

Succinct bit vectors consistently ensure best storage reductions

The **main reason**: succinct designs work well for sparse bit vectors, and graphs „that matter" are sparse

2 Log (Offset structure) Performance

Accessing randomly selected neighbors



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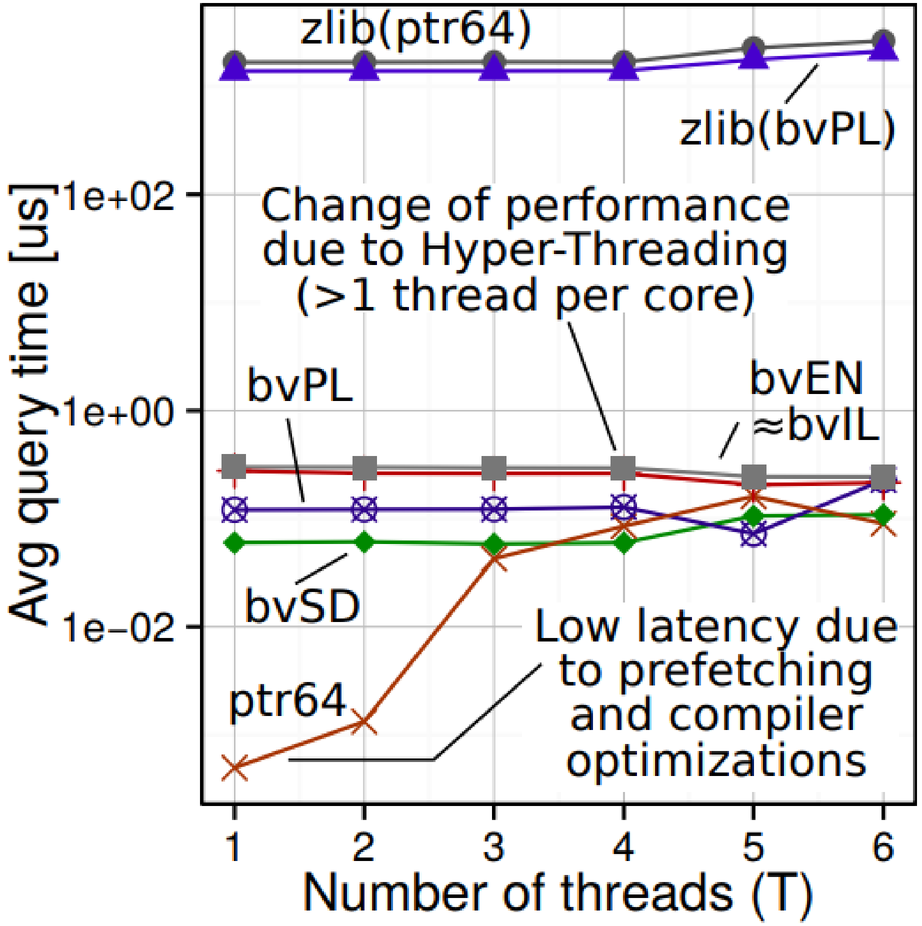

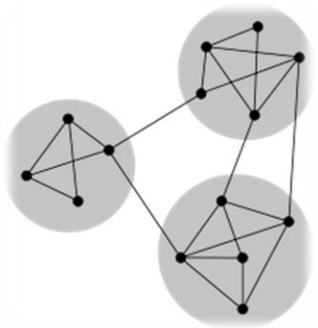

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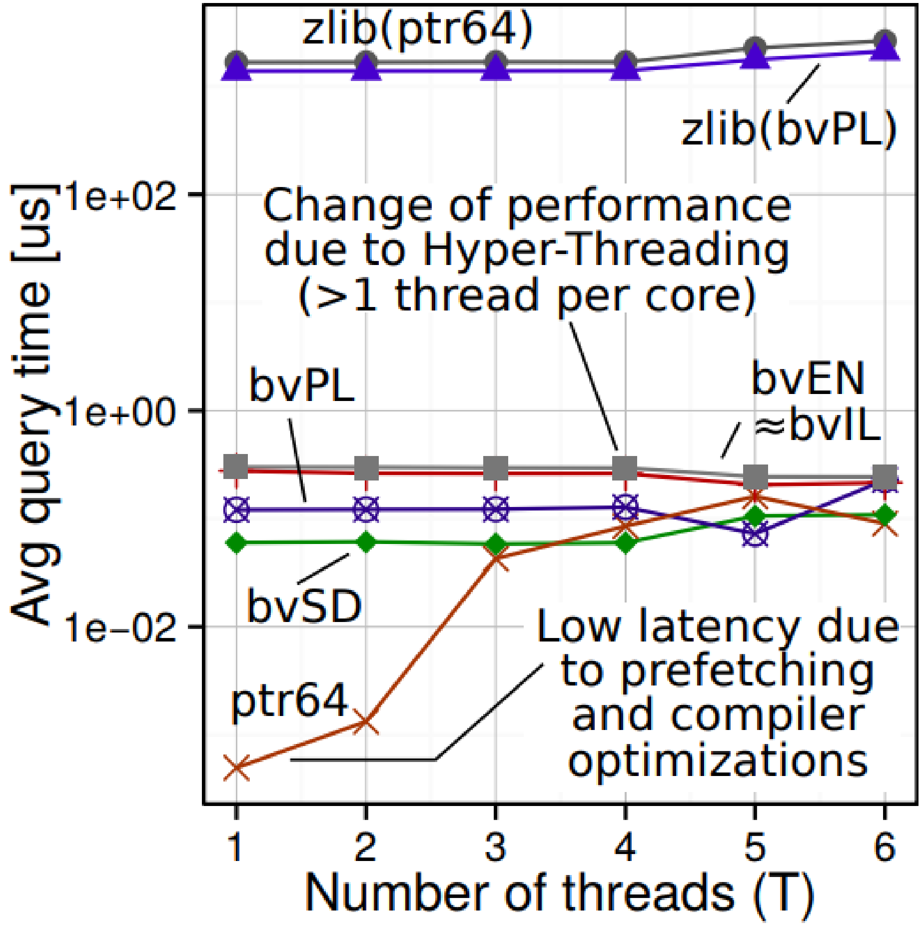

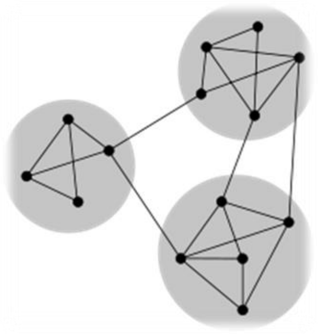

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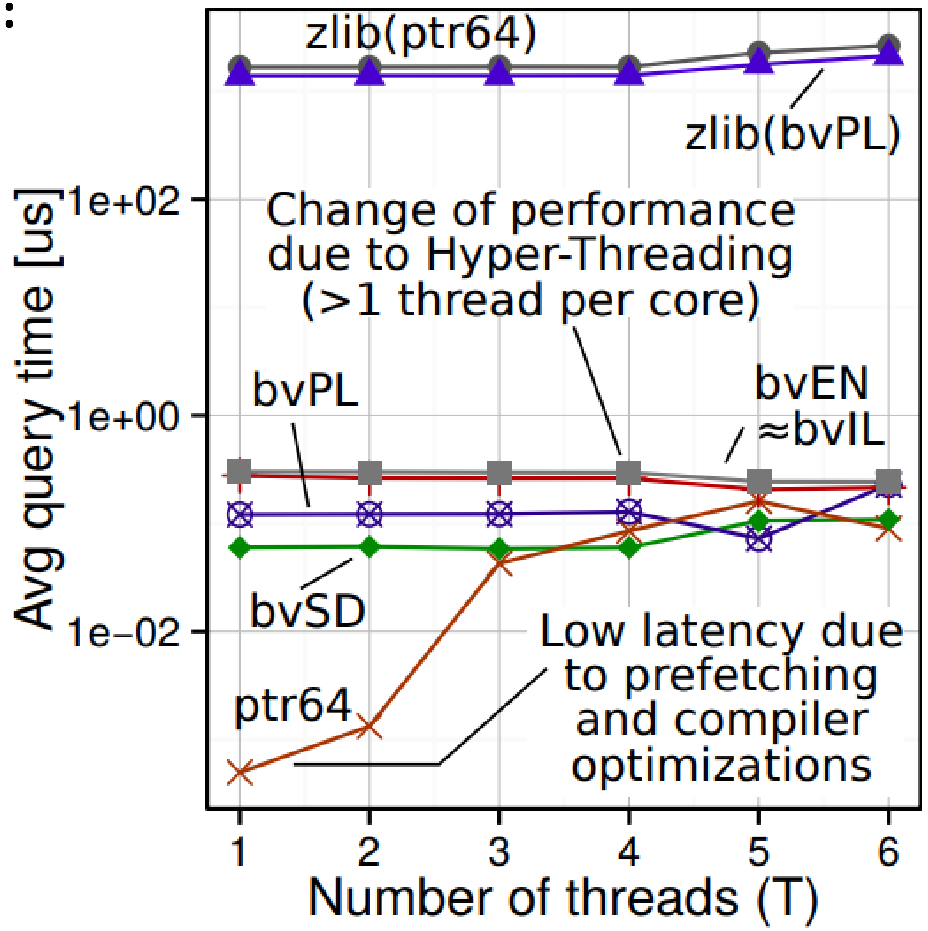

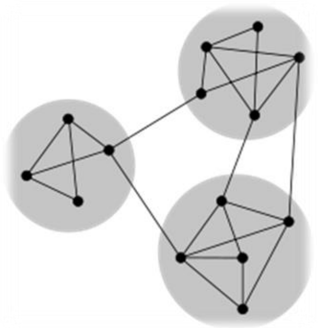

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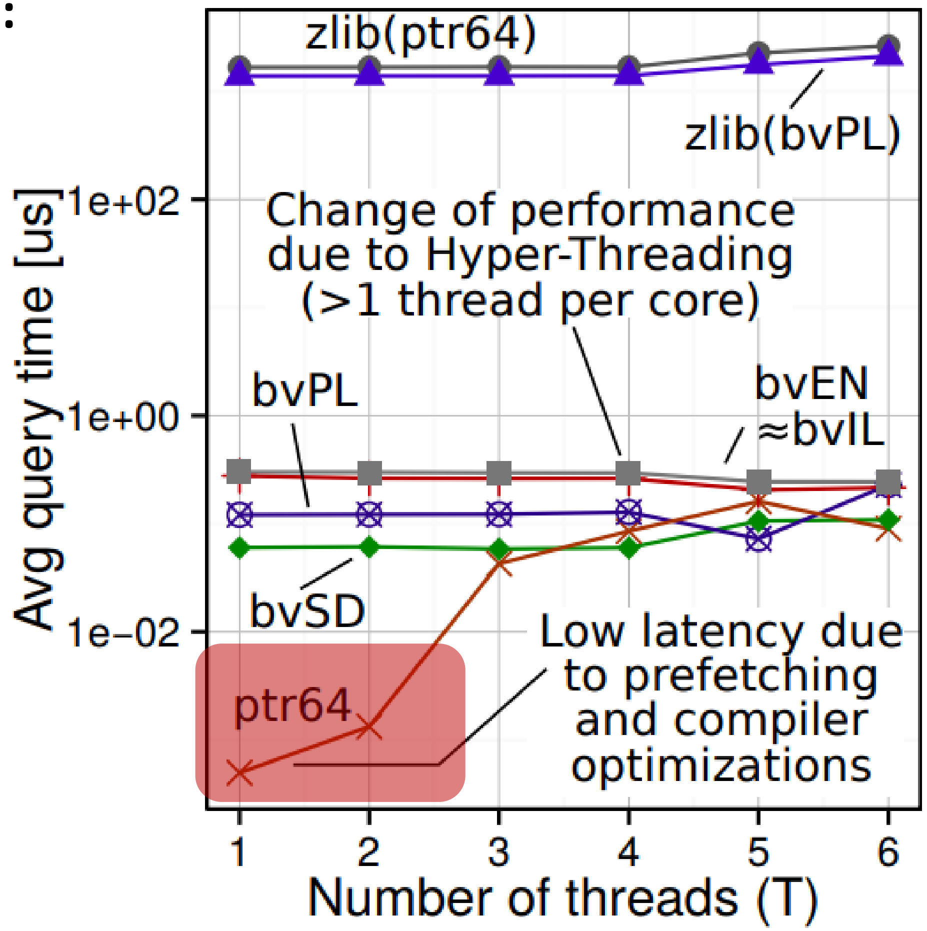
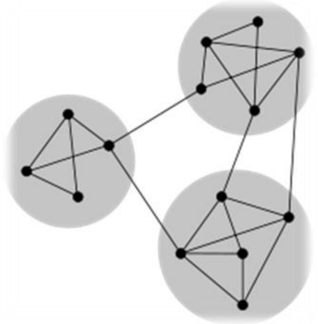
In sequential settings (or settings with low parallelism), **simple offset arrays are the fastest**





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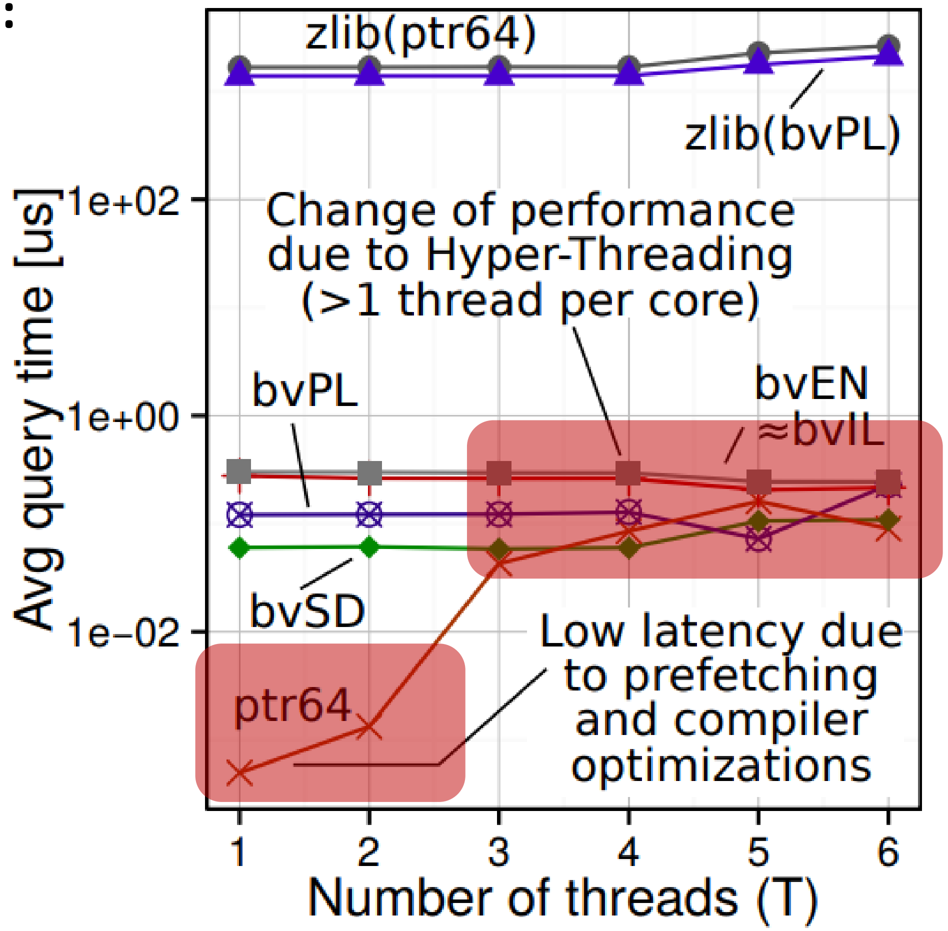
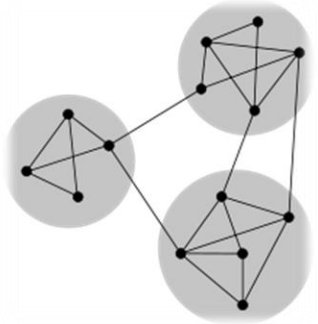
Once parallelism overheads kick in, performance of accessing succinct bit vectors and offset arrays **becomes comparable**





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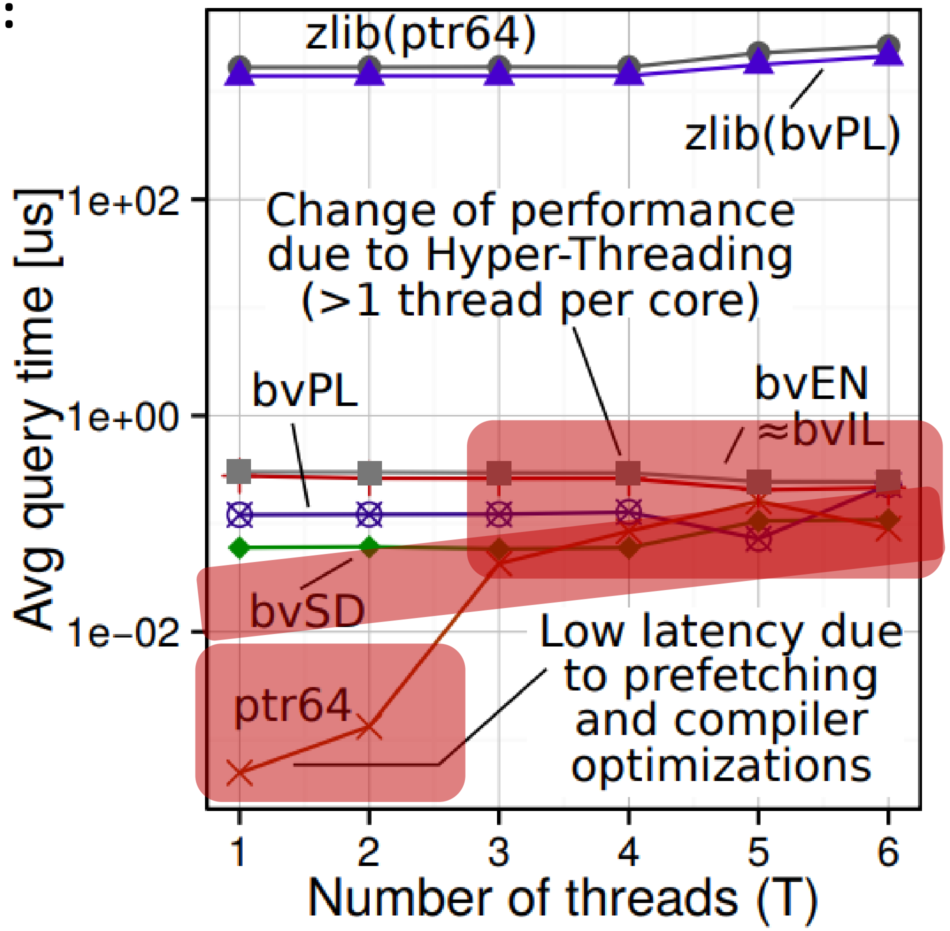
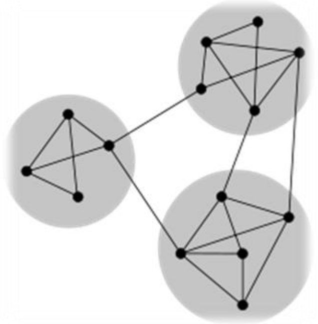


bvSD: the fastest and (usually) the smallest



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3 Log (Adjacency structure) Storage, Performane

3 **Log (** Adjacency structure **)** **Storage, Performane**

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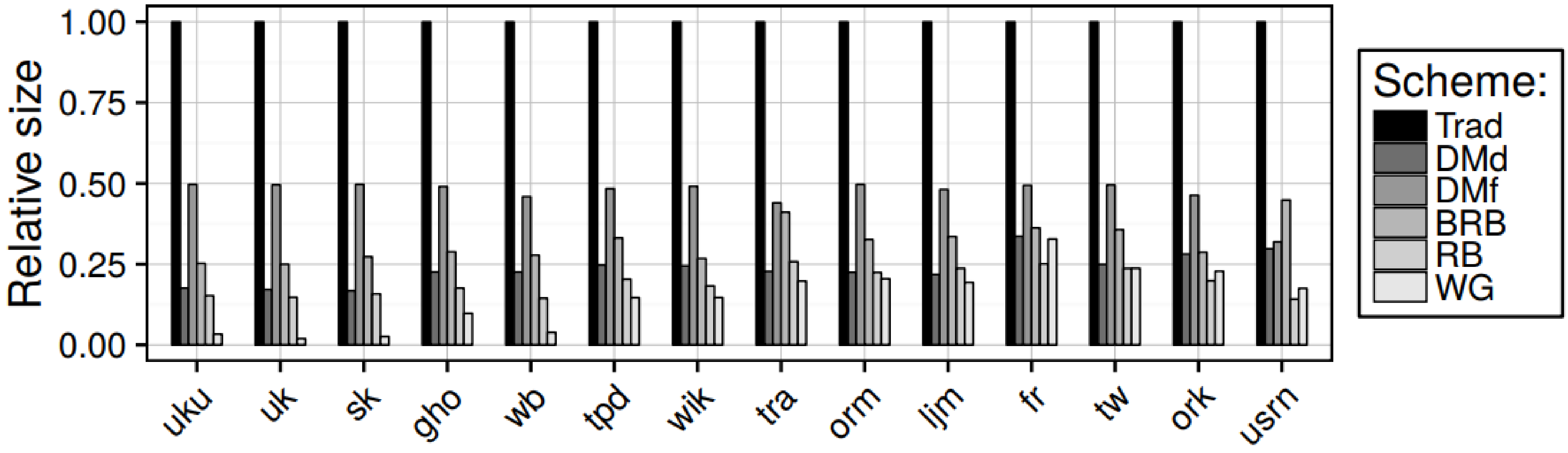
WG: WebGraph compression
BRB, RB: Schemes targeting certain specific classes of graphs



Various real-world graphs

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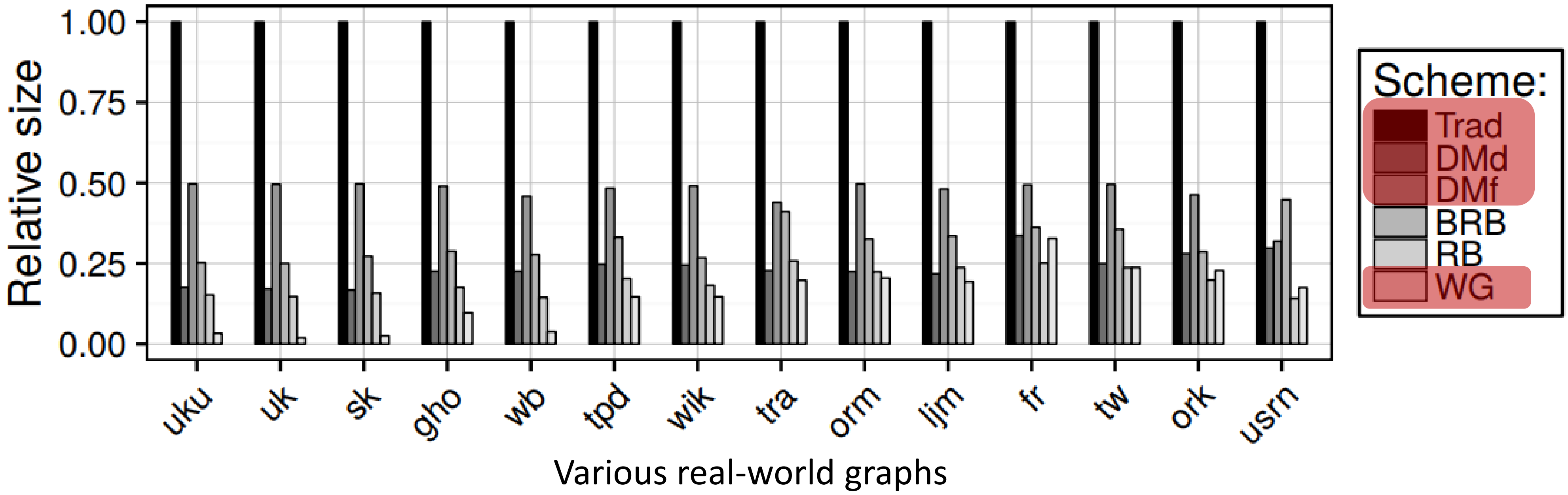

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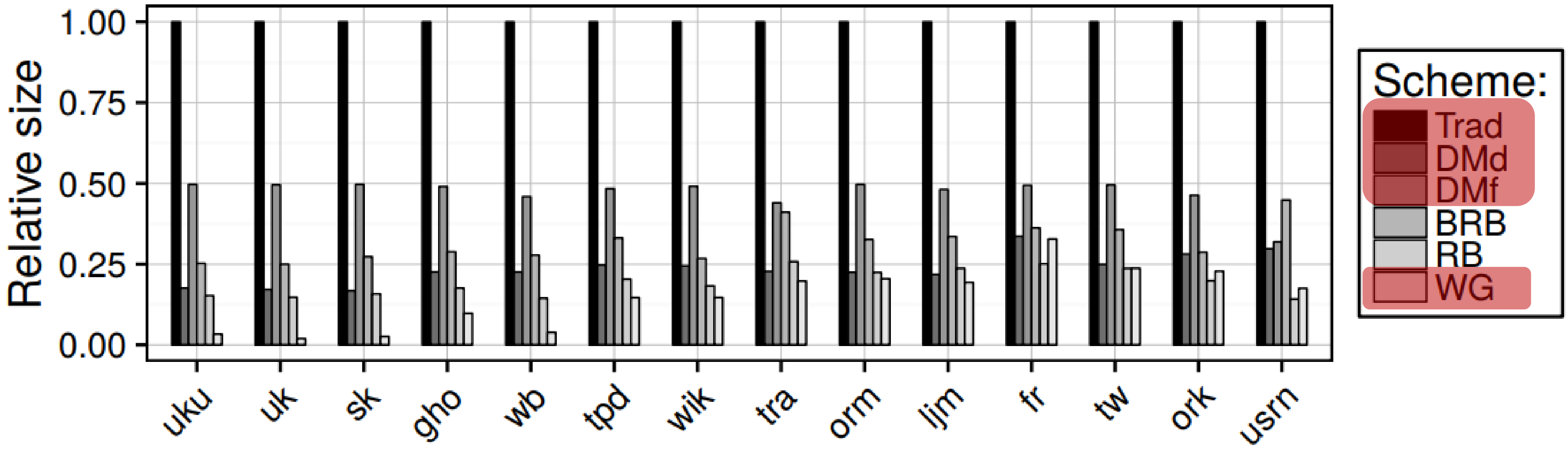

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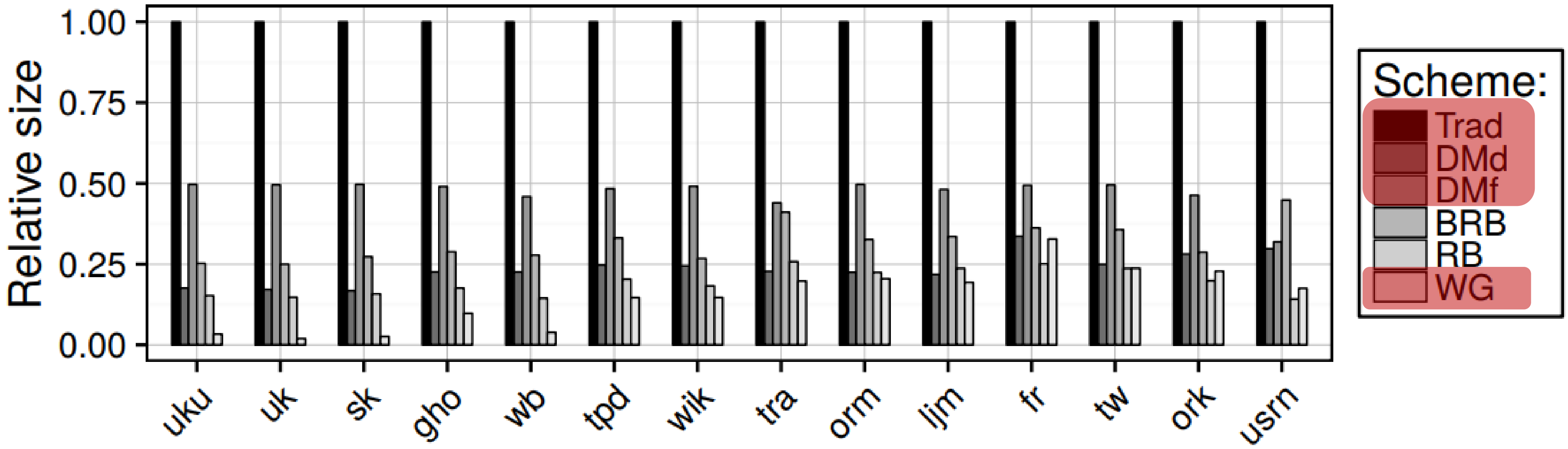

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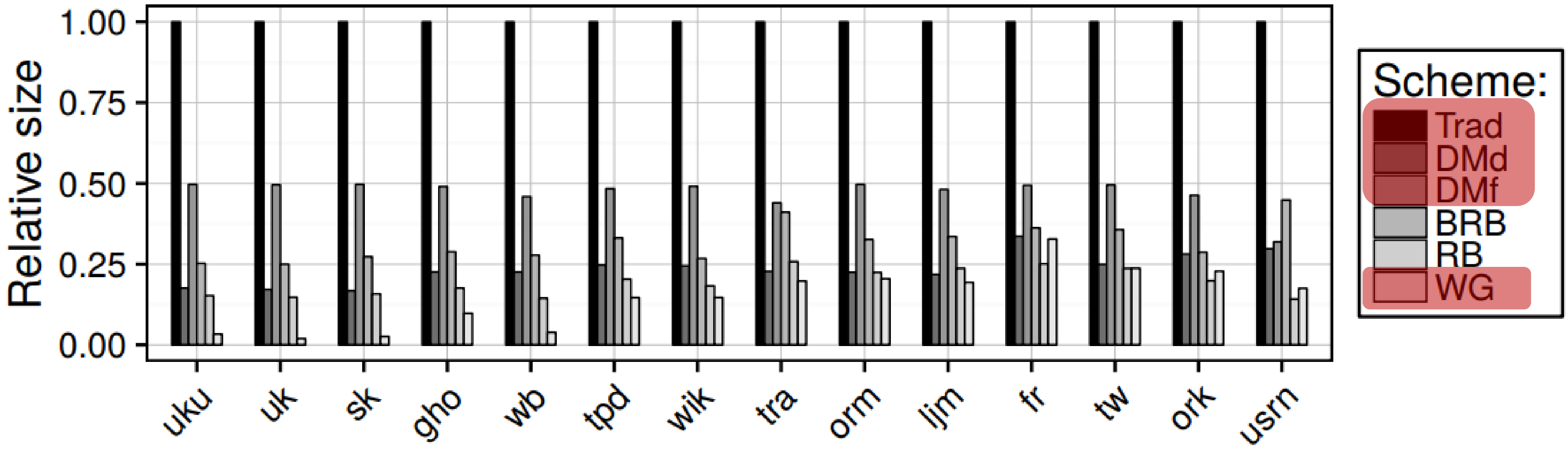

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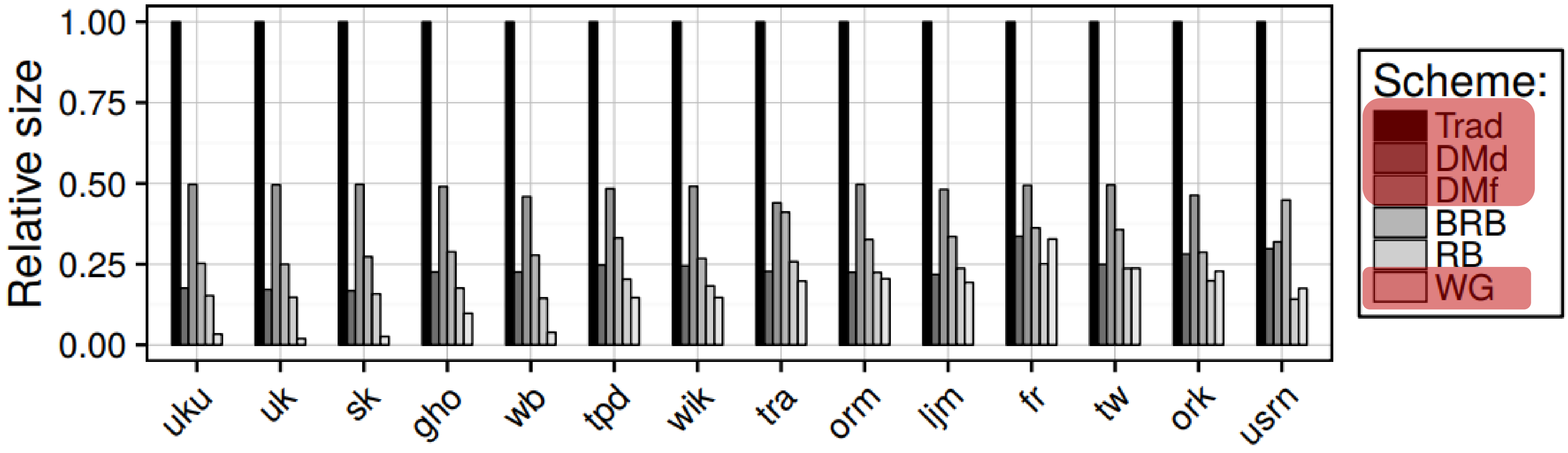

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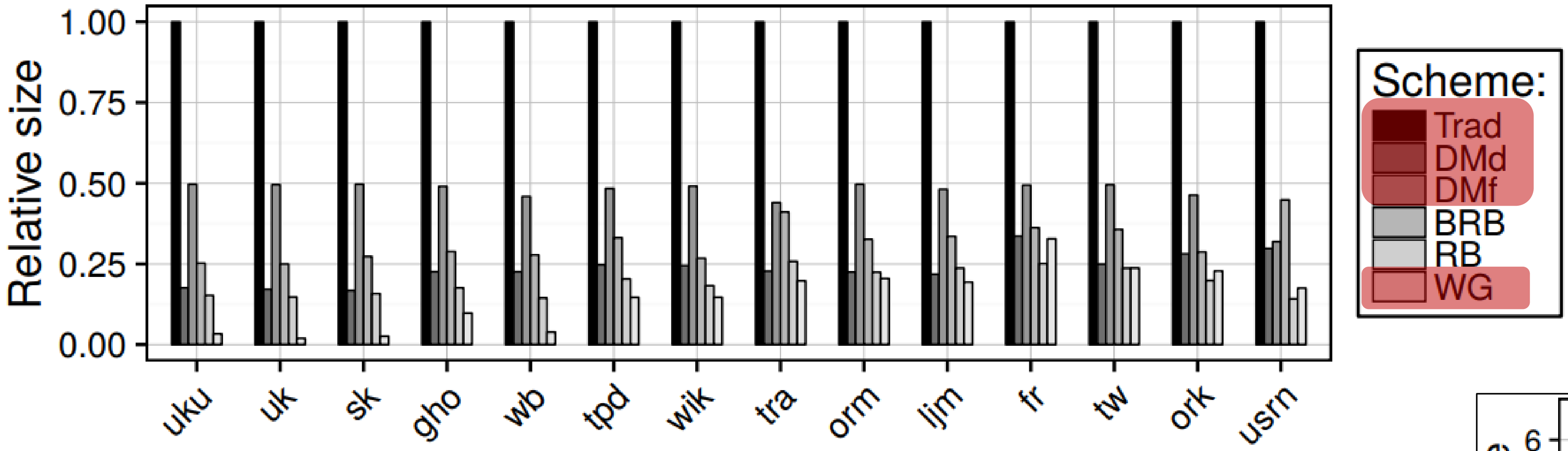

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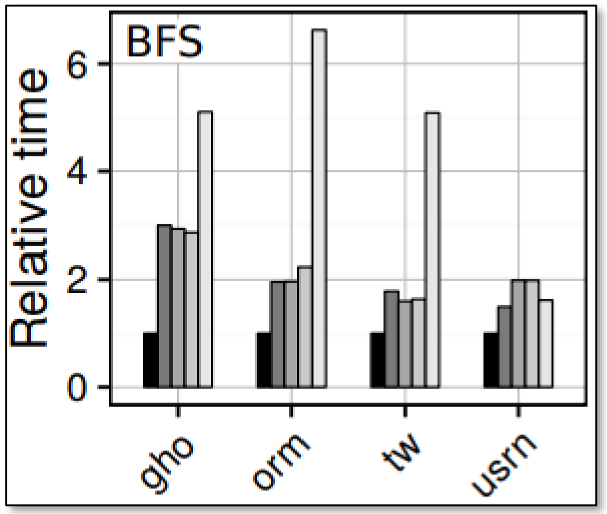
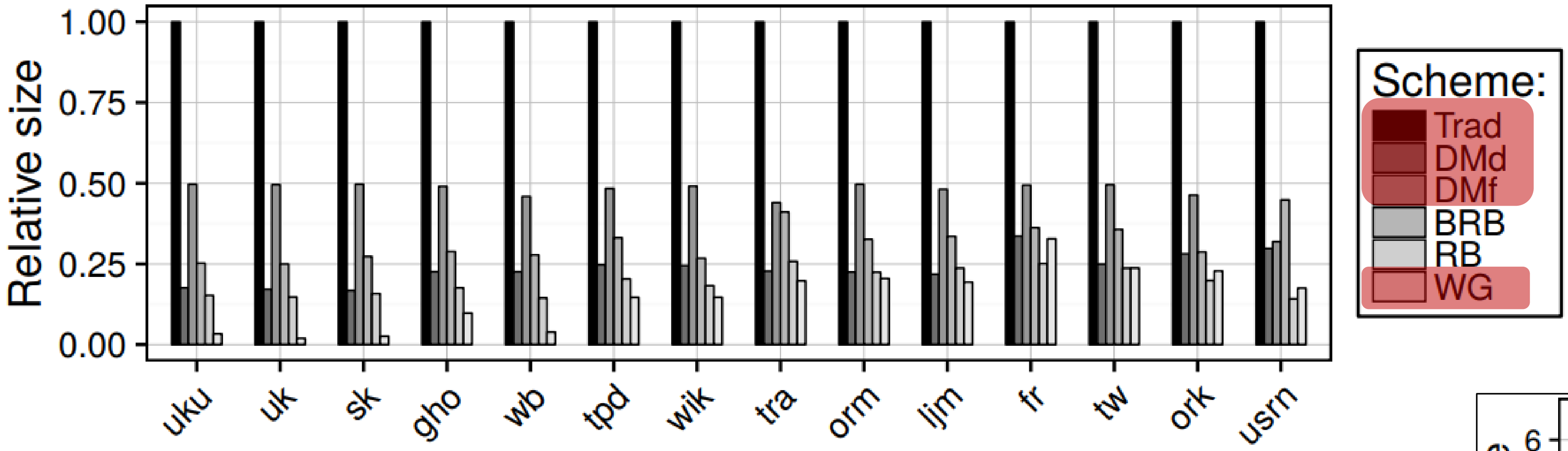


3 Log (Adjacency structure) Storage, Performane

Trad: Traditional adjacency array
DMd / DMf: Degree Minimizing (without / with gap encoding)
WG: WebGraph compression
BRB, RB: Schemes targeting certain specific classes of graphs



Various real-world graphs

WebGraph is the slowest, DM somewhat slower than Trad

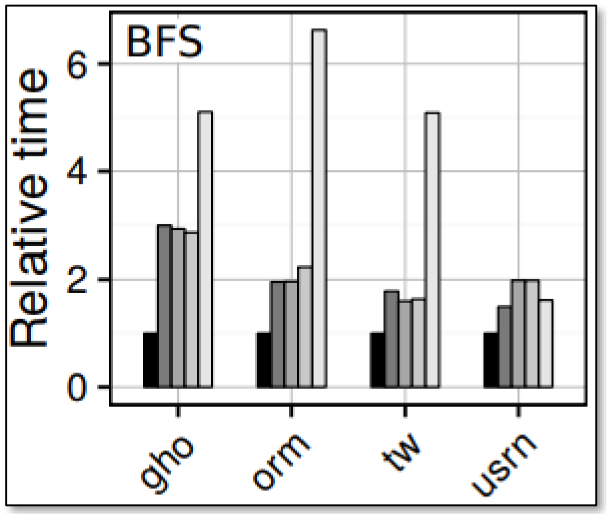
Lots of data ☺

Conclusions:

WebGraph **best for web graphs** ☺

DMd: much better than DMf, often comparable to WG

BRB, RB: various tradeoffs but very expensive preprocessing (details in the paper)



Takeaway (Results): Log(Graph) ensures Space-Performance sweetspot (tunable!)

Key insight (vertex labels)

20-35% storage reductions (compared to uncompressed data) and **negligible** decompression overheads

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Key insight (offsets)

Up to >90% storage reductions (compared to uncompressed data) and comparable performance to that of uncompressed data accesses (in parallel environments)

Key insight (vertex labels)

20-35% storage reductions (compared to uncompressed data) and **negligible** decompression overheads

Key insight (adjacency data)

80% storage reductions (compared to uncompressed data) and up to >2x speedup over modern graph compression schemes (Webgraph)

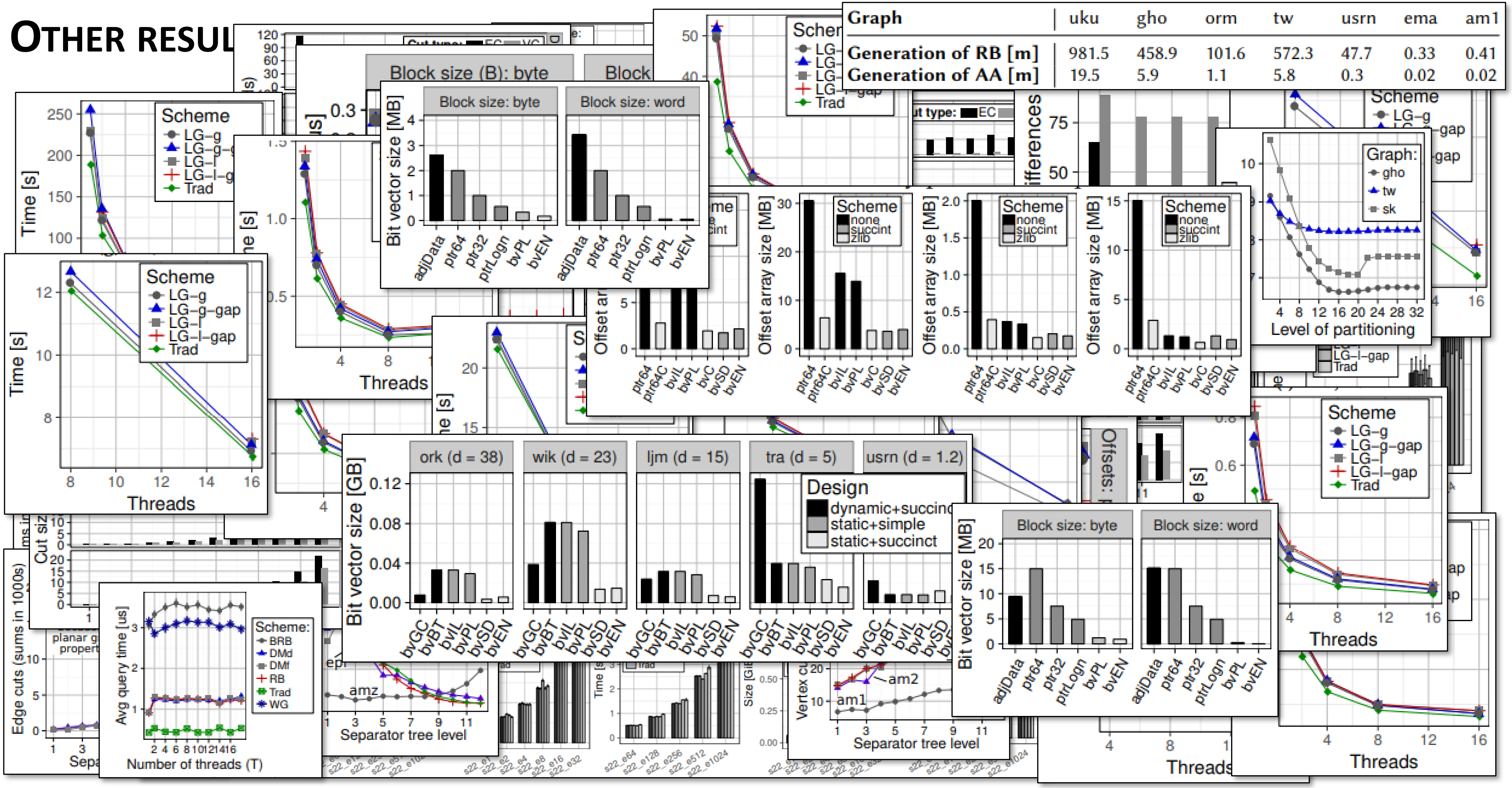
Takeaway (Results): Log(Graph) ensures Space-Performance sweetspot (tunable!)

Key insight (offsets)

Up to >90% storage reductions (compared to uncompressed data) and comparable performance to that of uncompressed data accesses (in parallel environments)

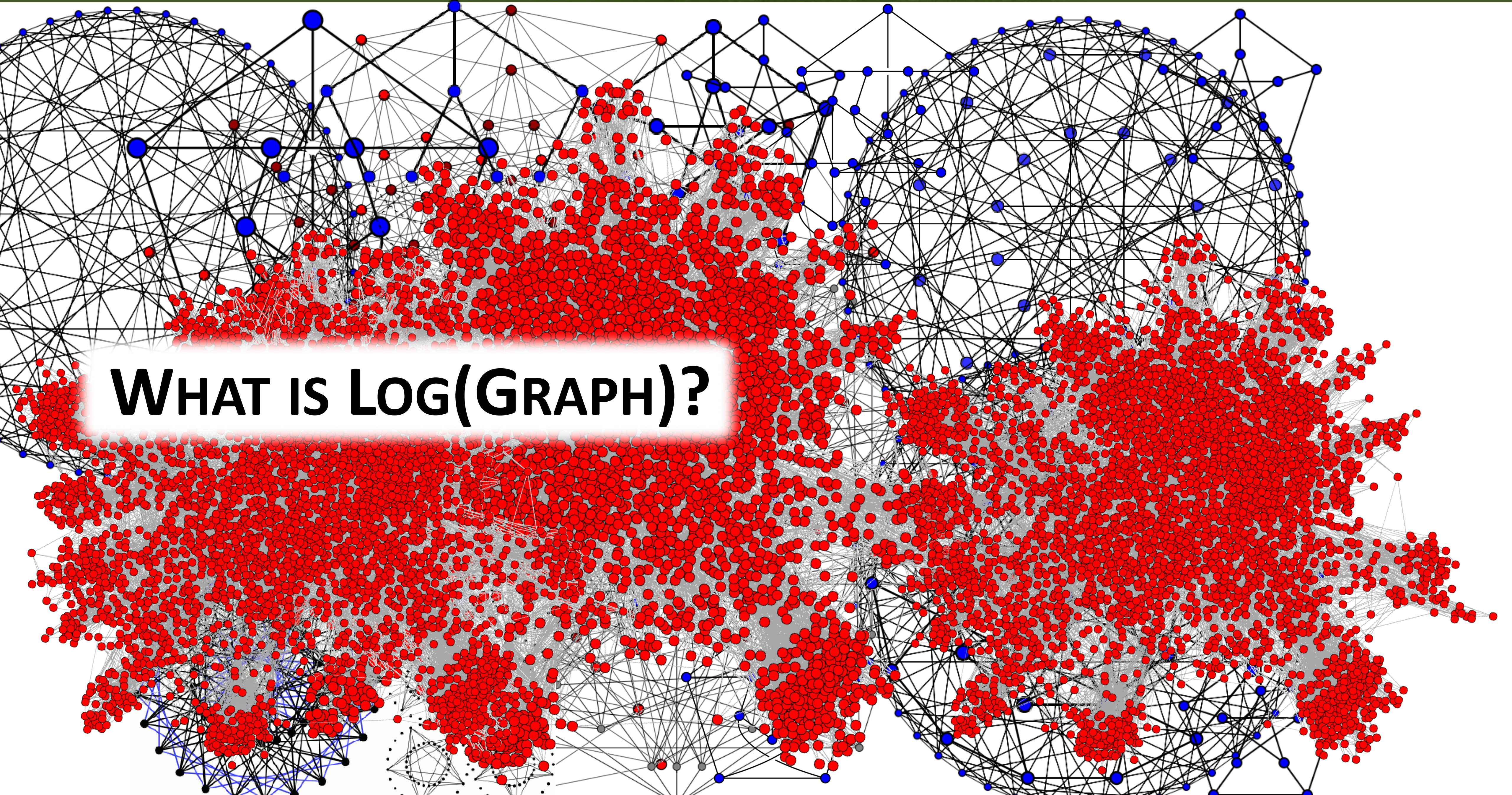
Other results

OTHER RESUL

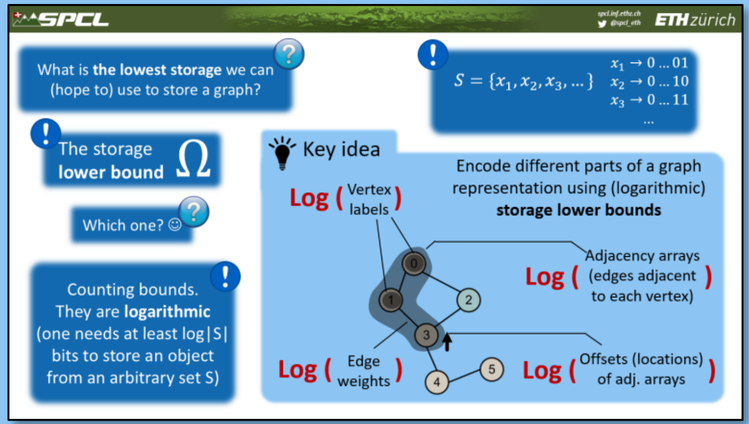


Graph	uku	gho	orm	tw	usrn	ema	am1
Generation of RB [m]	981.5	458.9	101.6	572.3	47.7	0.33	0.41
Generation of AA [m]	19.5	5.9	1.1	5.8	0.3	0.02	0.02

What is Log(Graph)?



A Near-Optimal Graph Representation



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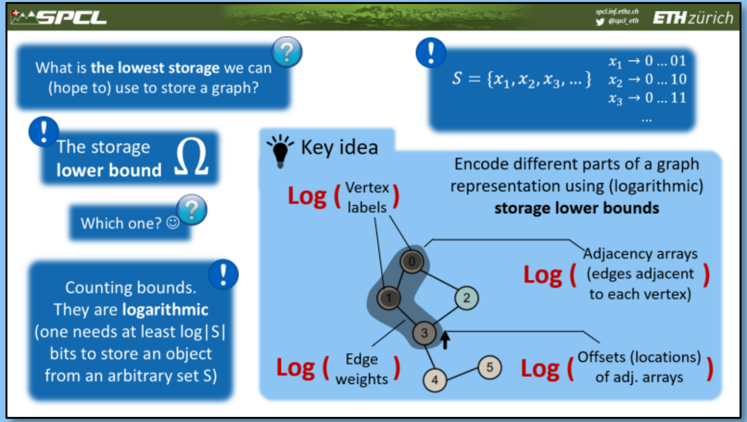
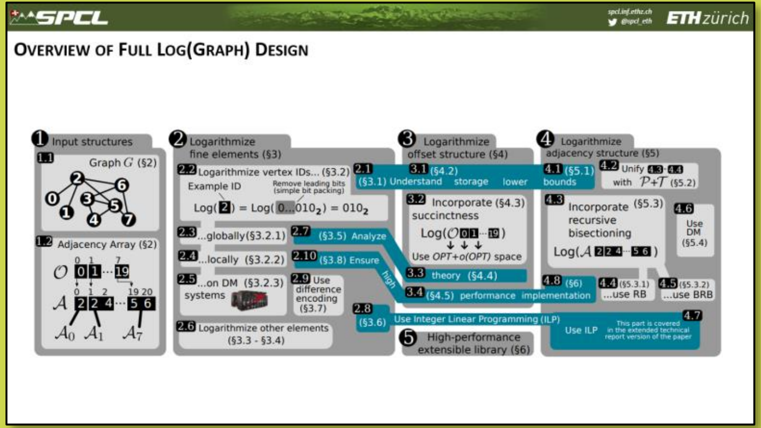


An Extensible Graph Representation



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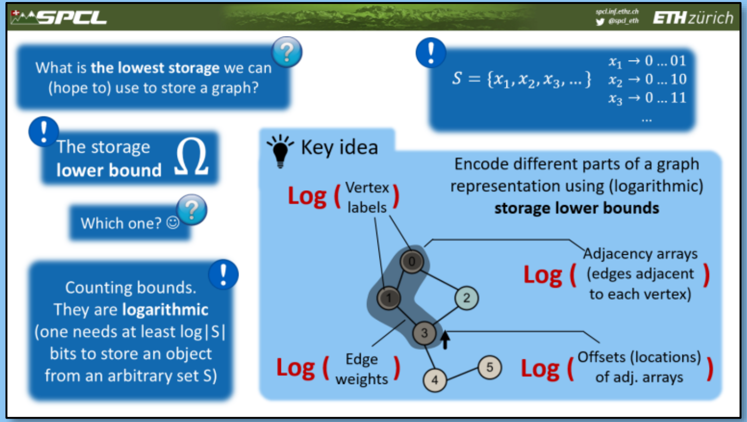

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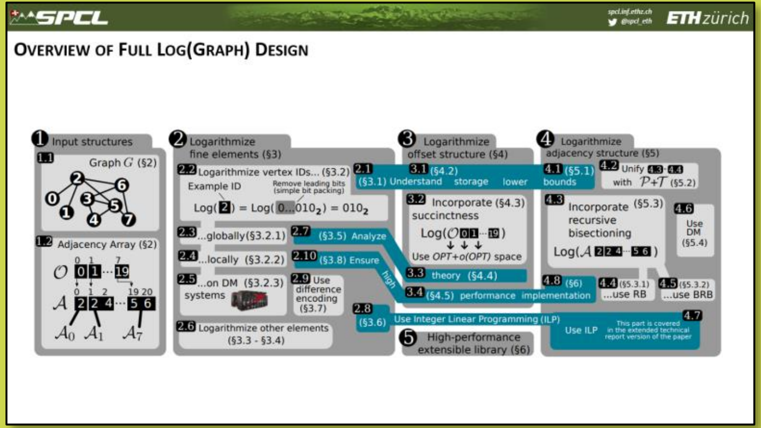

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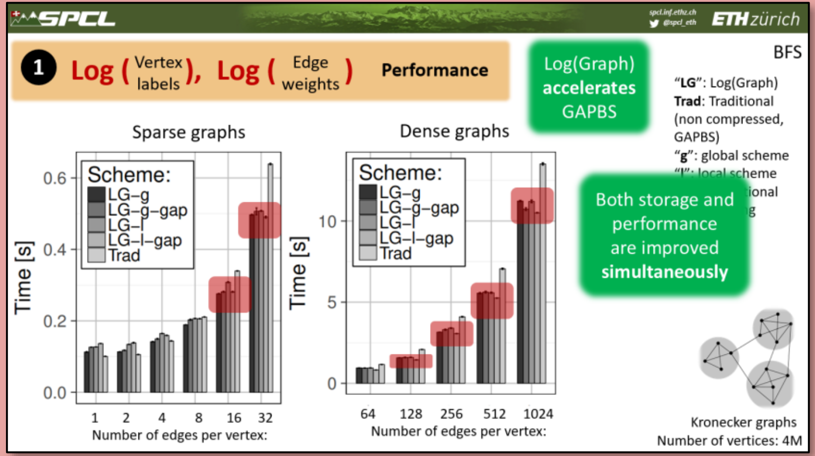
A High-Performance Graph Representation



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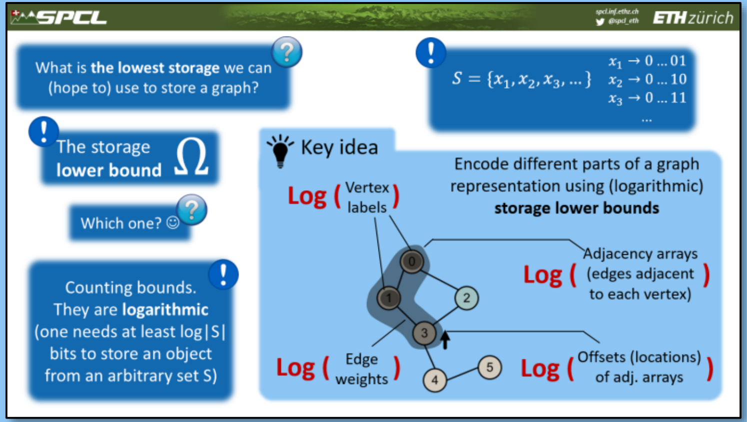

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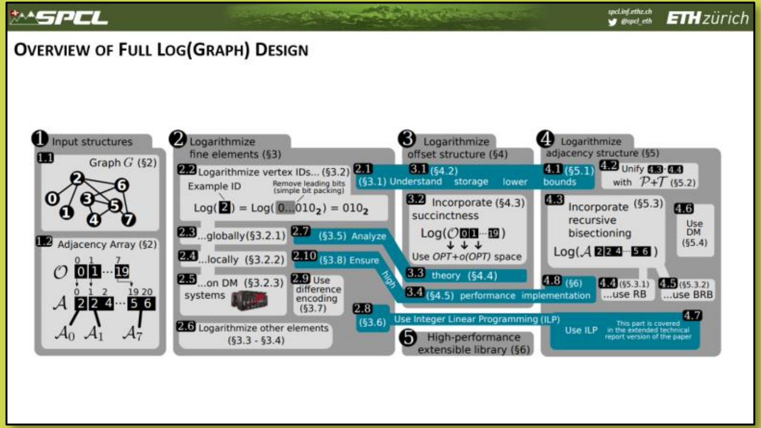

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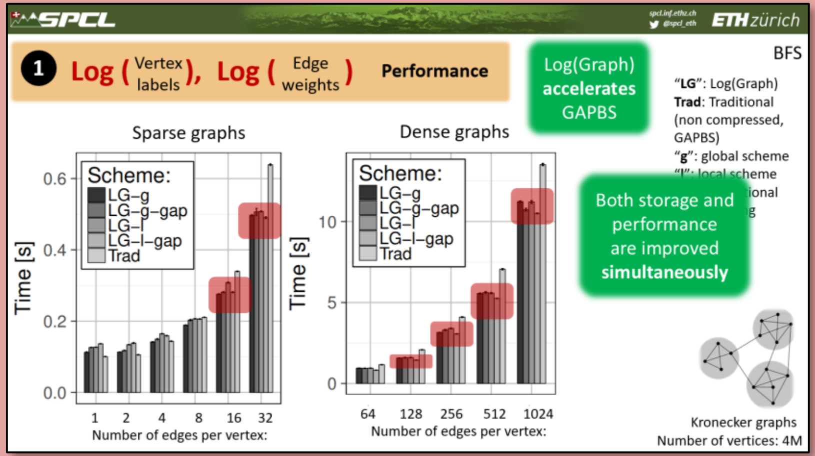
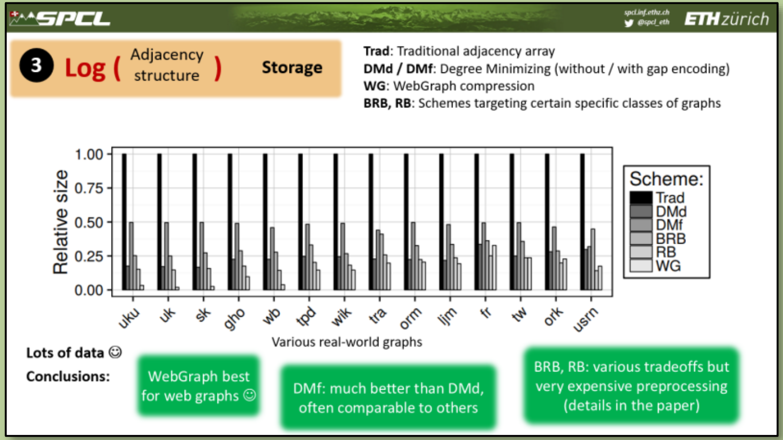


A Condensed Graph Representation



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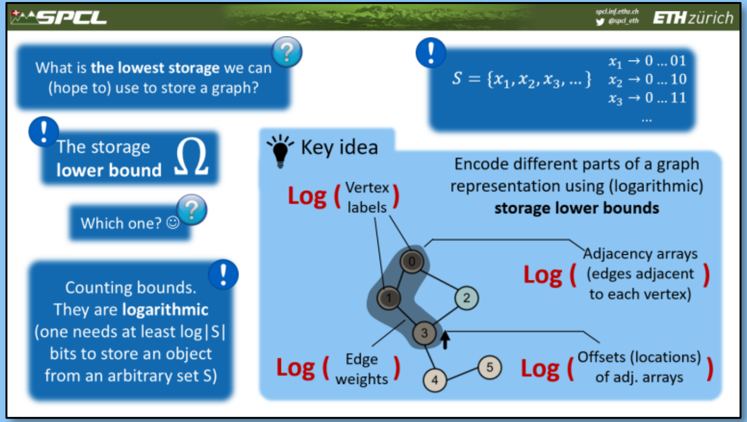

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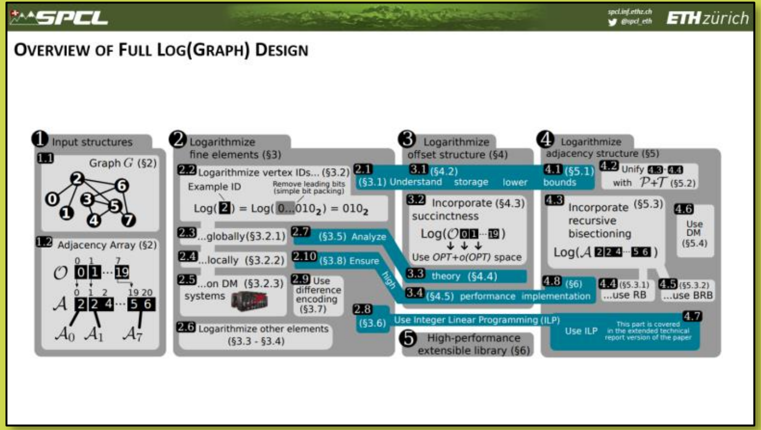

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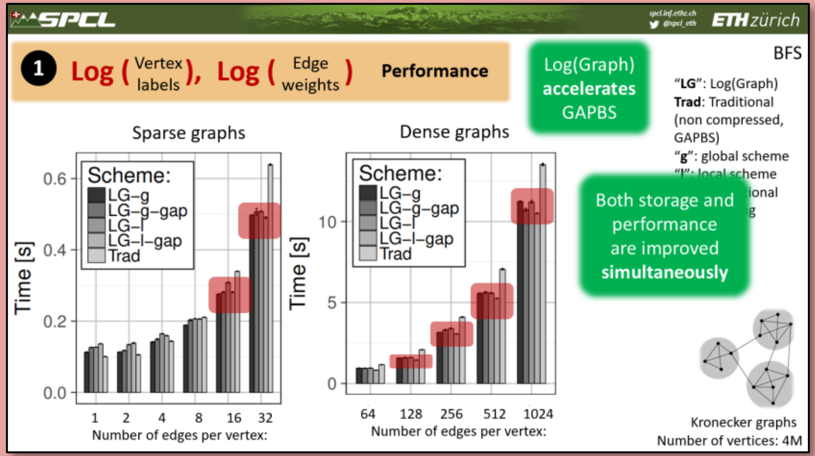

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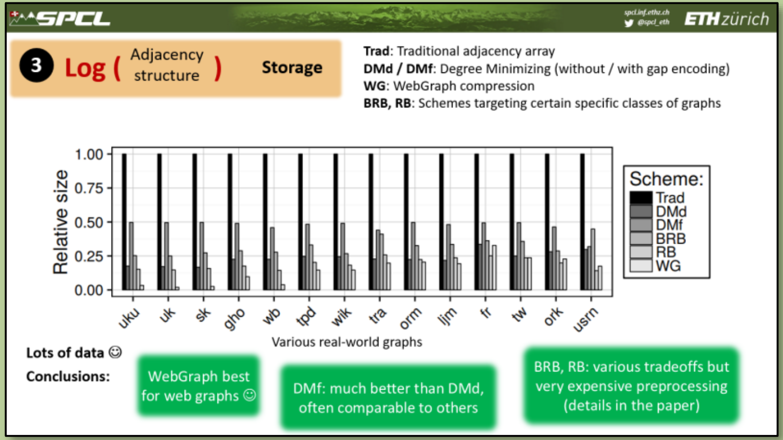

Website:

http://spcl.inf.ethz.ch/Research/Performance/LogGraph

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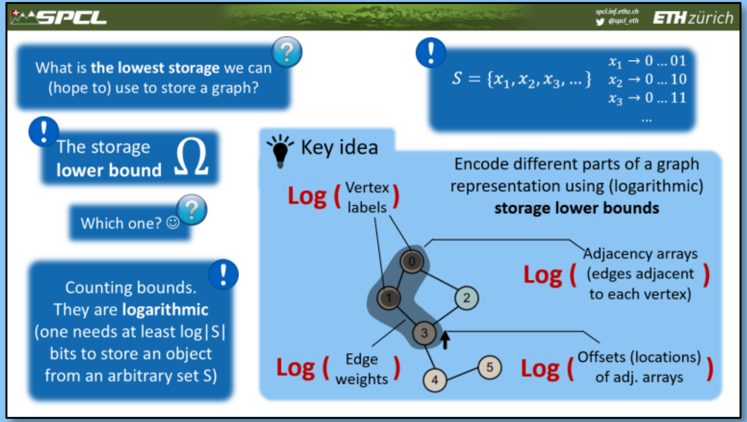

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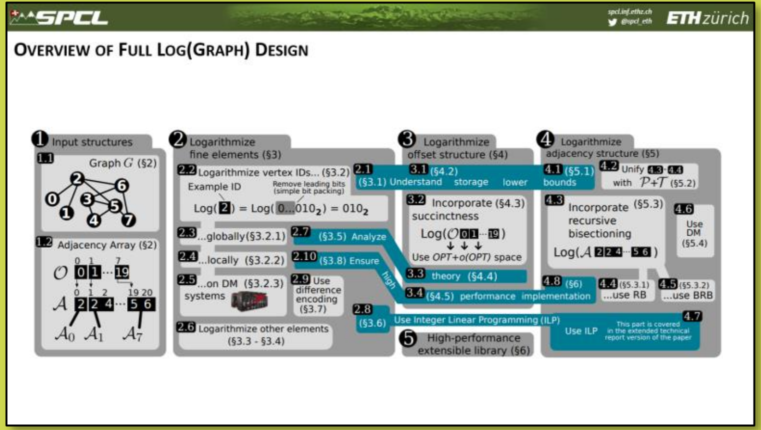

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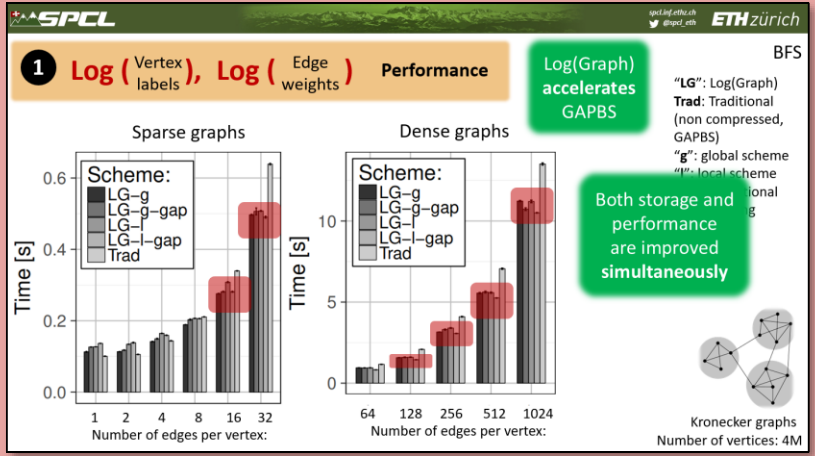

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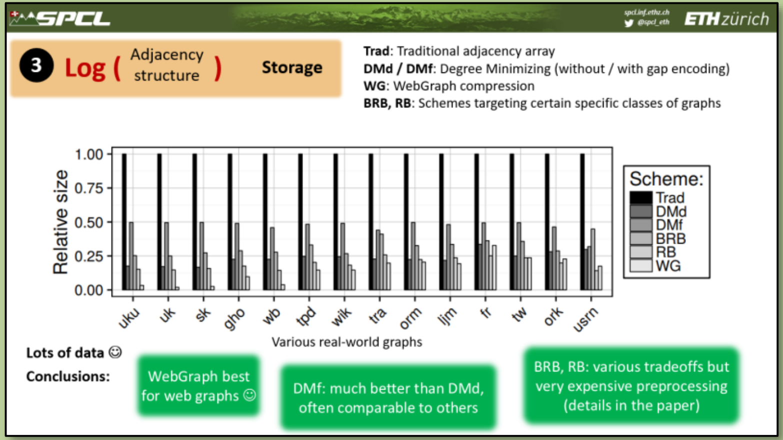

COMING SOON

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SPCL

spcl.inf.ethz.ch
@spcl_eth

ETH zürich

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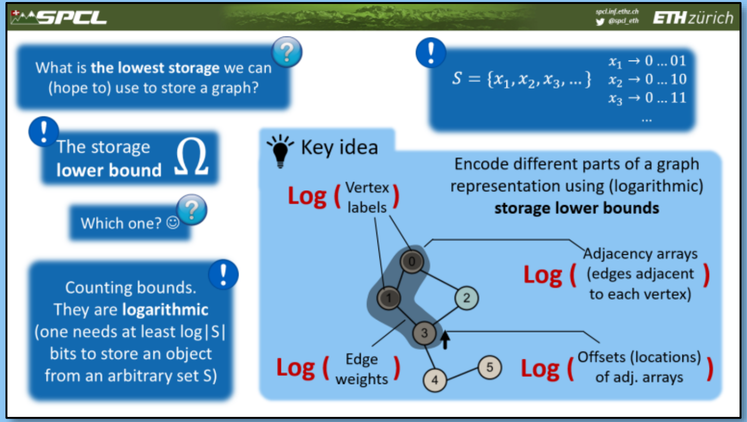

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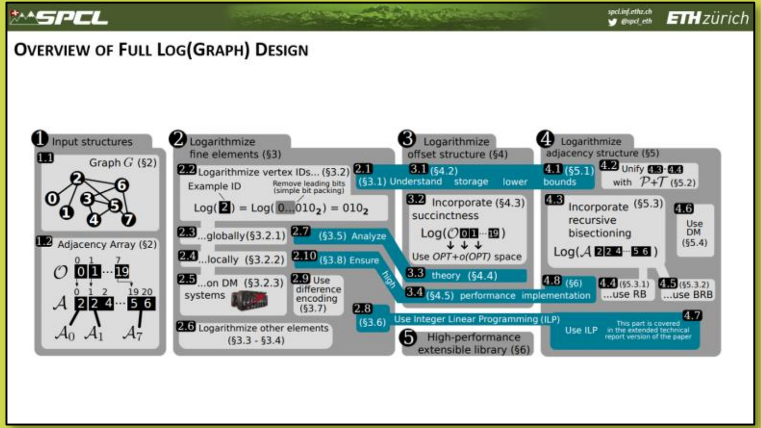

What is Log(Graph)?

Thank you for your attention

Website: COMING SOON

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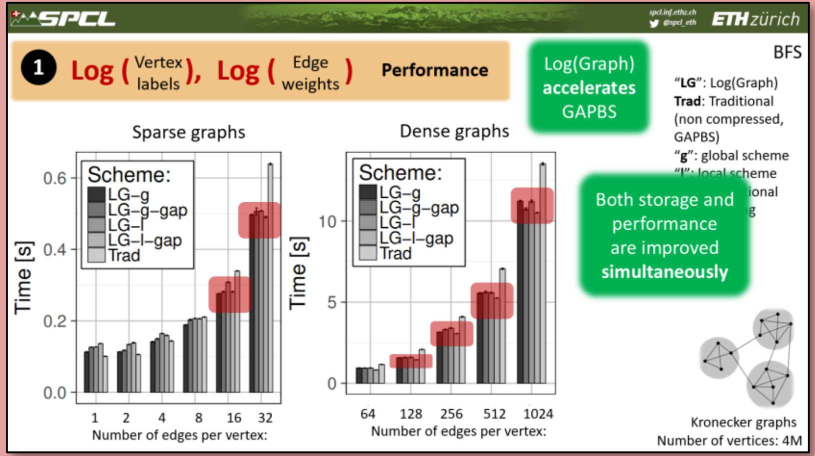

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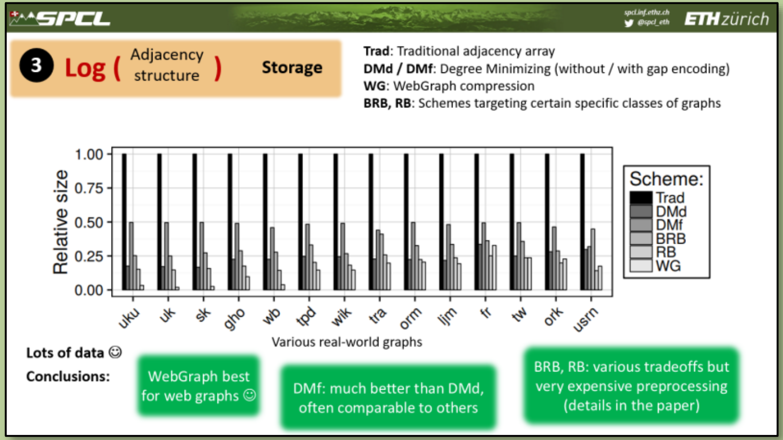

1 **Log (** Vertex labels **), Log (** Edge weights **)**

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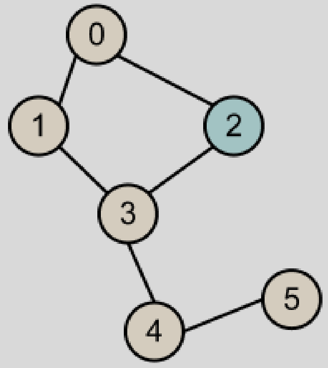

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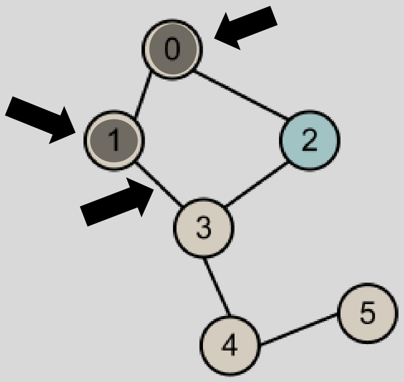

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Symbols

$\widehat{W}$: max edge weight,
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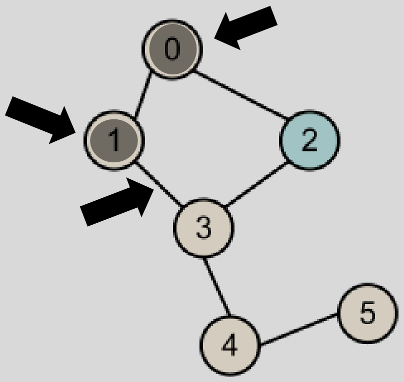

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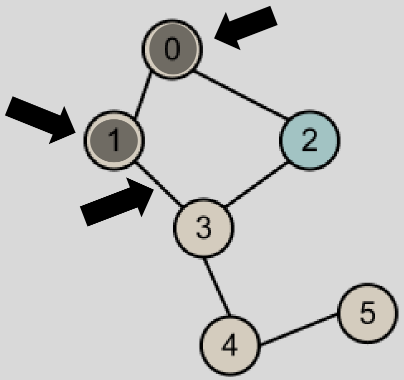

Lower bounds (global)

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$\lceil \log n \rceil$

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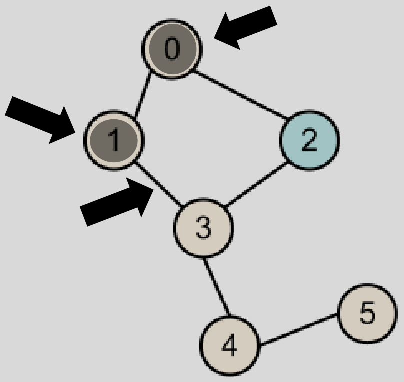

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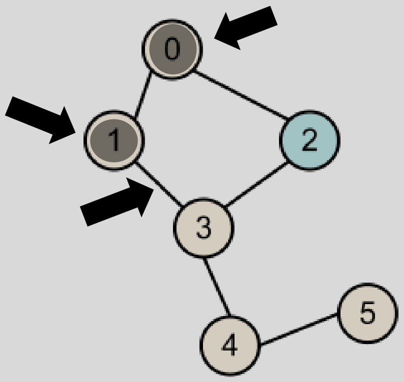

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Not really ☺

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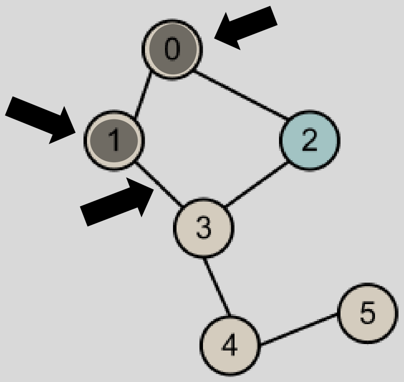

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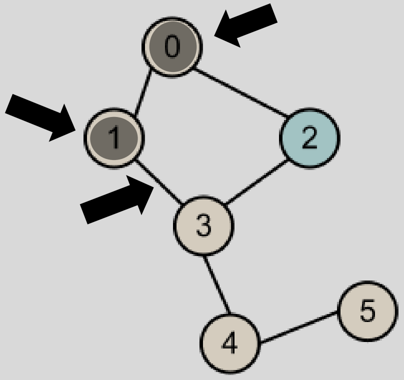

Lower bounds (global)

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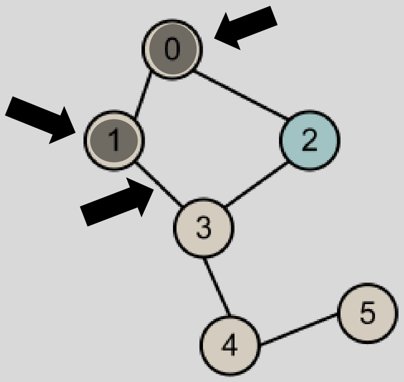

Lower bounds (global)

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Lower bounds (local)

Assume:

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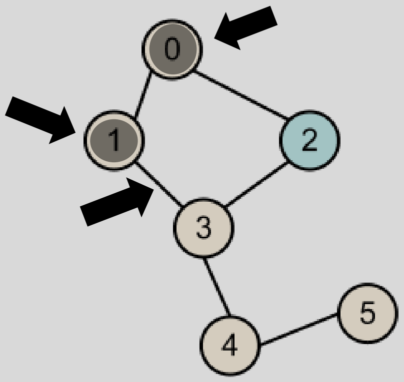

Lower bounds (local)

Assume:

- a graph, e.g., $V = \{1, \ldots, 2^{22}\}$

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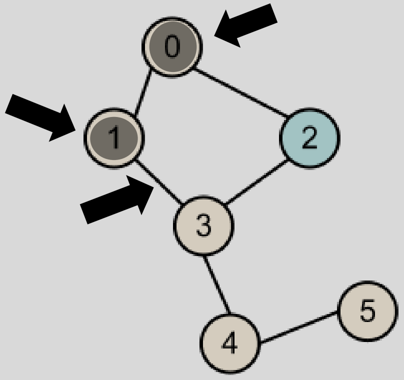

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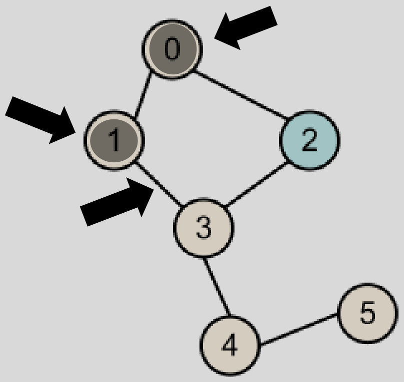

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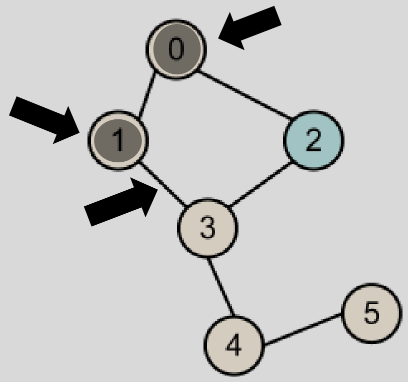

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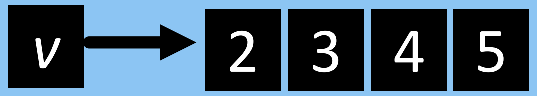

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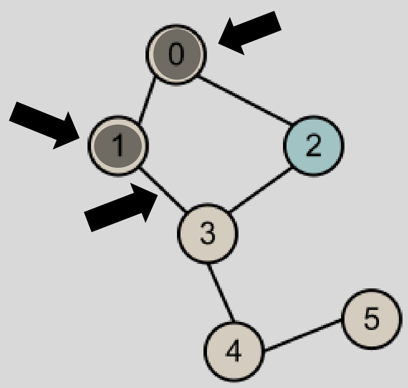

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$$\lceil \log 2^{22} \rceil = 22$$

v → | 2 | 3 | 4 | 5 |

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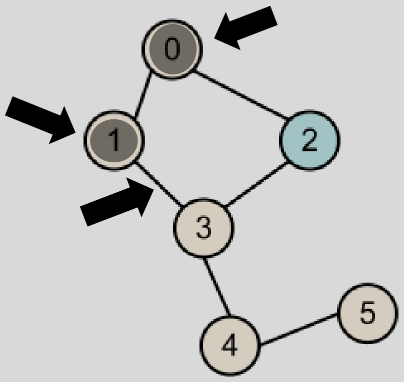

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$\lceil \log 2^{22} \rceil = 22$

v → | 0...10 | 0...11 | 0...100 | 0...101 |

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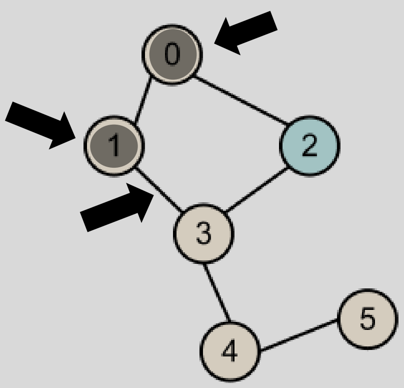

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v → 2 | 3 | 4 | 5

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v → 0...10 | 0...11 | 0...100 | 0...101

19 zeros!

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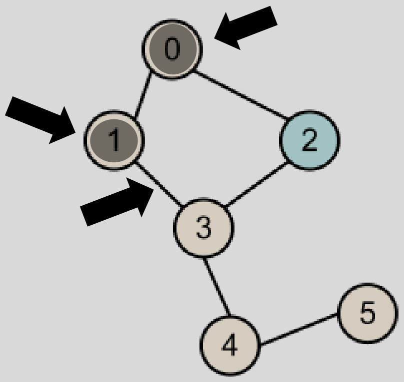

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- ...all these neighbors have small labels: $\widehat{N_v} \ll n$

v → 2 | 3 | 4 | 5

$\lceil \log 2^{22} \rceil = 22$

v → 0...10 | 0...11 | 0...100 | 0...101

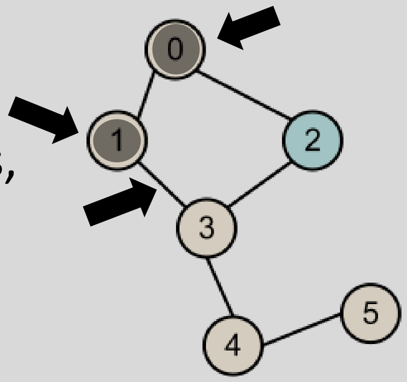
19 zeros!

Thus, use the local bound $\lceil \log \widehat{N_v} \rceil$

1 **Log (Vertex labels), Log (Edge weights)**

Symbols

n : #vertices,
m : #edges,
H : number of compute nodes,
H_i : number of machine elements at level i,
N : number of machine levels



1 **Log (Vertex labels), Log (Edge weights)**

This is it? Still not really ☺ ?

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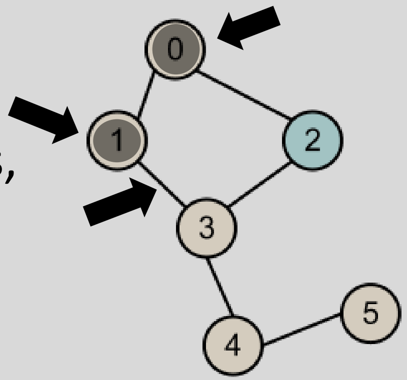

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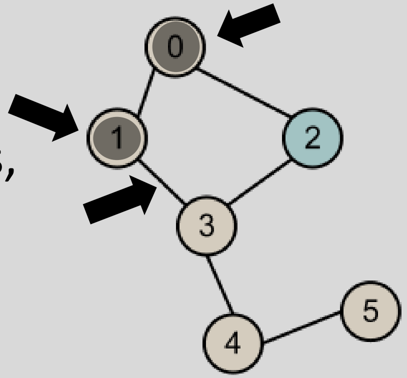

Lower bounds (local): distributed memories

1 **Log (** Vertex labels **), Log (** Edge weights **)**

This is it? Still not really ☺

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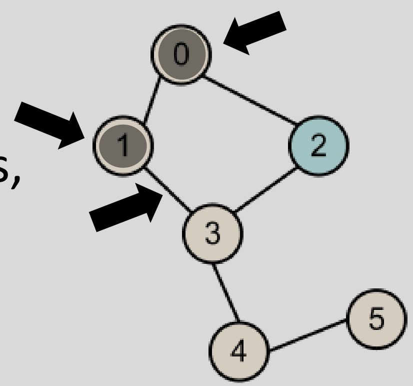

Lower bounds (local): distributed memories

A Cray XE/XT supercomputer

1 Log (Vertex labels), Log (Edge weights)

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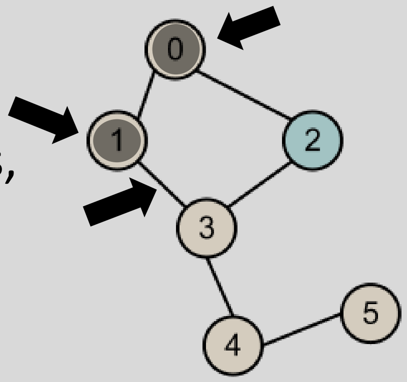

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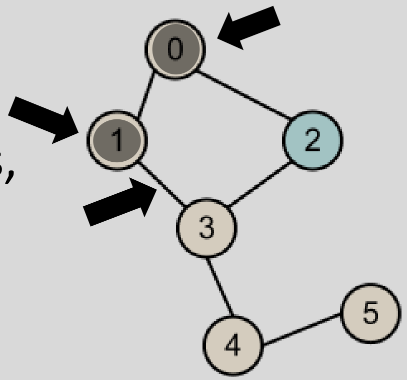

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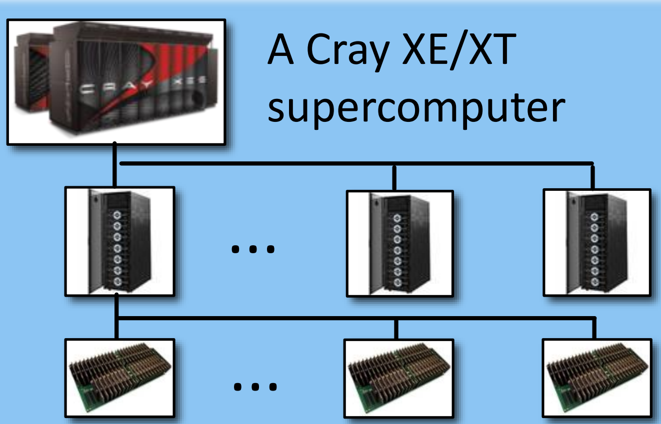

4 cabinets:

3 chassis:

1 Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

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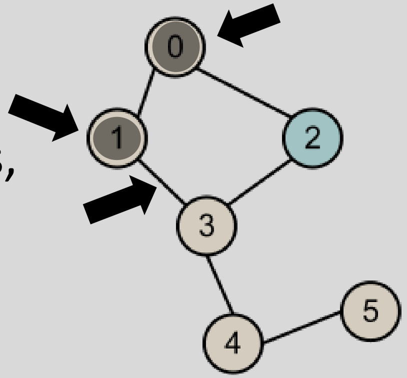
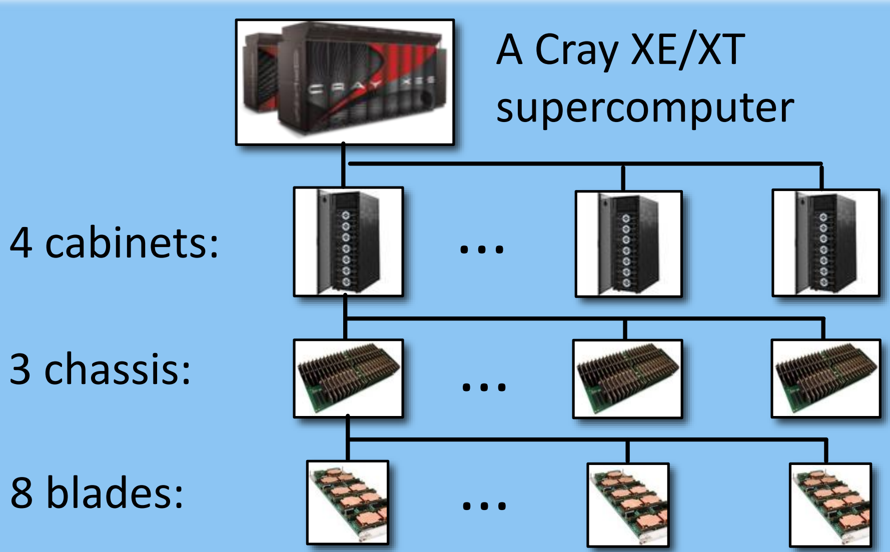


Lower bounds (local): distributed memories



1 **Log (Vertex labels), Log (Edge weights)**

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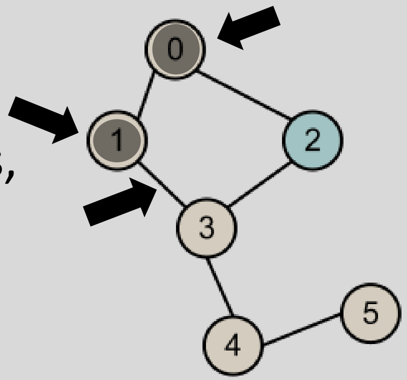
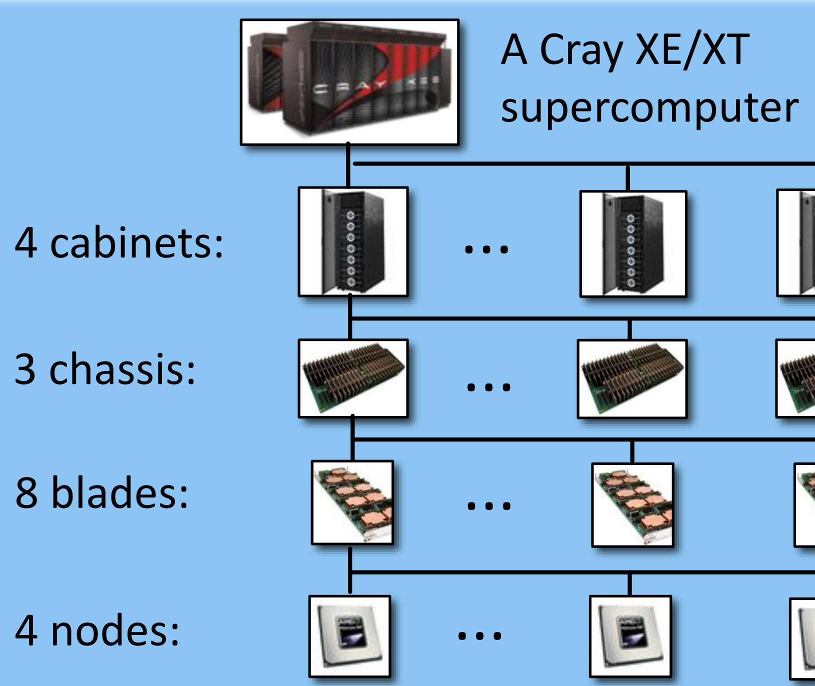


Lower bounds (local): distributed memories



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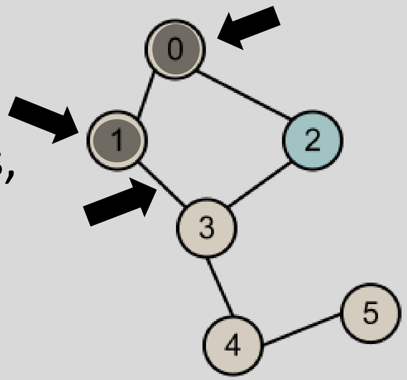

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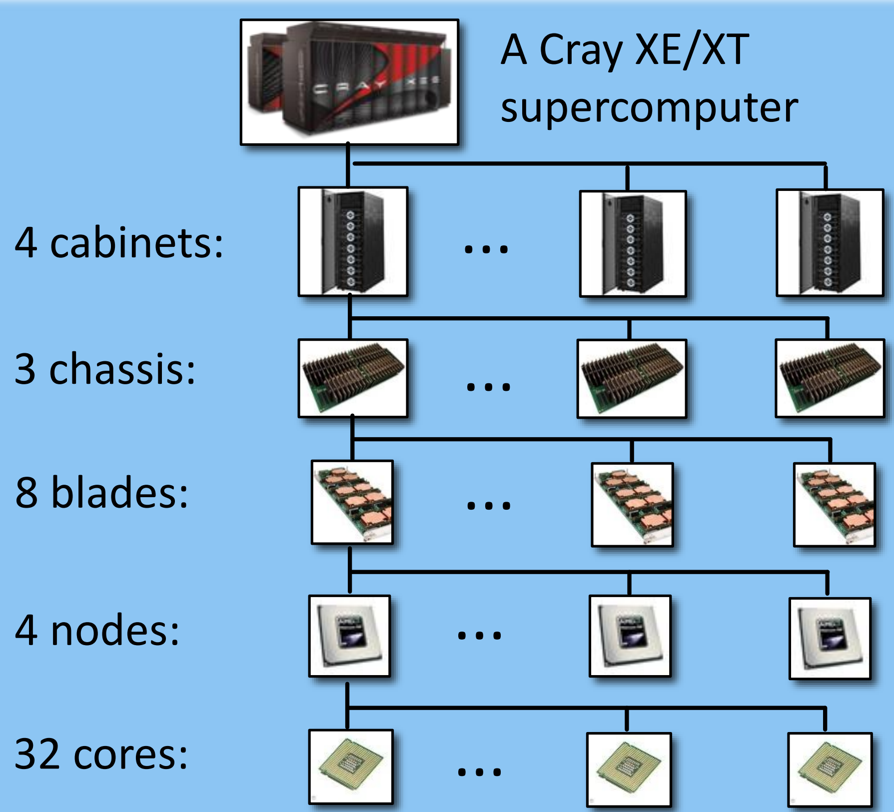

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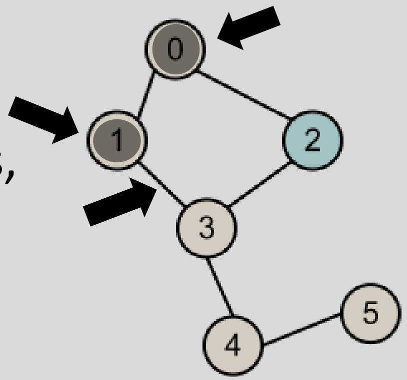

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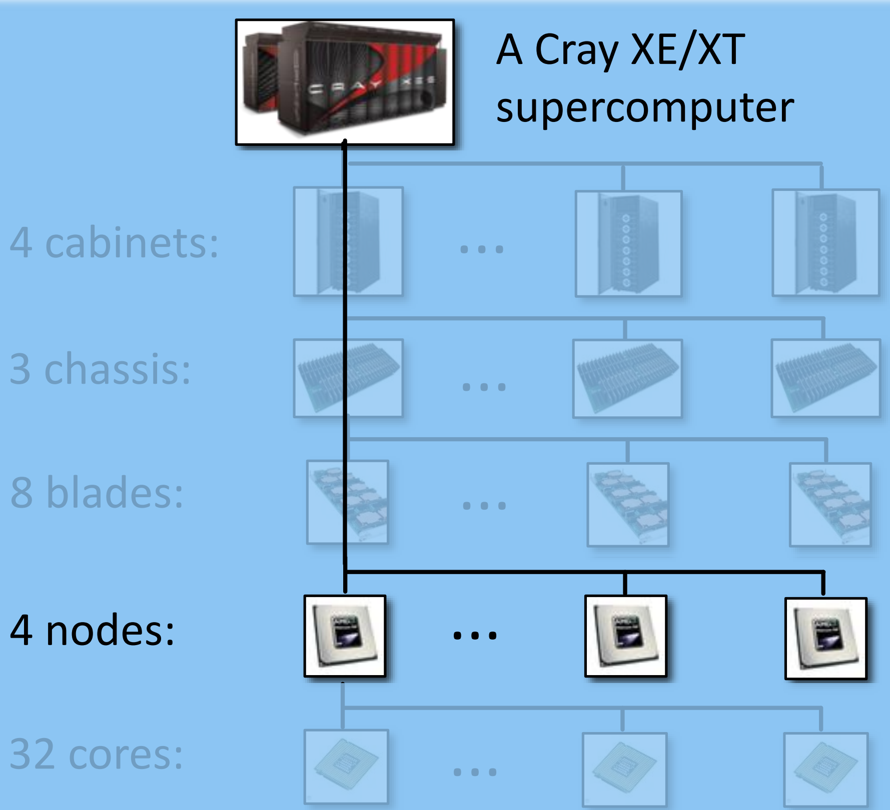

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❶ Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

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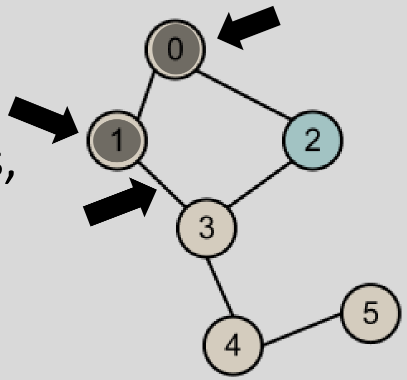

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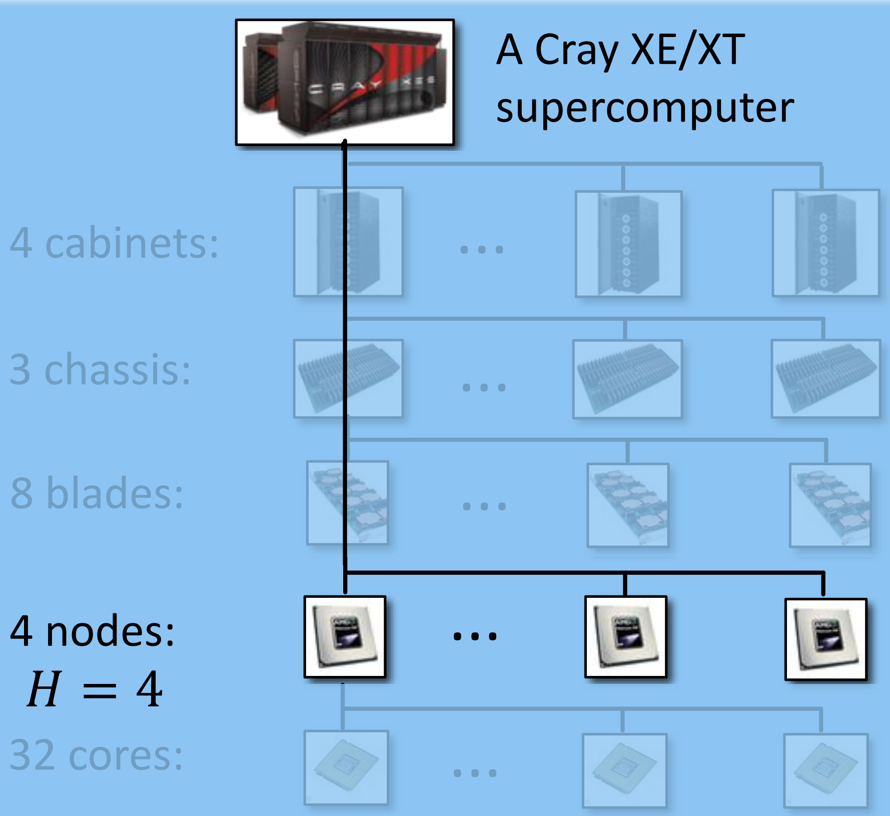

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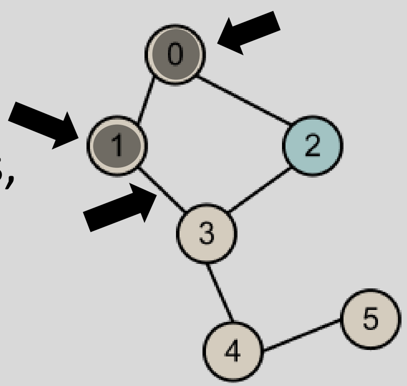
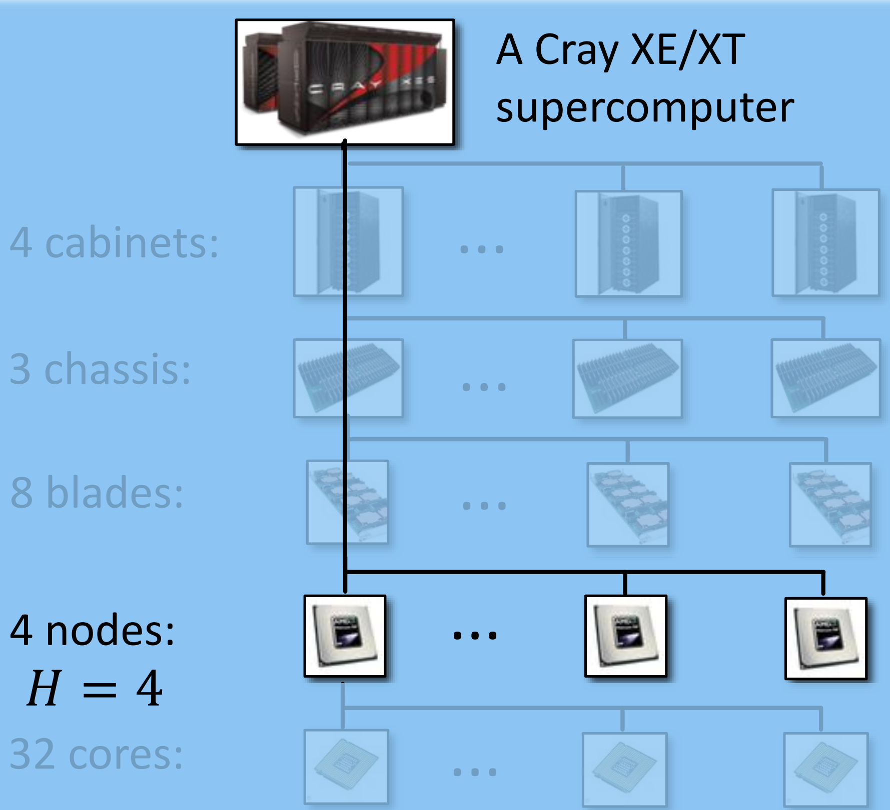


Lower bounds (local): distributed memories

The number of vertices that can be stored in the memory of one node:



1 Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

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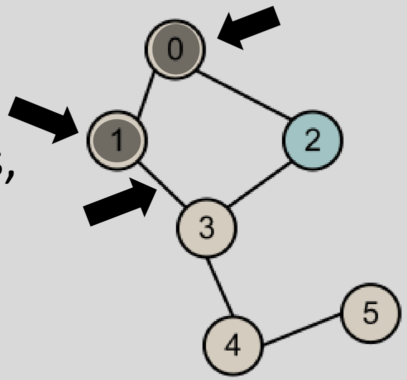
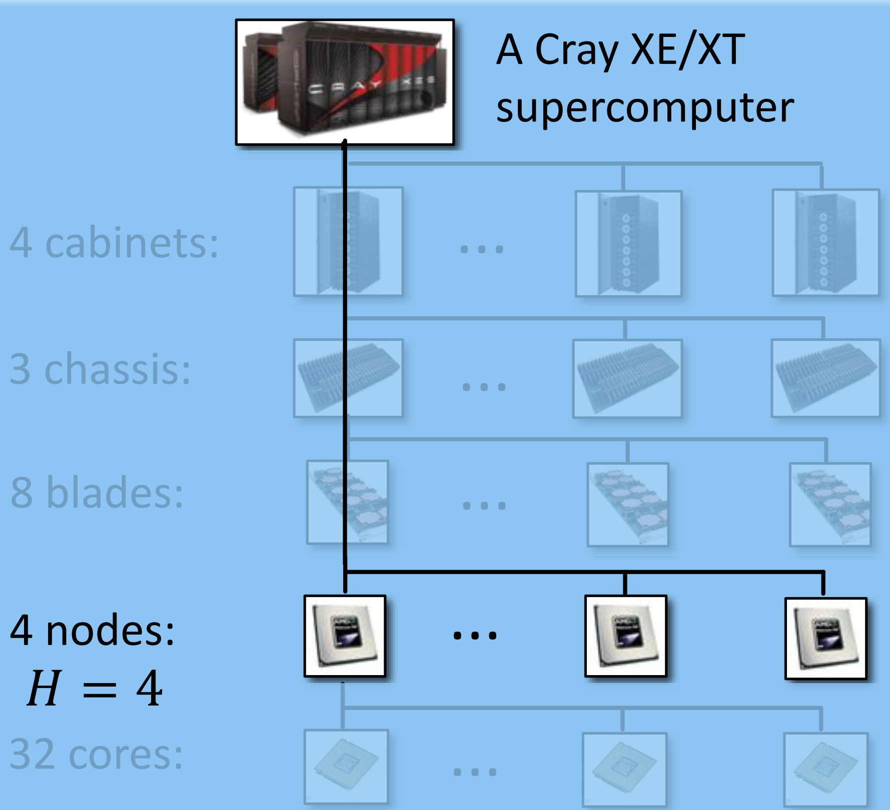


Lower bounds (local): distributed memories

The number of vertices that can be stored in the memory of one node: $\frac{n}{H}$



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1 Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

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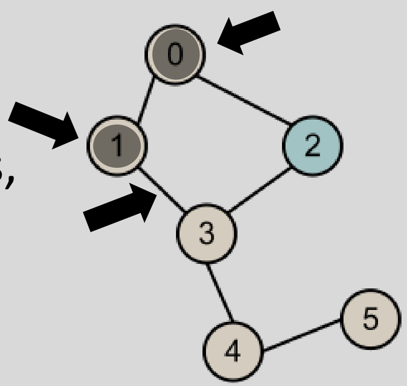
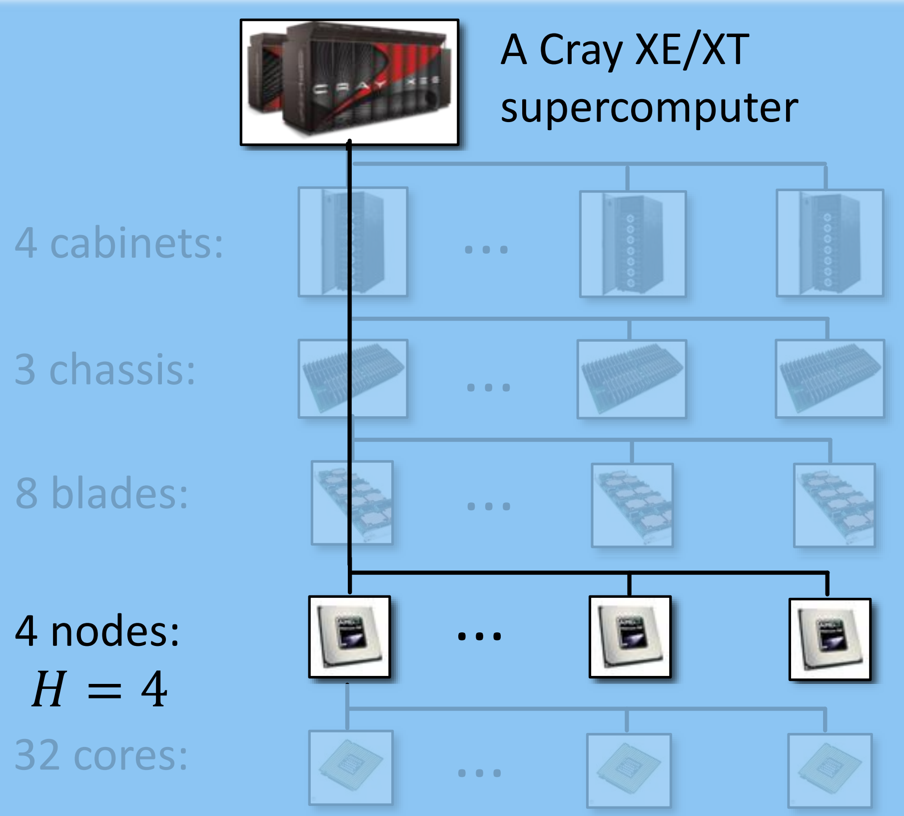


Lower bounds (local): distributed memories

The number of vertices that can be stored in the memory of one node: $\frac{n}{H}$

The „**intra-node**" vertex label thus takes [bits]: $\left\lceil \log \frac{n}{H} \right\rceil$



1 Log (Vertex labels), Log (Edge weights)

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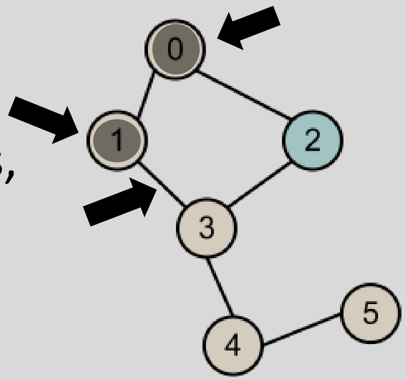
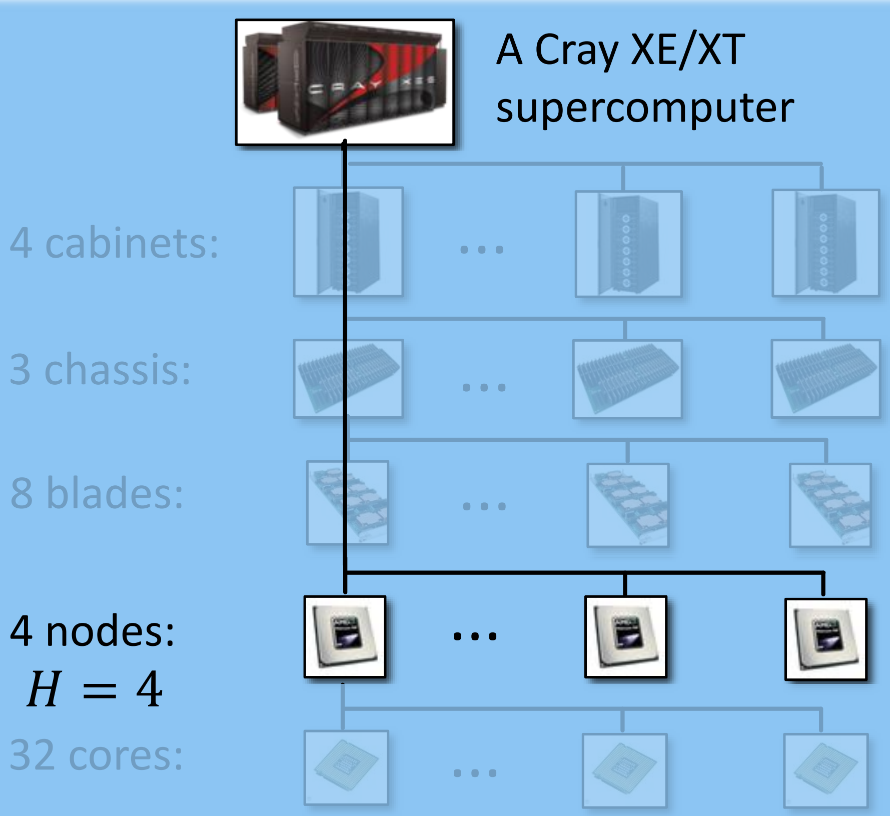


Lower bounds (local): distributed memories

The number of vertices that can be stored in the memory of one node: $\frac{n}{H}$

The „**intra-node**" vertex label thus takes [bits]: $\left\lceil \log \frac{n}{H} \right\rceil$

The „**inter-node**" vertex label is unique for a whole node and it takes [bits]: $\lceil \log H \rceil$



1 Log (Vertex labels), Log (Edge weights)

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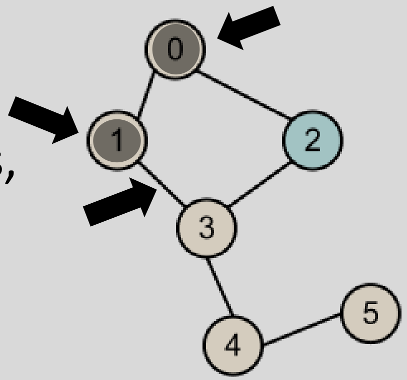

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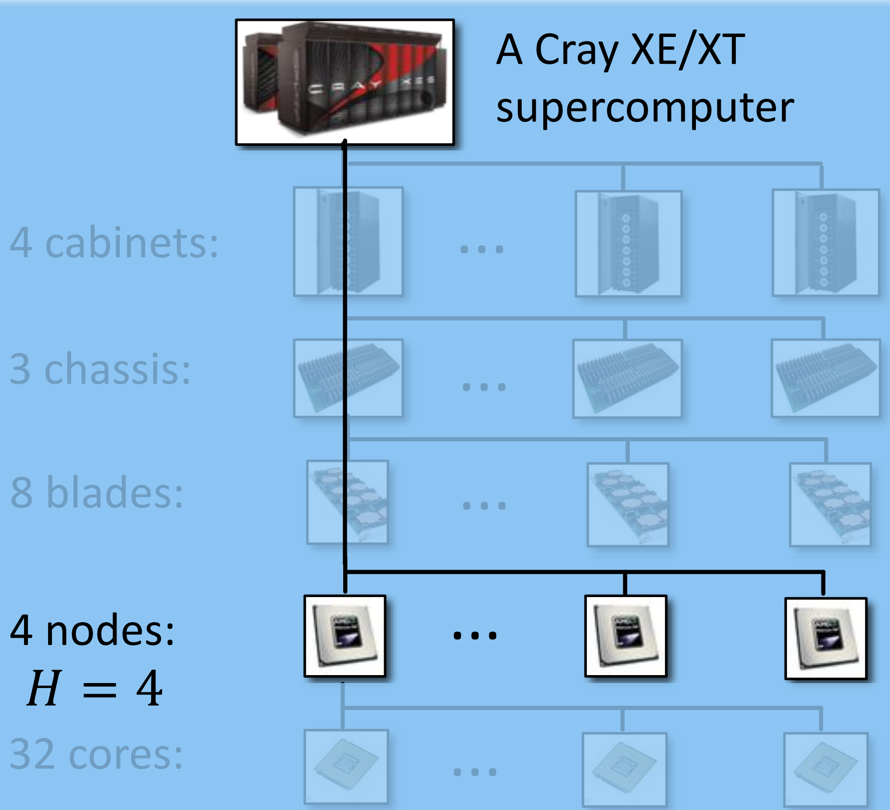

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The **total size of the adjacency arrays** is thus [bits]:

$$n \left\lceil \log \frac{n}{H} \right\rceil + H \lceil \log H \rceil$$

1 Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

Symbols

n : #vertices,
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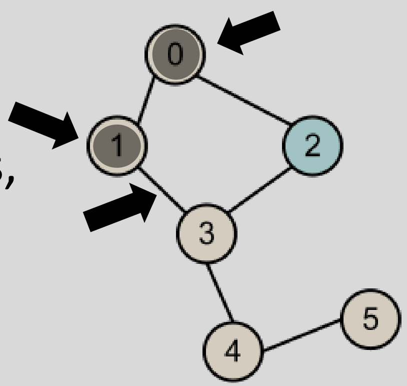

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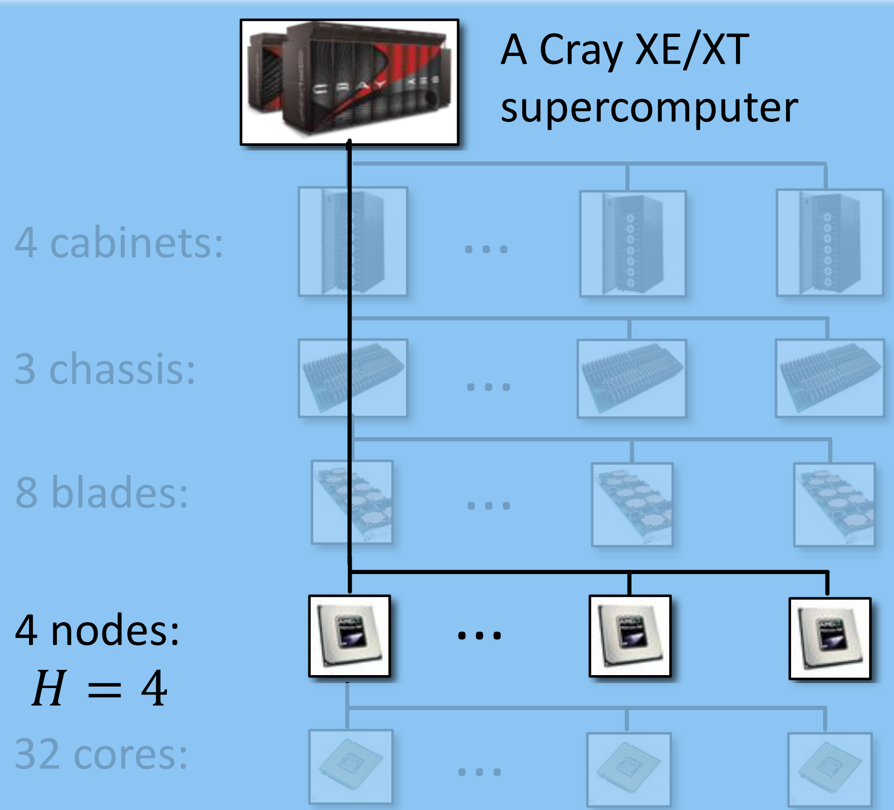

A Cray XE/XT supercomputer

The **total size of the adjacency arrays** is thus [bits]:

$$n \left\lceil \log \frac{n}{H} \right\rceil + H \lceil \log H \rceil$$

We also generalize this to arbitrarily many levels (details in the paper ☺) and derive the total size:

1 Log (Vertex labels), Log (Edge weights)

This is it? Still not really ☺

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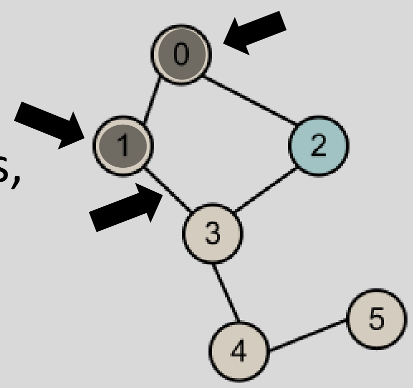

Lower bounds (local): distributed memories

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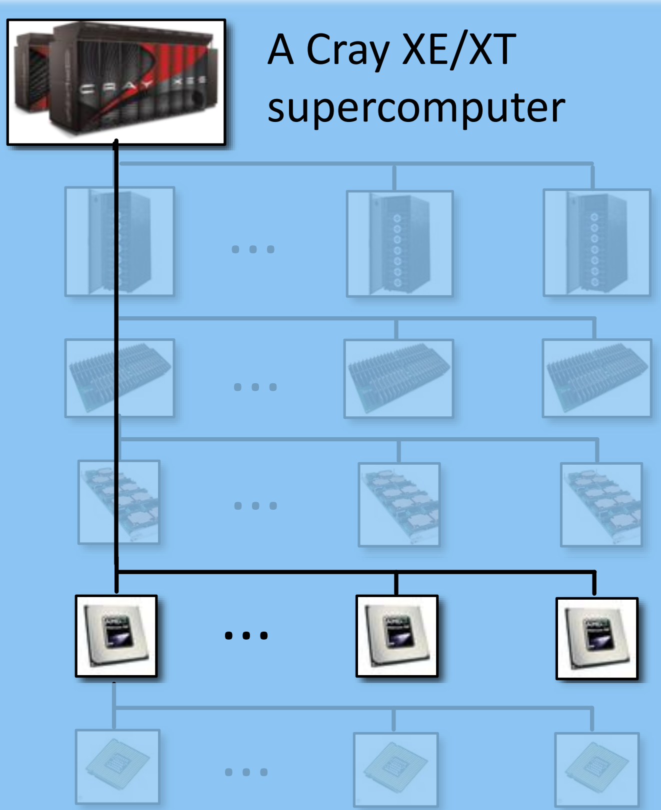
A Cray XE/XT supercomputer



4 cabinets:

3 chassis:

8 blades:

4 nodes: $H = 4$

32 cores:

The **total size of the adjacency arrays** is thus [bits]:

$$n \left\lceil \log \frac{n}{H} \right\rceil + H \lceil \log H \rceil$$

We also generalize this to arbitrarily many levels (details in the paper ☺) and derive the total size:

$$n \left\lceil \log \frac{n}{H_N} \right\rceil + \sum_{j=2}^{N-1} H_j \lceil \log H_j \rceil$$

1 **Log (** Vertex labels **), Log (** Edge weights **)**

∰ Formal analyses: more (check the paper ☺)

1 **Log (** Vertex labels **), Log (** Edge weights **)**

Formal analyses: more (check the paper ☺)

$$E[|\mathcal{O}|] = n\left\lceil \log\left(2pn^2\right)\right\rceil = n\lceil \log 2p + 2\log n\rceil$$

$$\forall_{v,u\in V}\,(u \in N_v) \Rightarrow \left[\mathcal{N}(u) \le \widehat{N}_v\right]$$

$$|\mathscr{A}| = \sum_{v\in V}\left(d_v\left\lceil \log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{N}_v\right\rceil\right)$$

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$$|\mathcal{A}| = n\left\lceil \log \frac{n}{\mathcal{H}}\right\rceil + \mathcal{H}\lceil \log \mathcal{H}\rceil$$

$$|\mathcal{A}| = 2m\left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right)$$

$$|\mathcal{A}| = \sum_{v\in V}\left(d_v\left(\left\lceil \log \widehat{N}_v\right\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right) + \left\lceil \log\log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{\mathcal{W}}\right\rceil\right)$$

$$E[|\mathcal{A}|] \approx \frac{\alpha}{2-\beta}\left(\left(\frac{\alpha n \log n}{\beta - 1}\right)^{\frac{2-\beta}{\beta-1}} - 1\right)\left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right)$$

$$E[|\mathcal{A}|] = \left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right) pn^2$$

1 Log (Vertex labels), Log (Edge weights)

Formal analyses: more (check the paper ☺)

$$E[|\mathcal{O}|] = n\left\lceil \log\left(2pn^2\right)\right\rceil = n\left\lceil \log 2p + 2\log n\right\rceil$$

$$\forall_{v,u\in V}\left(u \in N_v\right) \Rightarrow \left[\mathcal{N}(u) \le \widehat{N}_v\right]$$

$$| = \sum_{v\in V}\left(d_v\left\lceil \log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{N}_v\right\rceil\right)$$

$$|\mathcal{A}| = 2m\left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right)$$

$$\widehat{N}_v\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right) + \left\lceil \log\log \widehat{N}_v\right\rceil + \left\lceil \log\log \widehat{\mathcal{W}}\right\rceil\right)$$

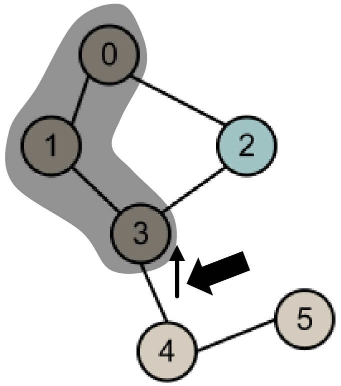
$$E[|\mathcal{A}|] = \left(\lceil \log n\rceil + \left\lceil \log \widehat{\mathcal{W}}\right\rceil\right) pn^2$$

```
/* Input: graph G, Output: a new relabeling N(v), ∀v ∈ V. */
void relabel(G) {
 ID[0..n − 1] = [0..n − 1]; //An array with vertex IDs.
 D[0..n − 1] = [d_0..d_{n−1}]; //An array with degrees of vertices.
 //An auxiliary array for determining if a vertex was relabeled:
 visit[0..n − 1] = [false..false];
 nl = 1; //An auxiliary variable ``new label''.
 sort(ID); sort(D);
 for(int i = 1; i < n; ++i) //For each vertex...
  for(int j = 0; j < D[i]; ++j) { //For each neighbor...
   int id = N_{j,ID[i]}; //N_{j,ID[i]} is jth neighbor of vertex with ID ID[i]
   if(visit[id] == false) {
    N(id) = nl++;
    visit[id] = true;
 }}
 for(int i = 1; i < n; ++i)
  if(visit[i] == false)
   N(id) = nl++;
}
```

2 Log (Offset structure)

...Encode the resulting bit vectors as **succinct bit vectors**

!



2 Log (Offset structure)

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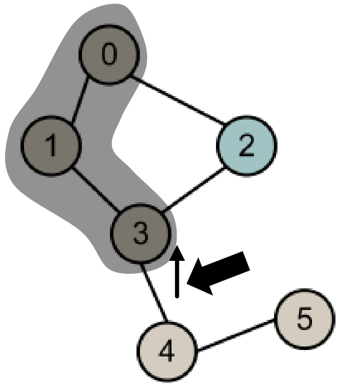
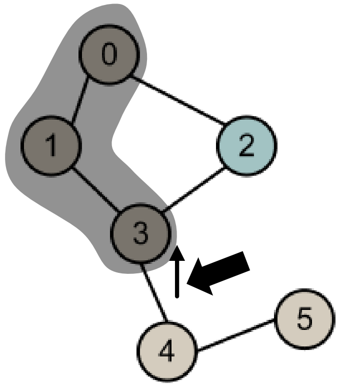


Formal analyses

2 Log (Offset structure)

...Encode the resulting bit vectors as **succinct bit vectors** !

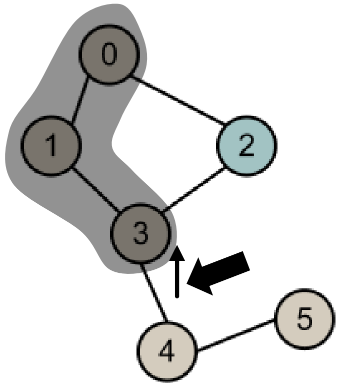


∰ Formal analyses

$\mathcal{O}$	ID	**Asymptotic size** [bits]	**Exact size** [bits]	*select* or $\mathcal{O}[v]$
Pointer array	ptr$W$	$O(Wn)$	$W(n+1)$	$O(1)$
Plain [44]	bvPL	$O\left(\frac{Wm}{B}\right)$	$\frac{2Wm}{B}$	$O(1)$
Interleaved [44]	bvIL	$O\left(\frac{Wm}{B}+\frac{Wm}{L}\right)$	$2Wm\left(\frac{1}{B}+\frac{64}{L}\right)$	$O\left(\log\frac{Wm}{B}\right)$
Entropy based [31, 78]	bvEN	$O\left(\frac{Wm}{B}\log\frac{Wm}{B}\right)$	$\approx \log\binom{\frac{2Wm}{B}}{n}$	$O\left(\log\frac{Wm}{B}\right)$
Sparse [76]	bvSD	$O\left(n+n\log\frac{Wm}{Bn}\right)$	$\approx n\left(2+\log\frac{2Wm}{Bn}\right)$	$O(1)$
B-tree based [1]	bvBT	$O\left(\frac{Wm}{B}\right)$	$\approx 1.1\cdot\frac{2Wm}{B}$	$O(\log n)$
Gap-compressed [1]	bvGC	$O\left(\frac{Wm}{B}\log\frac{Wm}{Bn}\right)$	$\approx 1.3\cdot\frac{2Wm}{B}\log\frac{2Wm}{Bn}$	$O(\log n)$

2 Log (Offset structure)

...Encode the resulting bit vectors as **succinct bit vectors** !



Formal analyses

Check the paper for details ☺

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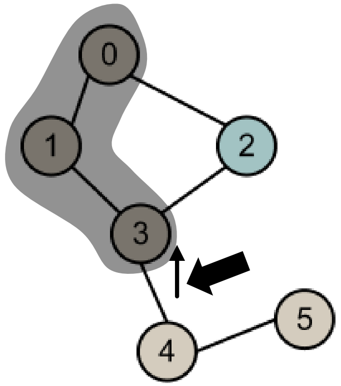
Interleaved [44]	bvIL	$O\left(\frac{Wm}{B} + \frac{Wm}{L}\right)$	$2Wm\left(\frac{1}{B} + \frac{64}{L}\right)$	$O\left(\log \frac{Wm}{B}\right)$
Entropy based [31, 78]	bvEN	$O\left(\frac{Wm}{B} \log \frac{Wm}{B}\right)$	$\approx \log \binom{\frac{2Wm}{B}}{n}$	$O\left(\log \frac{Wm}{B}\right)$
Sparse [76]	bvSD	$O\left(n + n \log \frac{Wm}{Bn}\right)$	$\approx n\left(2 + \log \frac{2Wm}{Bn}\right)$	$O(1)$
B-tree based [1]	bvBT	$O\left(\frac{Wm}{B}\right)$	$\approx 1.1 \cdot \frac{2Wm}{B}$	$O(\log n)$
Gap-compressed [1]	bvGC	$O\left(\frac{Wm}{B} \log \frac{Wm}{Bn}\right)$	$\approx 1.3 \cdot \frac{2Wm}{B} \log \frac{2Wm}{Bn}$	$O(\log n)$

2 Log (Offset structure)

...Encode the resulting bit vectors as **succinct bit vectors** !



Formal analyses

Check the paper for details ☺

$\mathcal{O}$	ID	**Asymptotic size** [bits]	**Exact size** [bits]	*select* or $\mathcal{O}[v]$
Pointer array	ptr$W$	$O(Wn)$	$W(n+1)$	$O(1)$
Plain [44]	bvPL	$O\left(\frac{Wm}{B}\right)$	$\frac{2Wm}{B}$	$O(1)$
Interleaved [44]	bvIL	$O\left(\frac{Wm}{B} + \frac{Wm}{L}\right)$	$2Wm\left(\frac{1}{B} + \frac{64}{L}\right)$	$O\left(\log \frac{Wm}{B}\right)$
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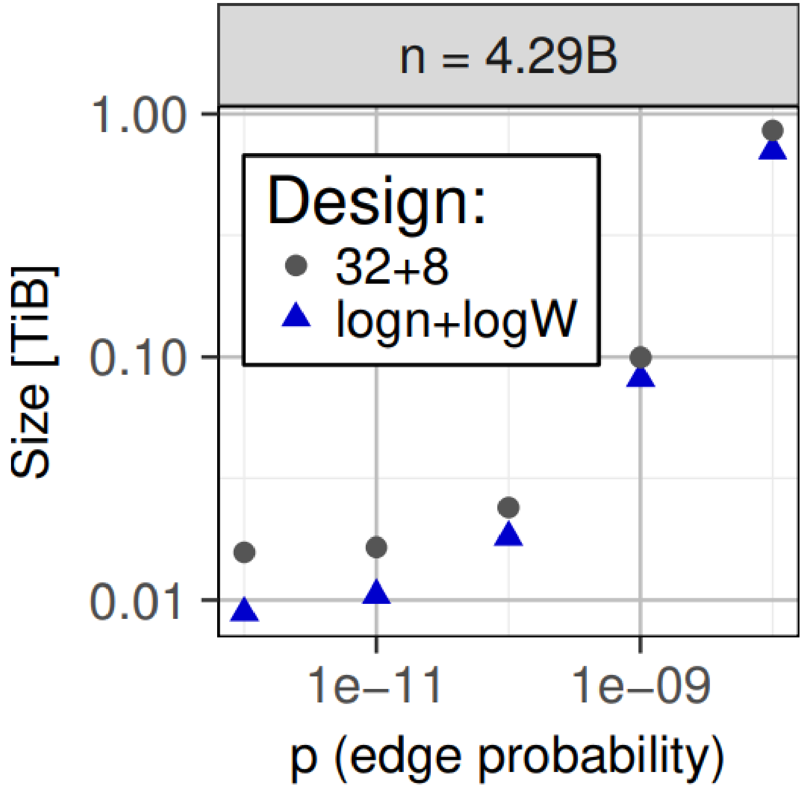
Key methods

Use the sdsl-lite sequential library of succinct bit vectors [1] and investigate if it fares well when being accessed by multiple threads

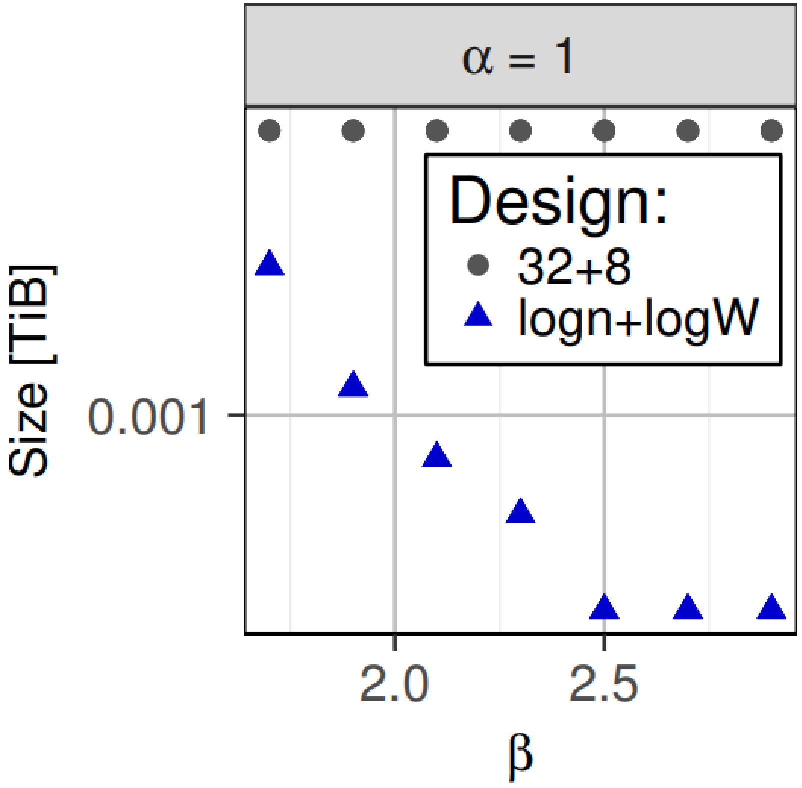
[1] S. Gog. SDSL-Lite Succinct Library. 2015.

1 Log (Vertex labels), Log (Edge weights) Storage

Power-law model



Random-uniform model



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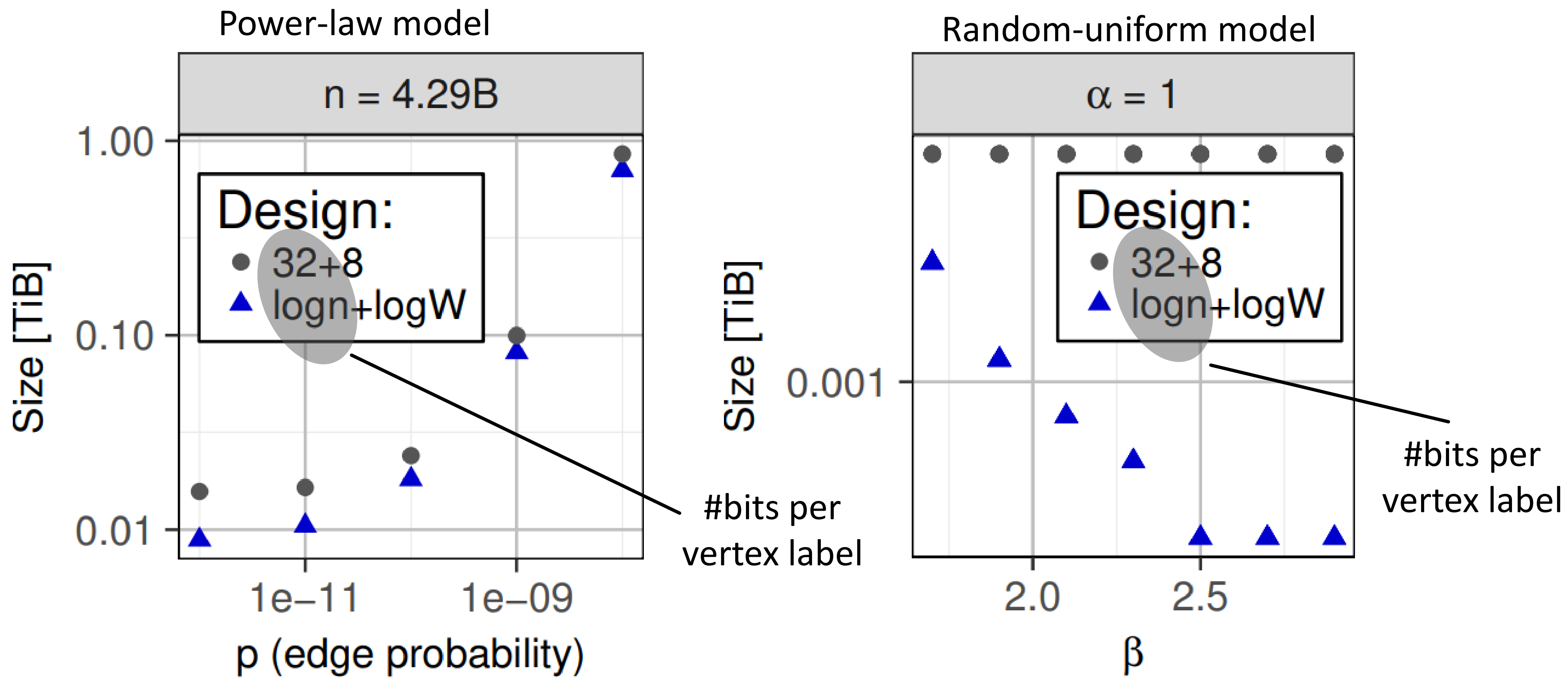

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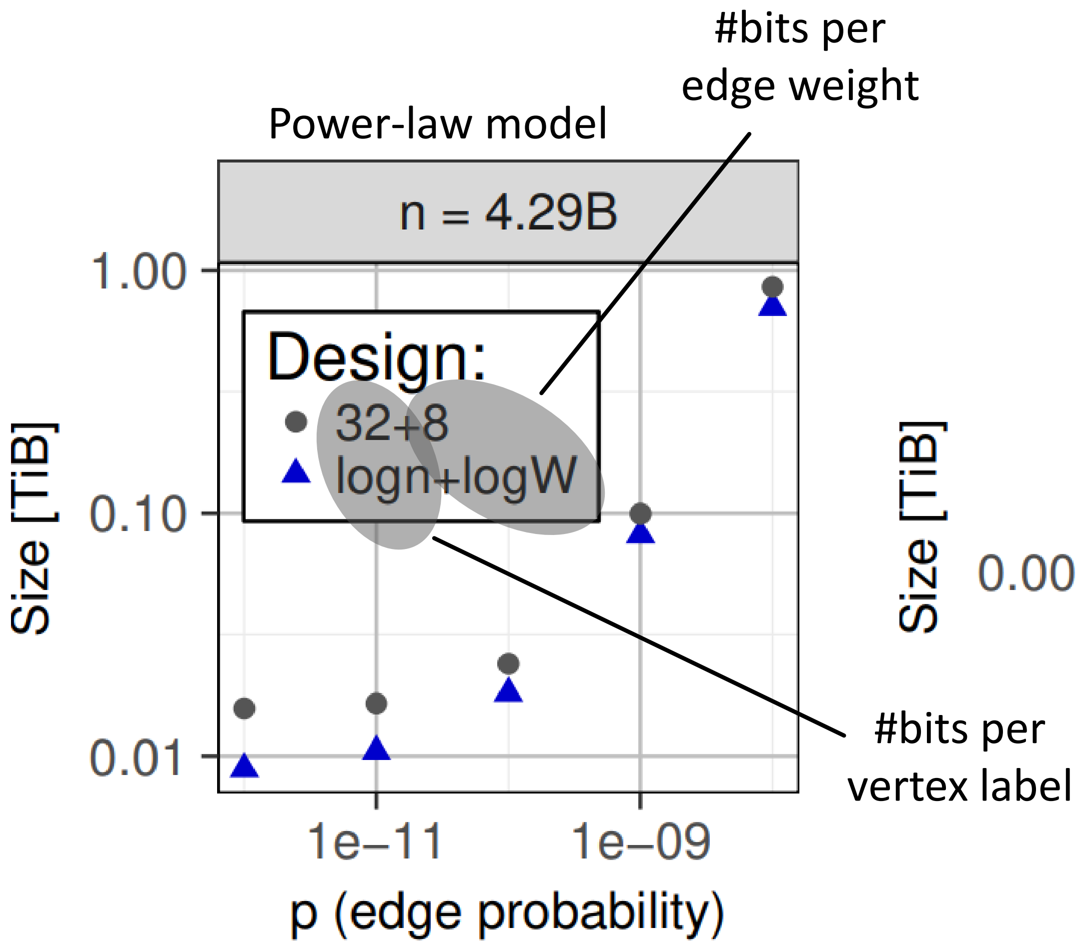
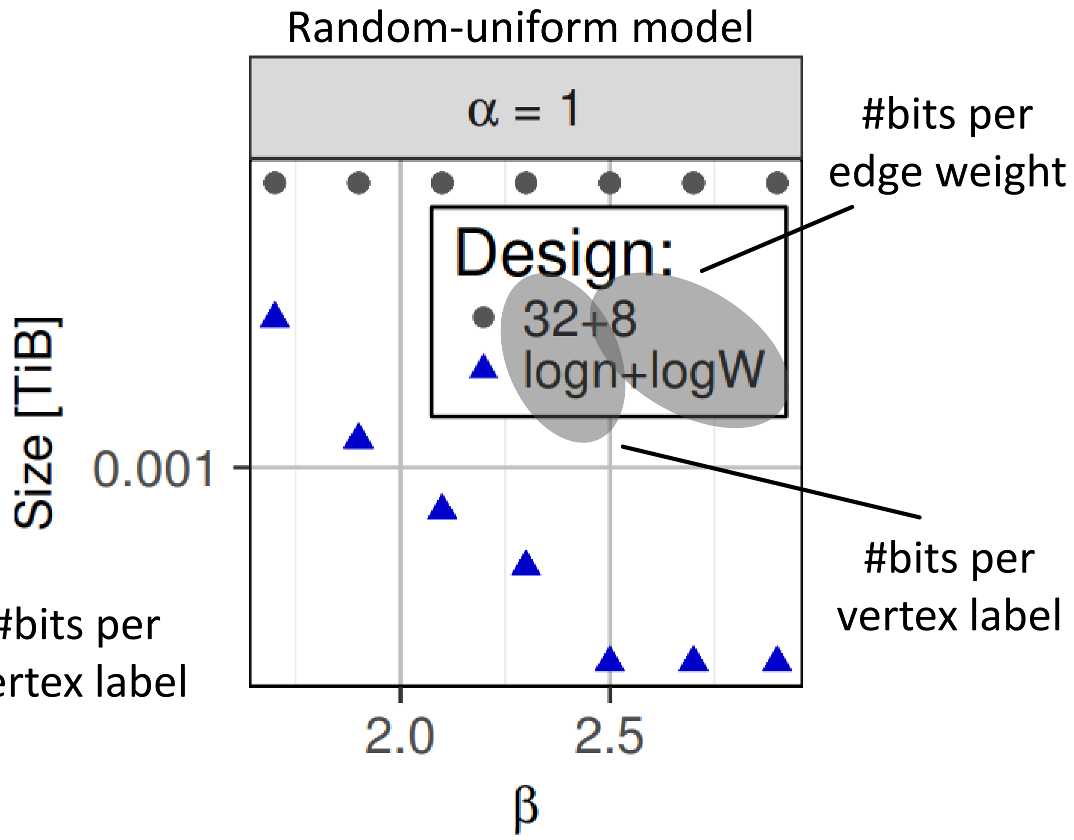
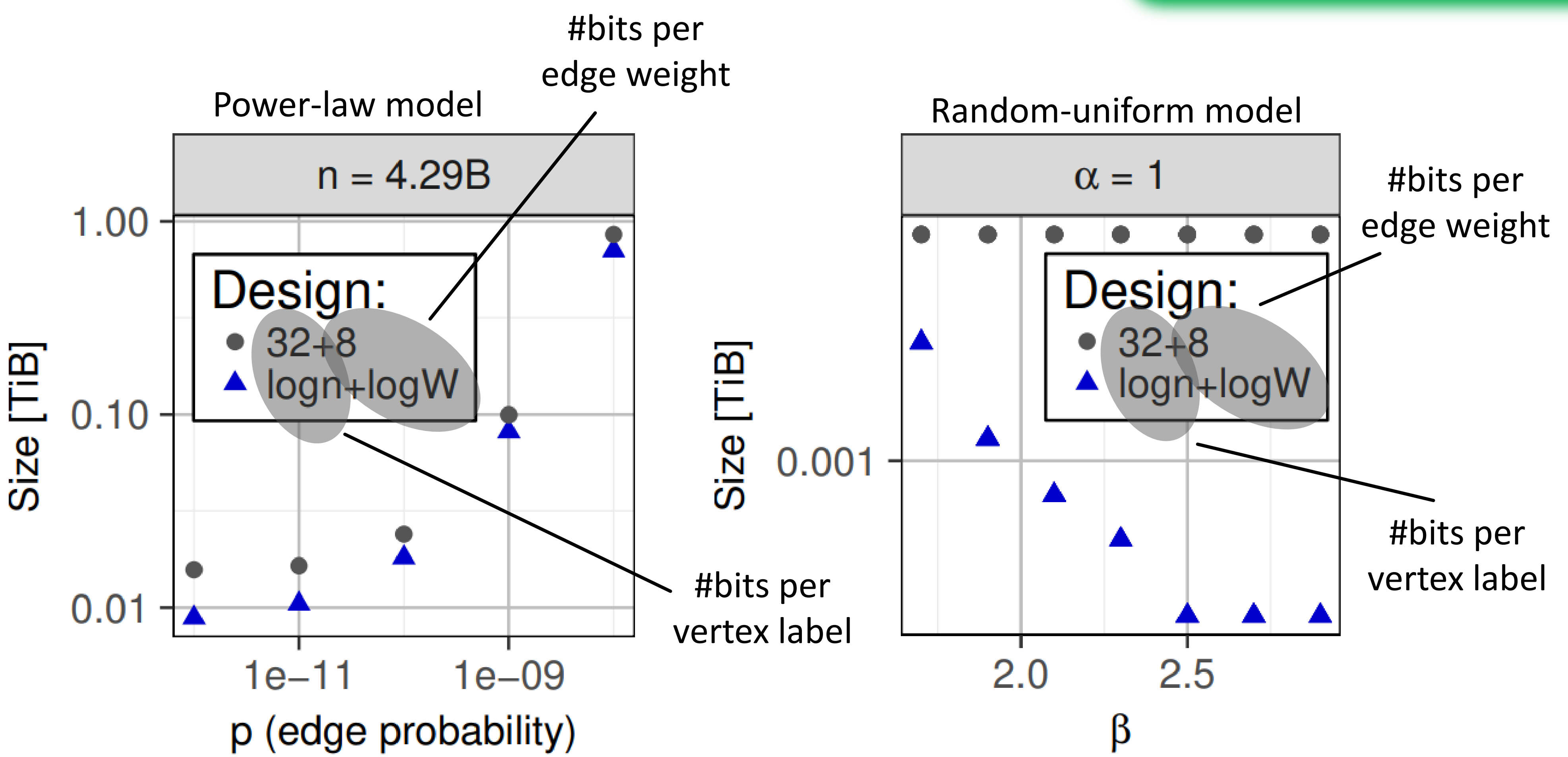




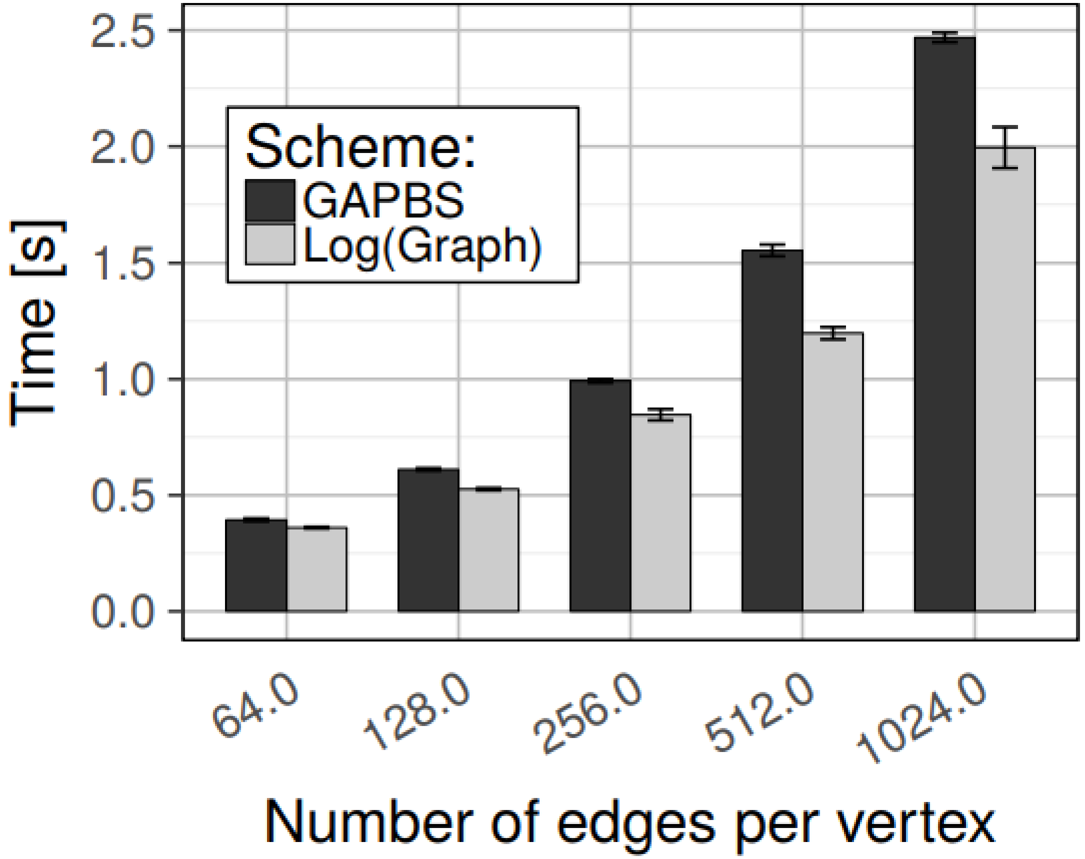
1 **Log (Vertex labels), Log (Edge weights) Storage**

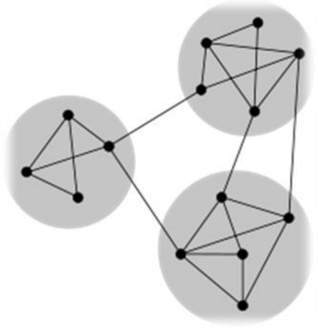
Log(Graph) consistently reduces storage overhead (by 20-35%)



1 Log (Vertex labels), Log (Edge weights) Performance

SSSP

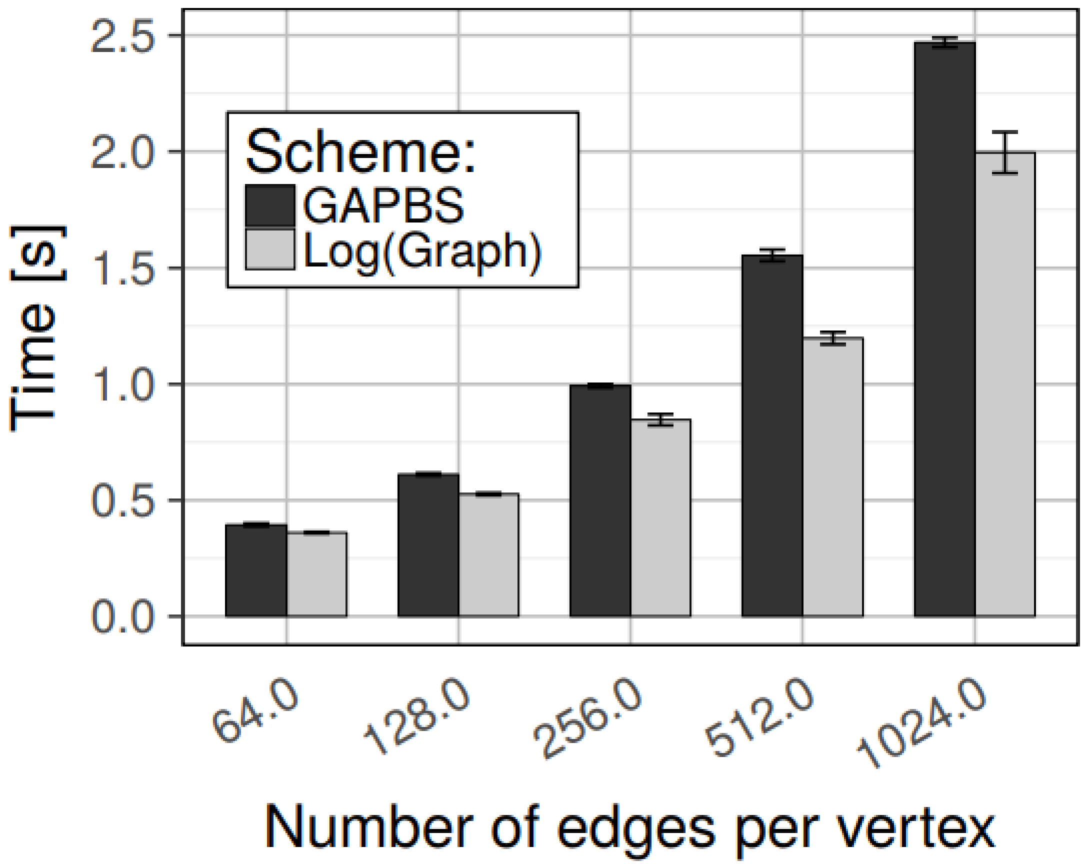




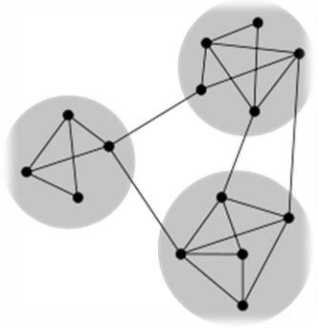
Kronecker graphs
Number of vertices: 4M

SSSP

1 **Log (Vertex labels), Log (Edge weights)** **Performance**



Log(Graph) **accelerates** GAPBS



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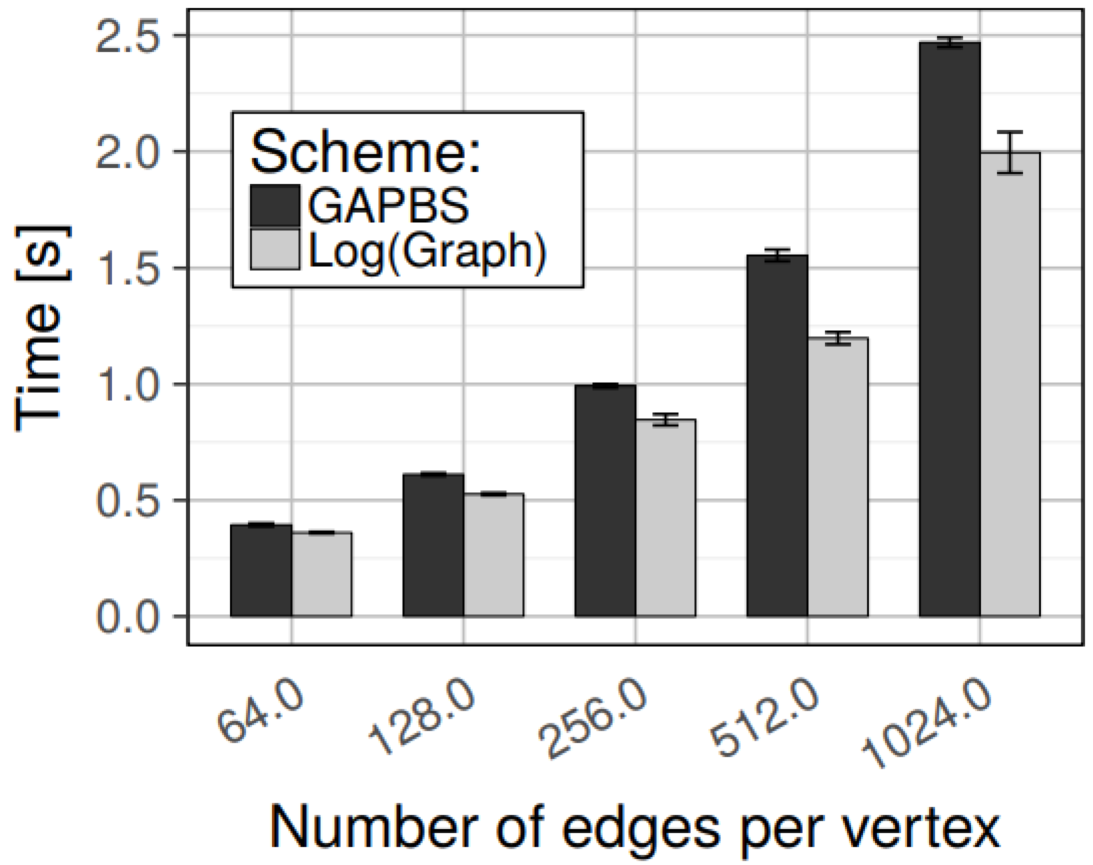

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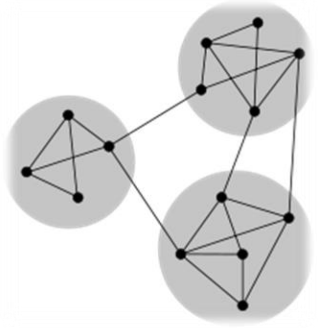


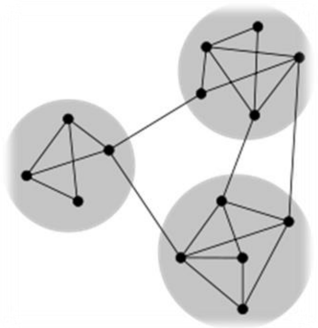
Kronecker graphs
Number of vertices: 4M

Both storage and performance are improved **simultaneously**

1 Log (Vertex labels), Log (Edge weights) Performance

Betweenness Centrality

"**LG**": Log(Graph)
Trad: Traditional (non compressed, GAPBS)
"**g**": global scheme
"**l**": local scheme
"**gap**": additional gap encoding



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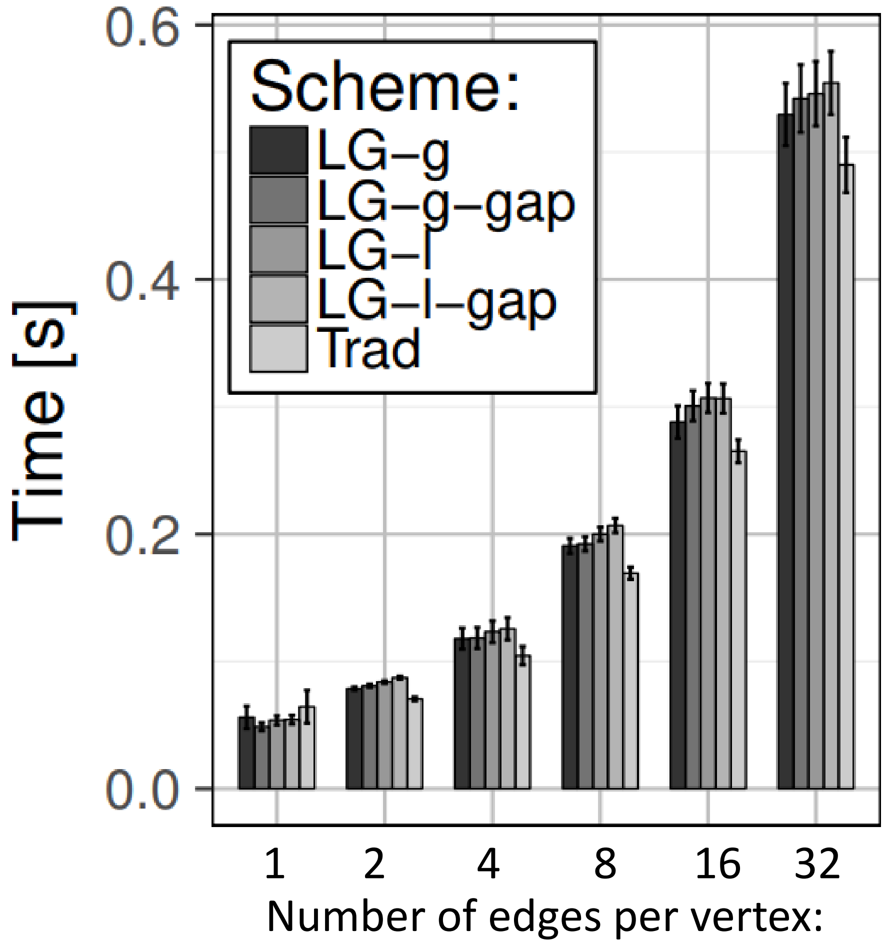
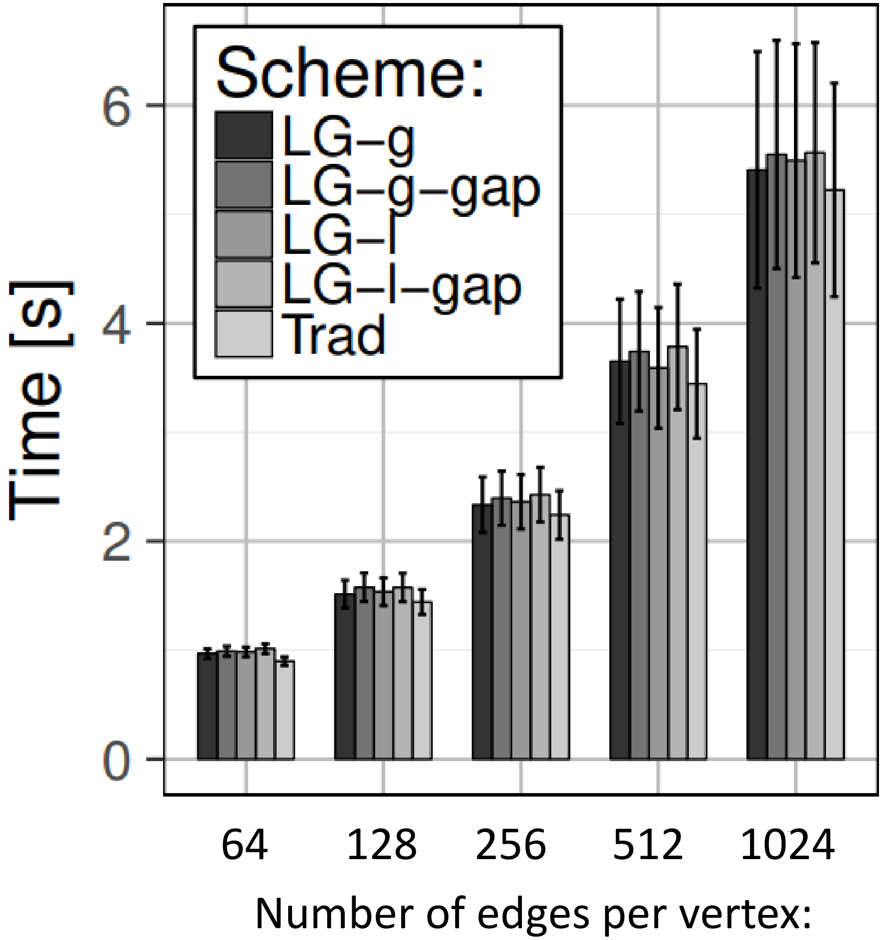
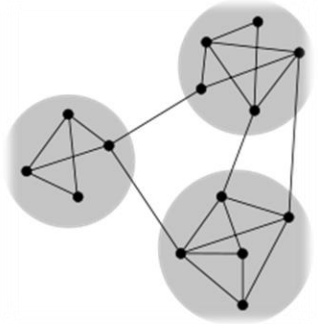


Dense graphs





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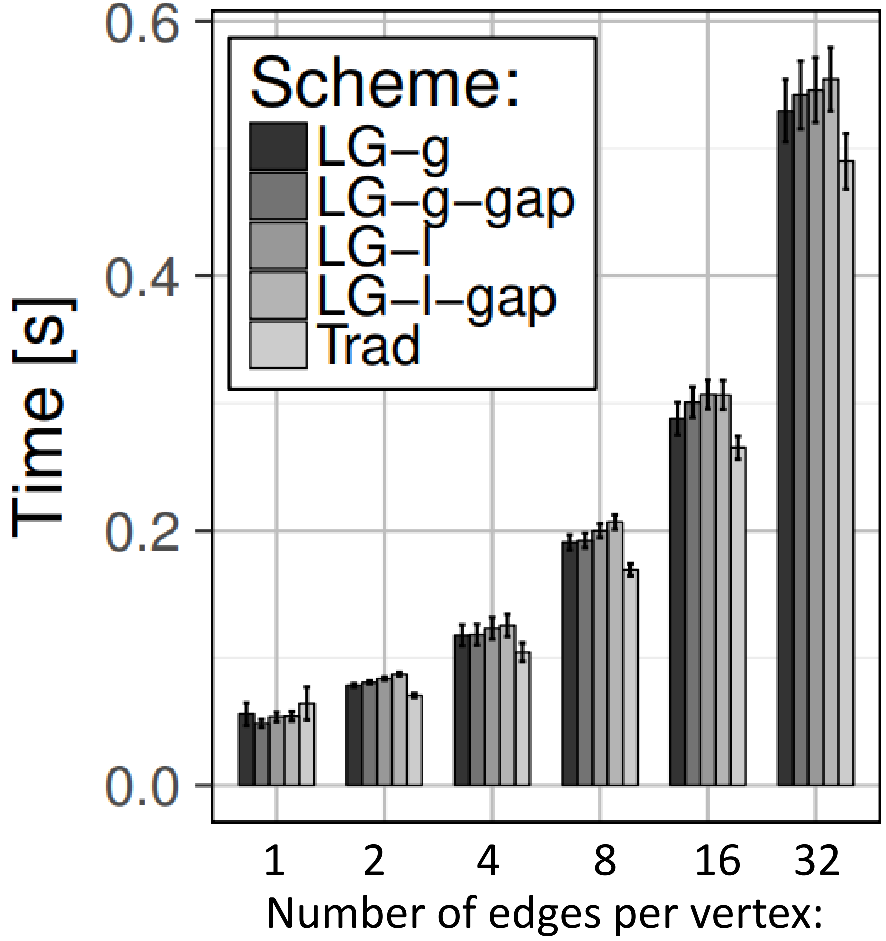

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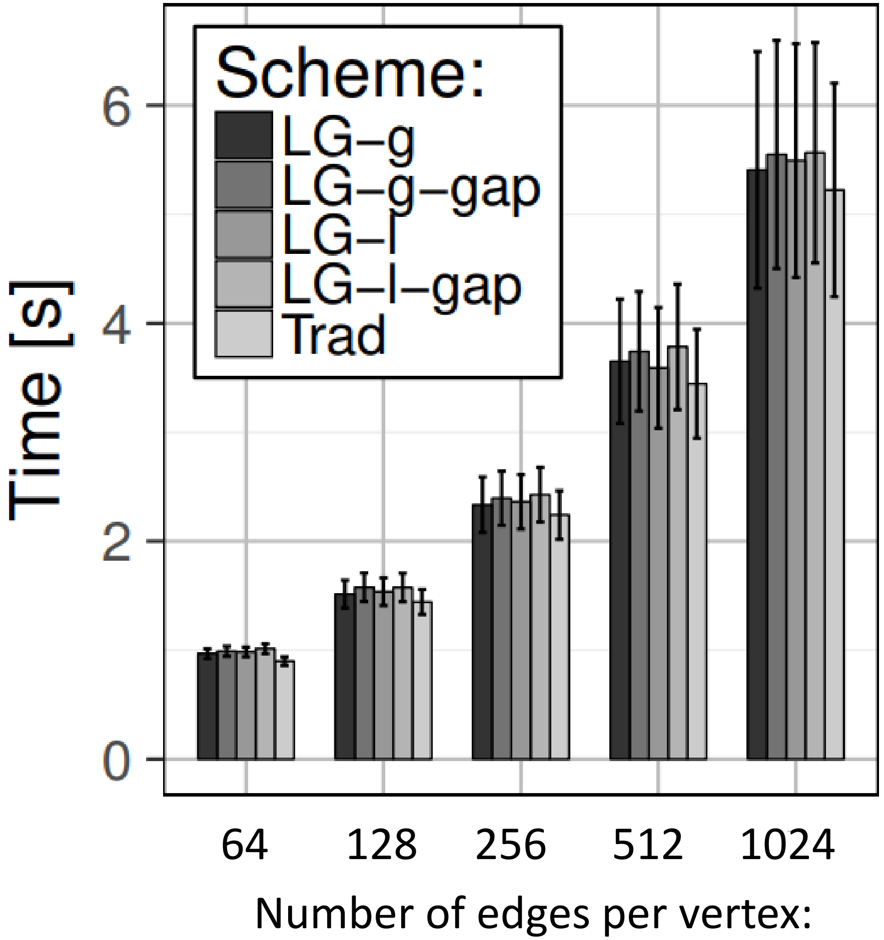
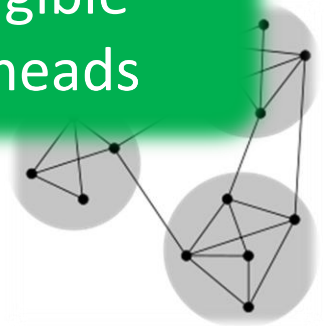


Log(Graph) incurs negligible overheads



Kronecker graphs
Number of vertices: 4M

1 **Log (Vertex labels), Log (Edge weights) Performance**

BFS

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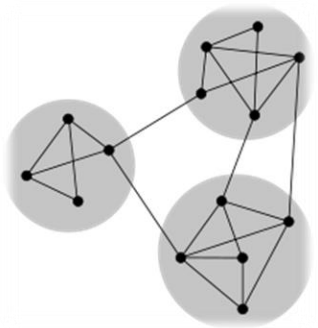

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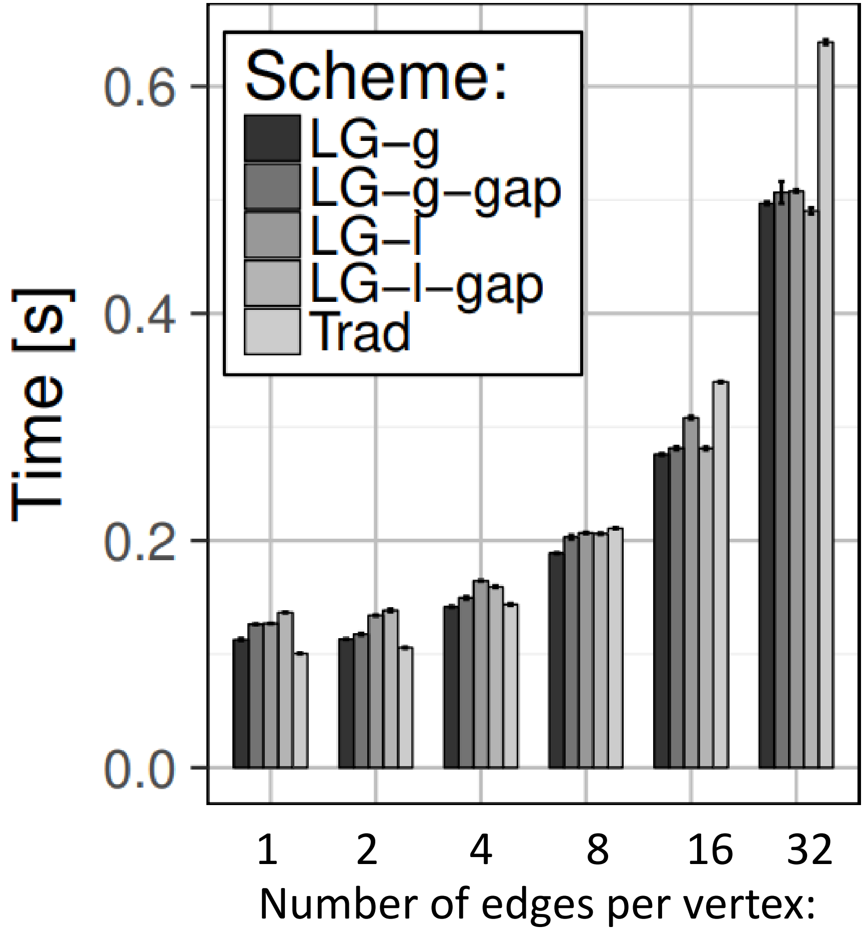

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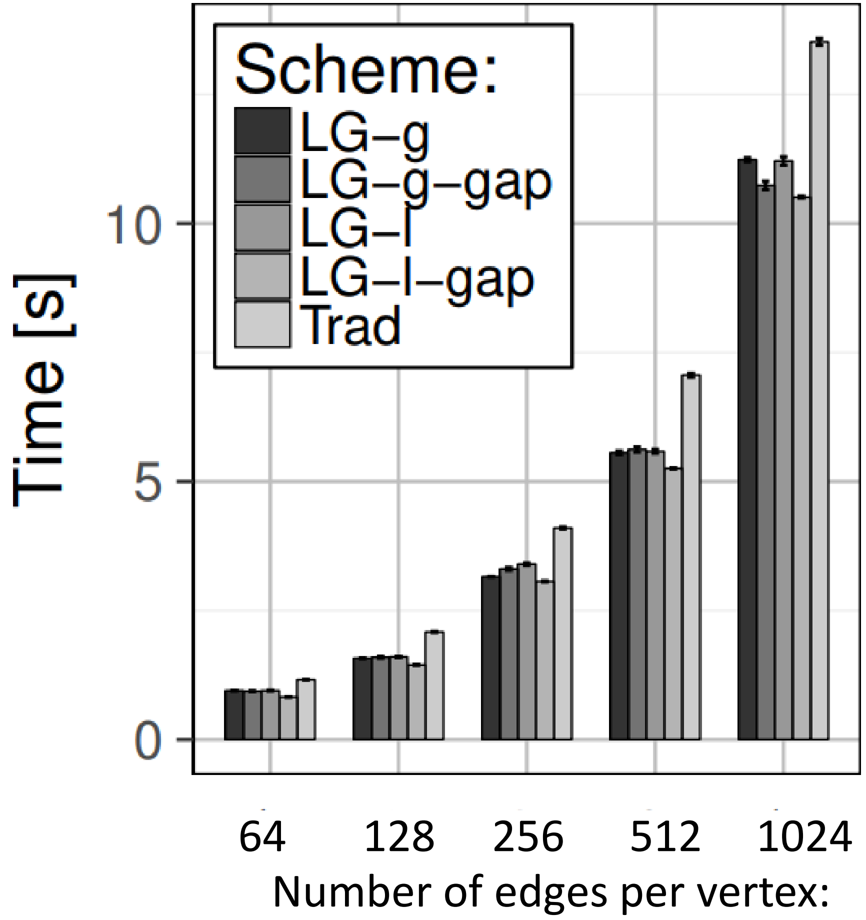

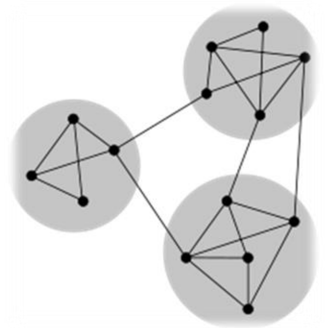

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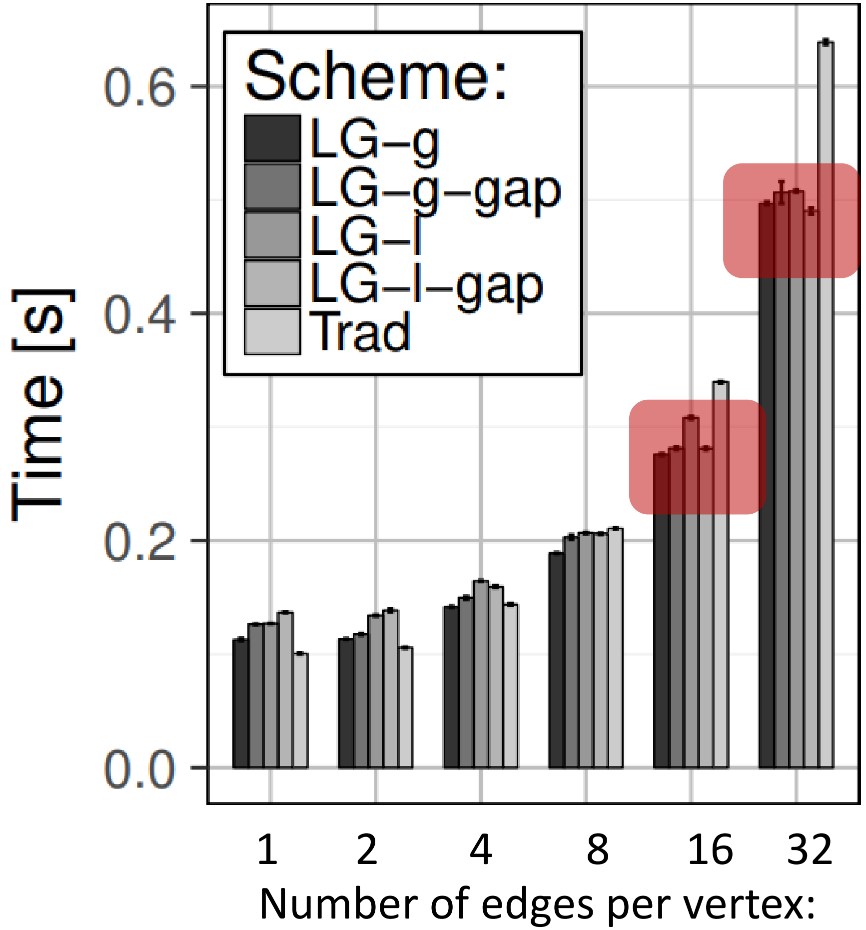

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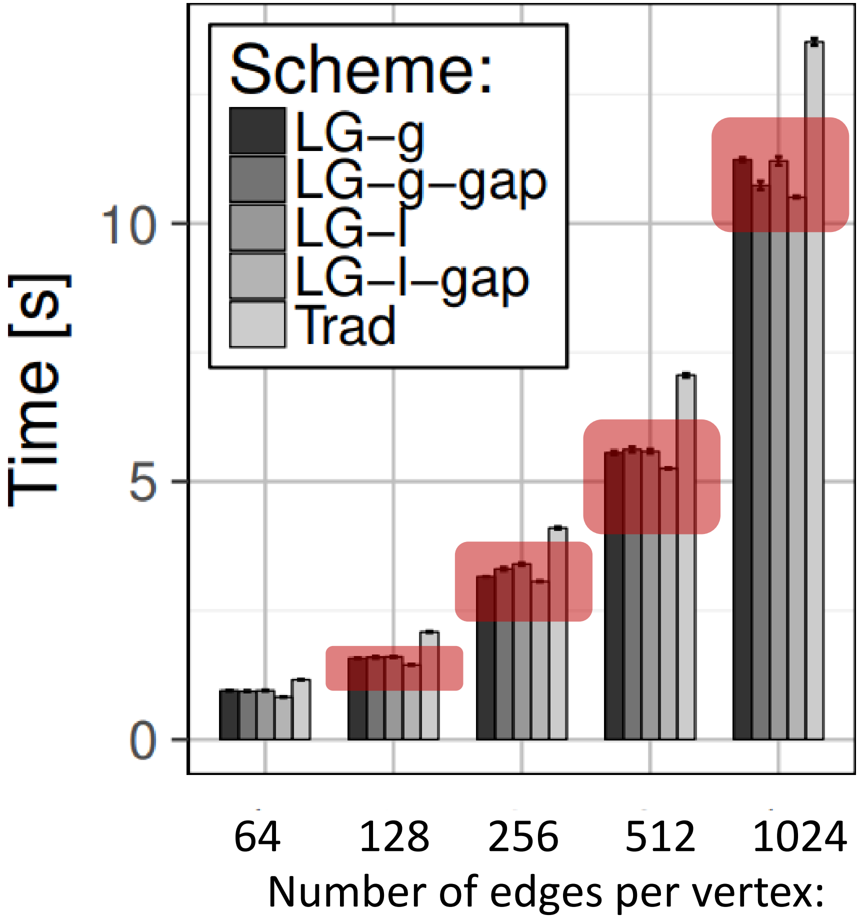

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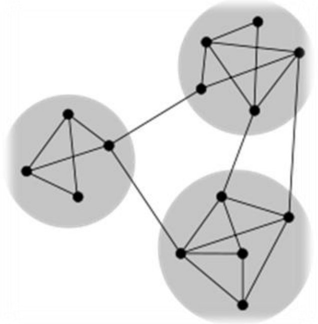

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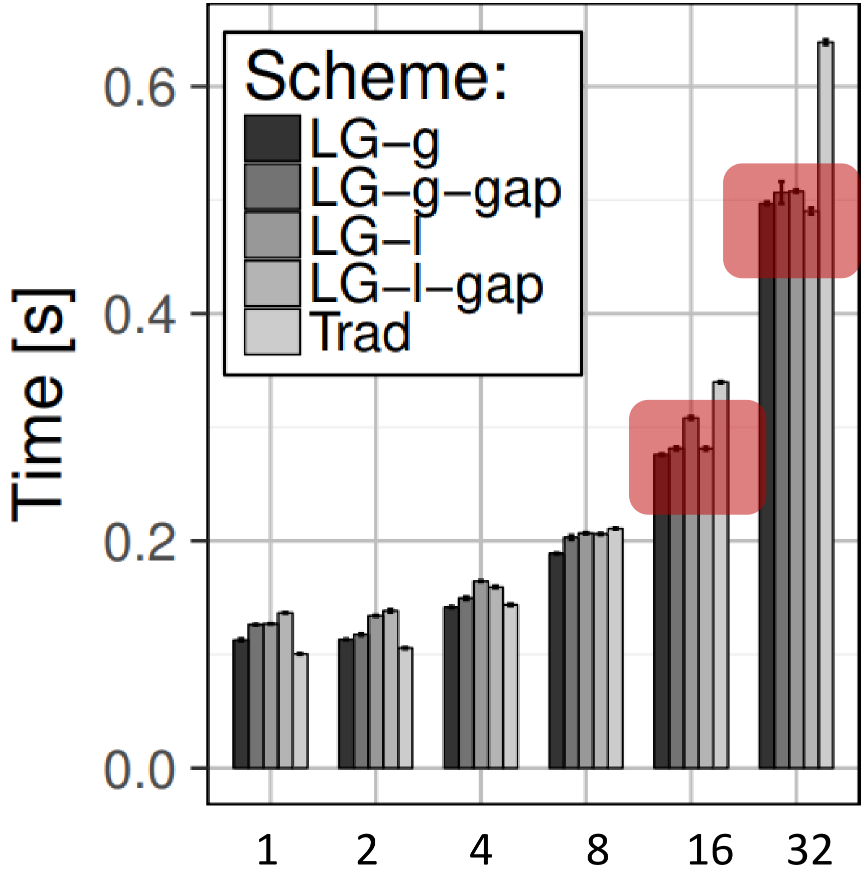
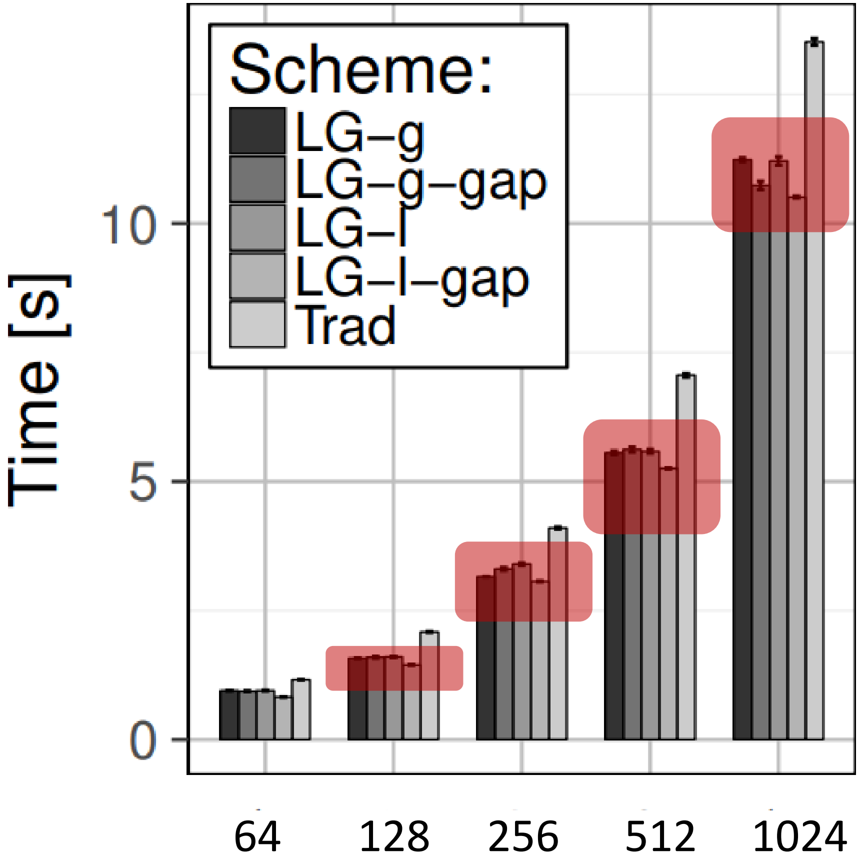


Number of edges per vertex:

Dense graphs



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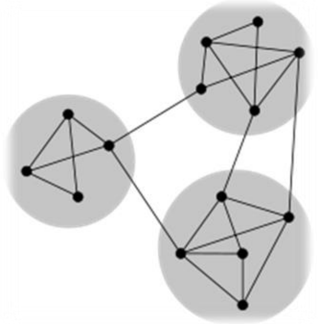
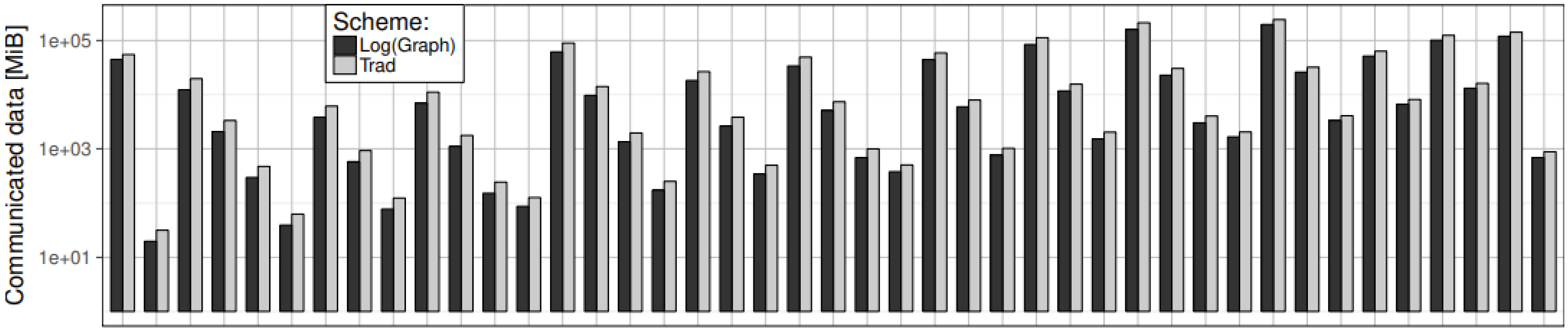


Kronecker graphs
Number of vertices: 4M

1 **Log (Vertex labels), Log (Edge weights)** **Communicated data**

PageRank

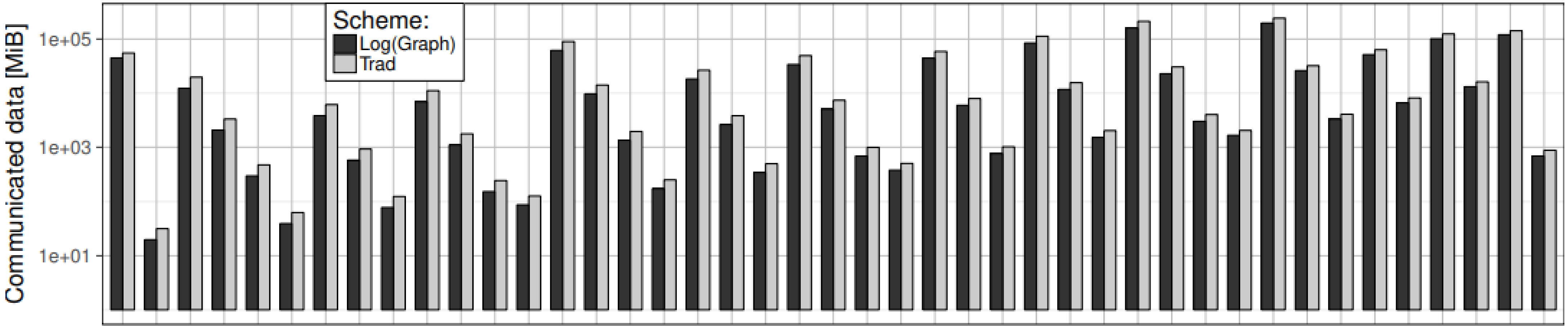


Various real-world and synthetic graphs

1024 compute nodes

PageRank

1 **Log (** Vertex labels **), Log (** Edge weights **)** **Communicated data**



Various real-world and synthetic graphs

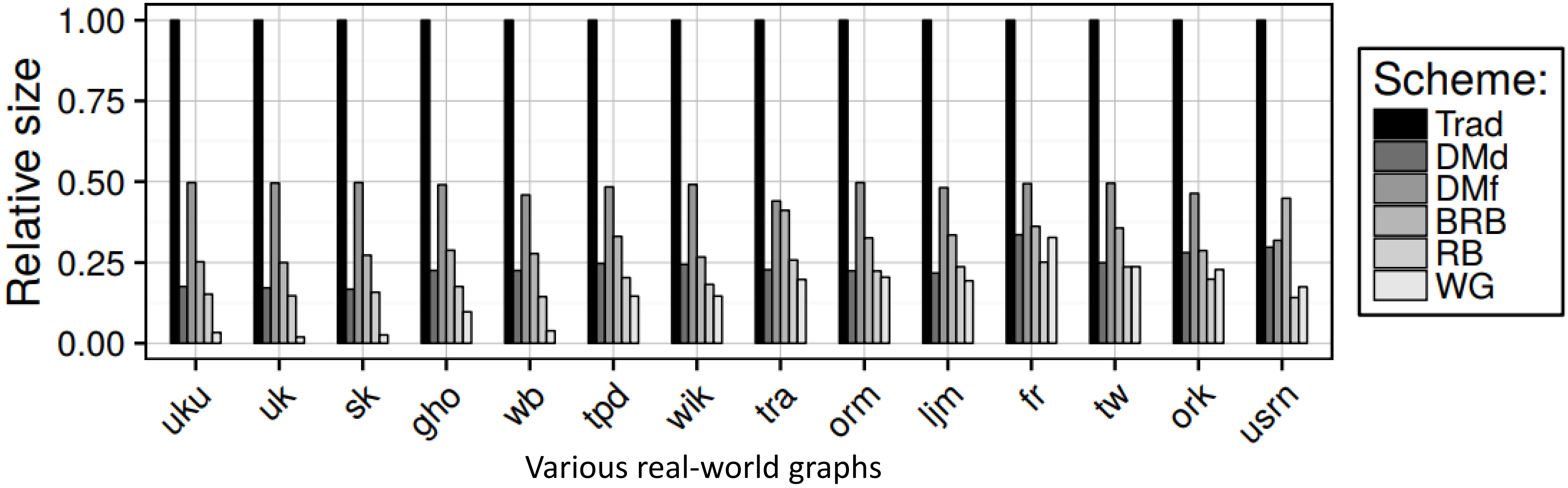
The amount of communicated data is consistently reduced by ~37%

1024 compute nodes

3 **Log (** Adjacency structure **)** **Storage**

3 **Log (Adjacency structure) Storage**

Trad: Traditional adjacency array
DMd / DMf: Degree Minimizing (without / with gap encoding)
WG: WebGraph compression
BRB, RB: Schemes targeting certain specific classes of graphs



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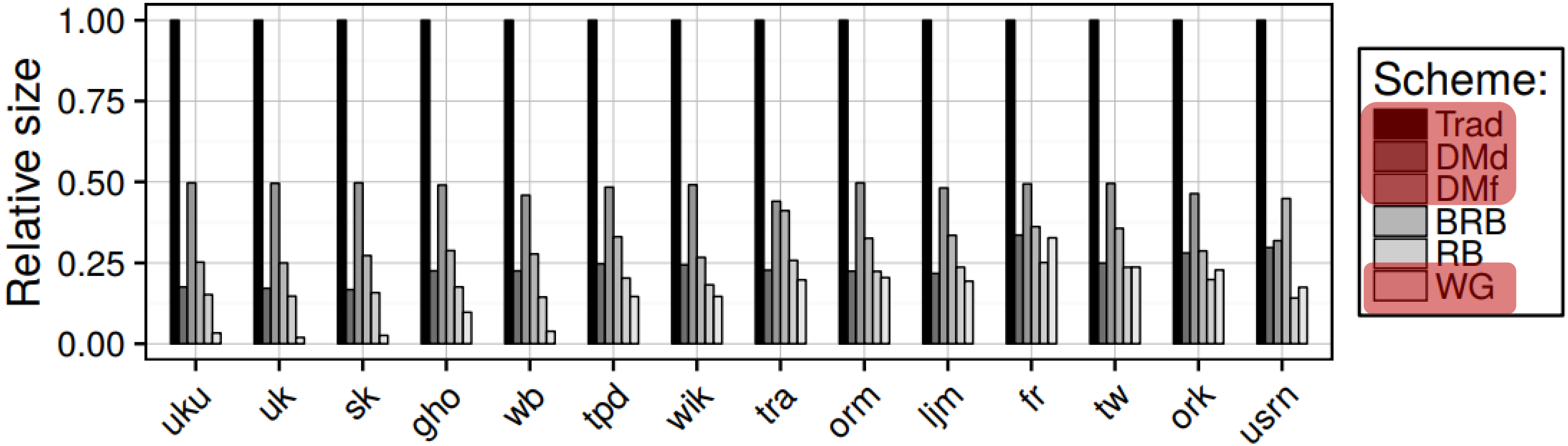

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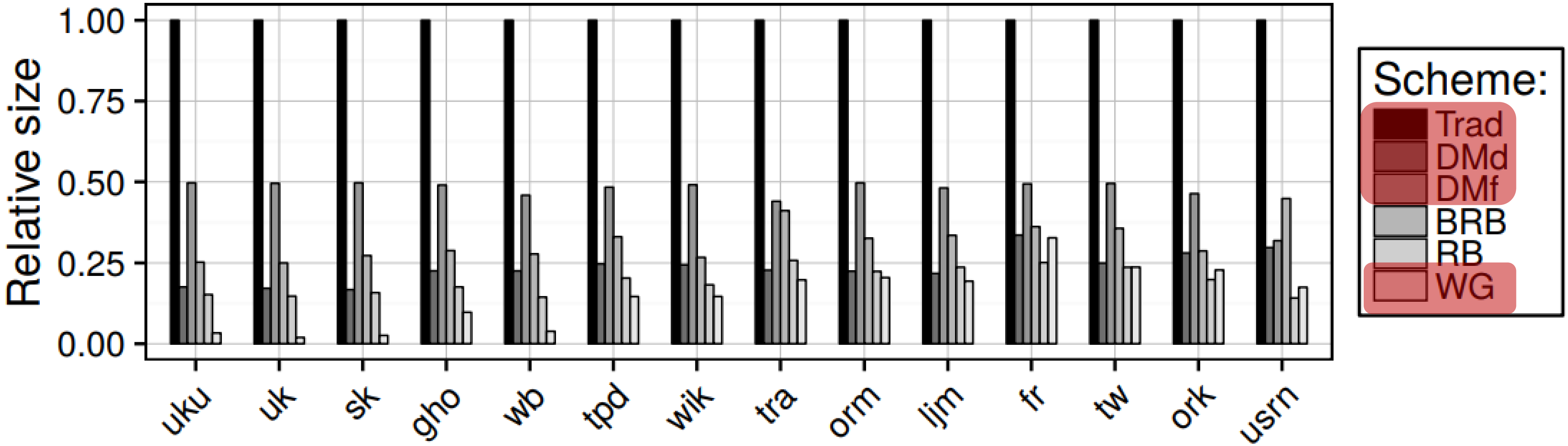

Various real-world graphs

Lots of data ☺

Conclusions:

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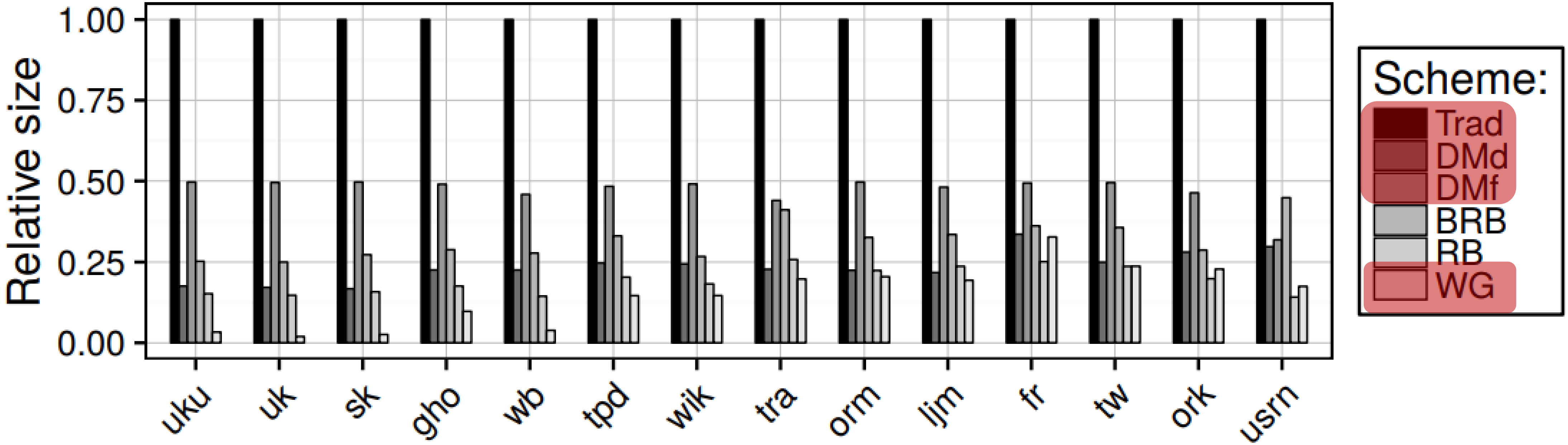

Lots of data ☺

Conclusions: WebGraph **best for web graphs ☺**

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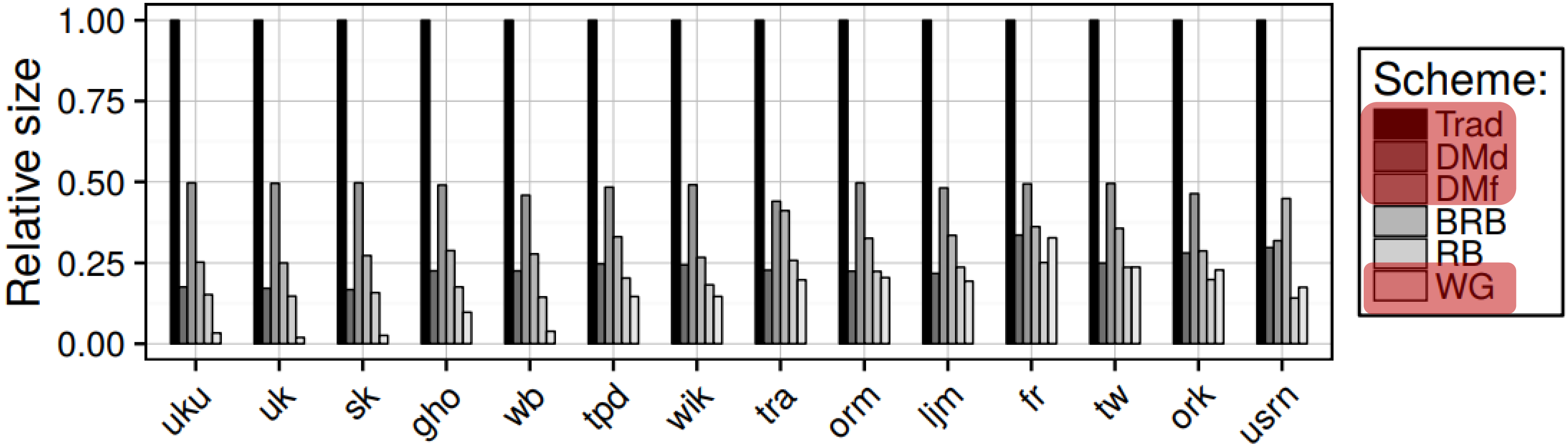

Various real-world graphs

Lots of data ☺

Conclusions:

WebGraph **best for web graphs** ☺

BRB, RB: various tradeoffs but very expensive preprocessing (details in the paper)

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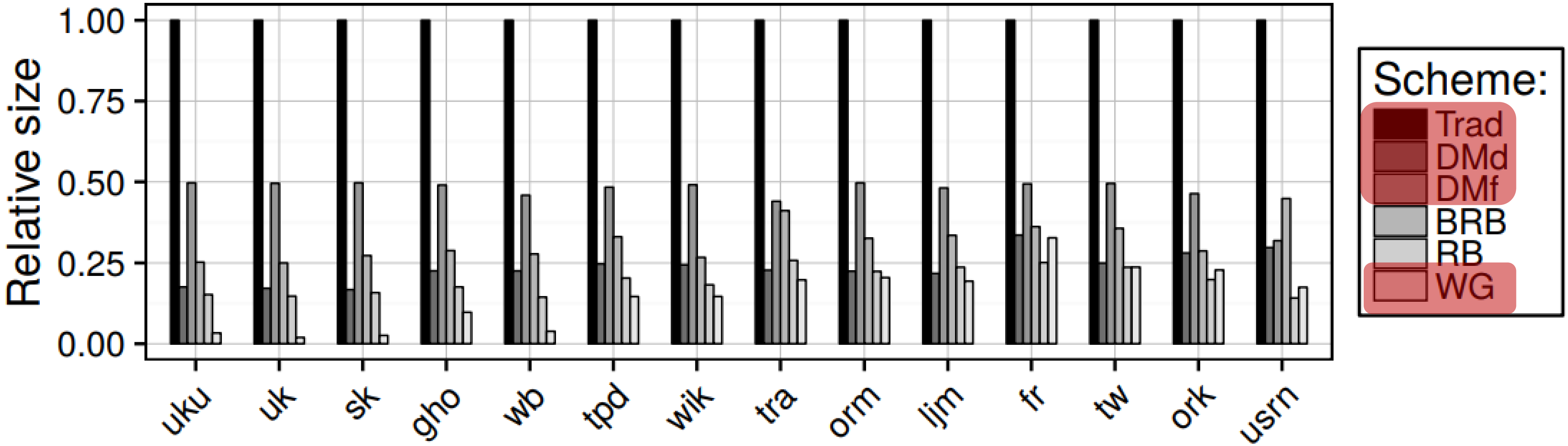

Various real-world graphs

Lots of data ☺

Conclusions:

WebGraph **best for web graphs ☺**

DMd: much better than DMf, often comparable to others

BRB, RB: various tradeoffs but very expensive preprocessing (details in the paper)

3 **Log (Adjacency structure) Performance**

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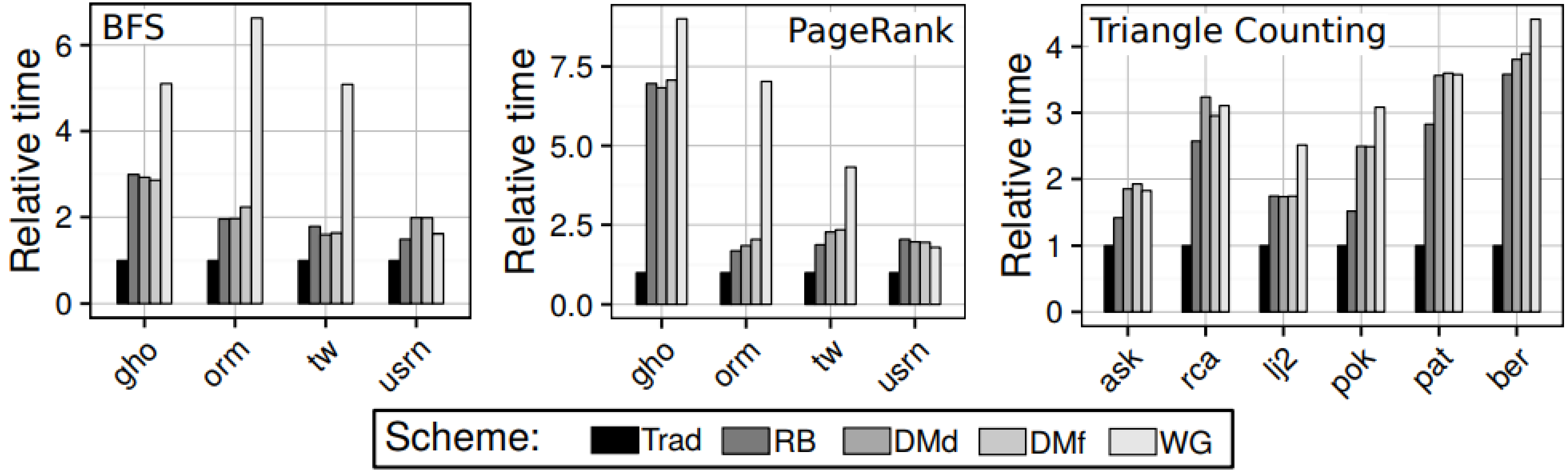

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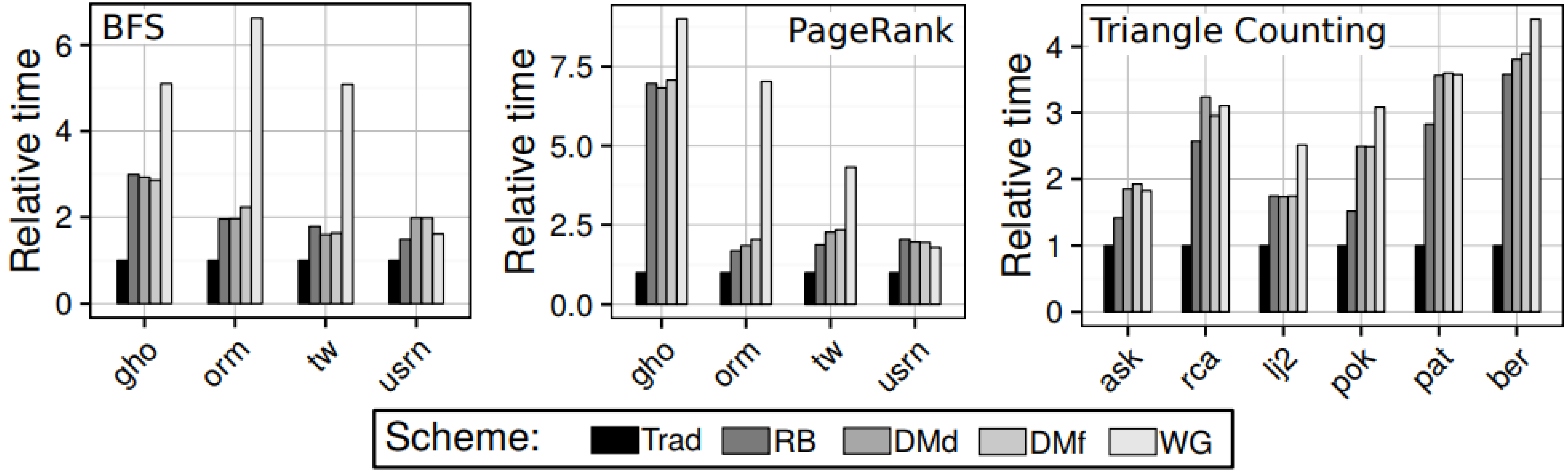

WebGraph is the slowest

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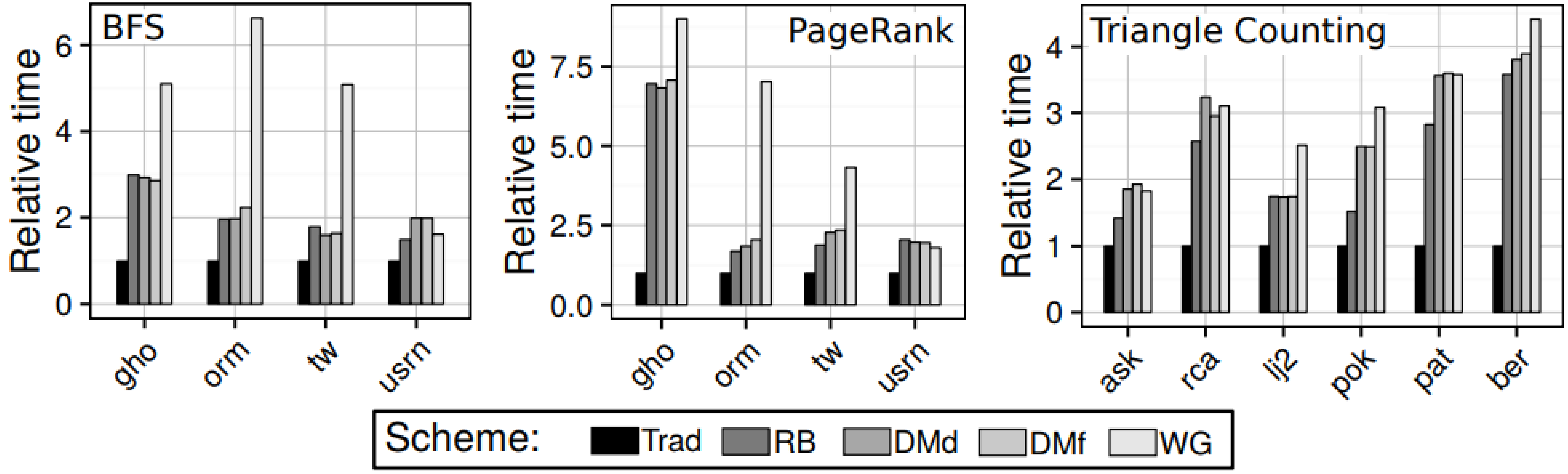

WebGraph is the slowest

DM, RB: comparable

Log(Graph) full design…

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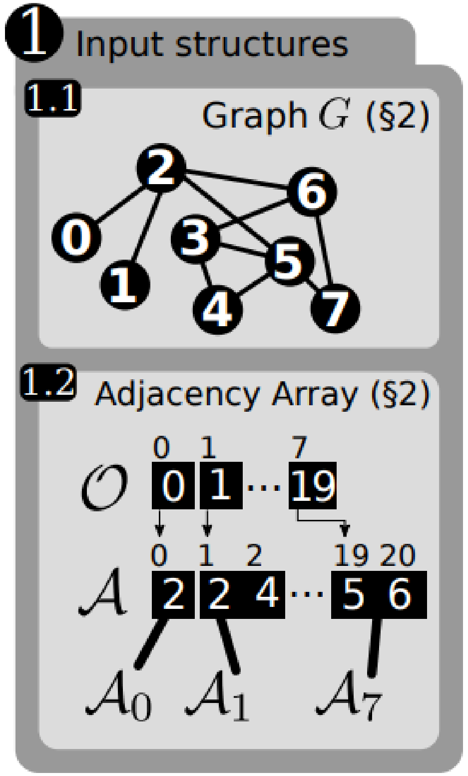

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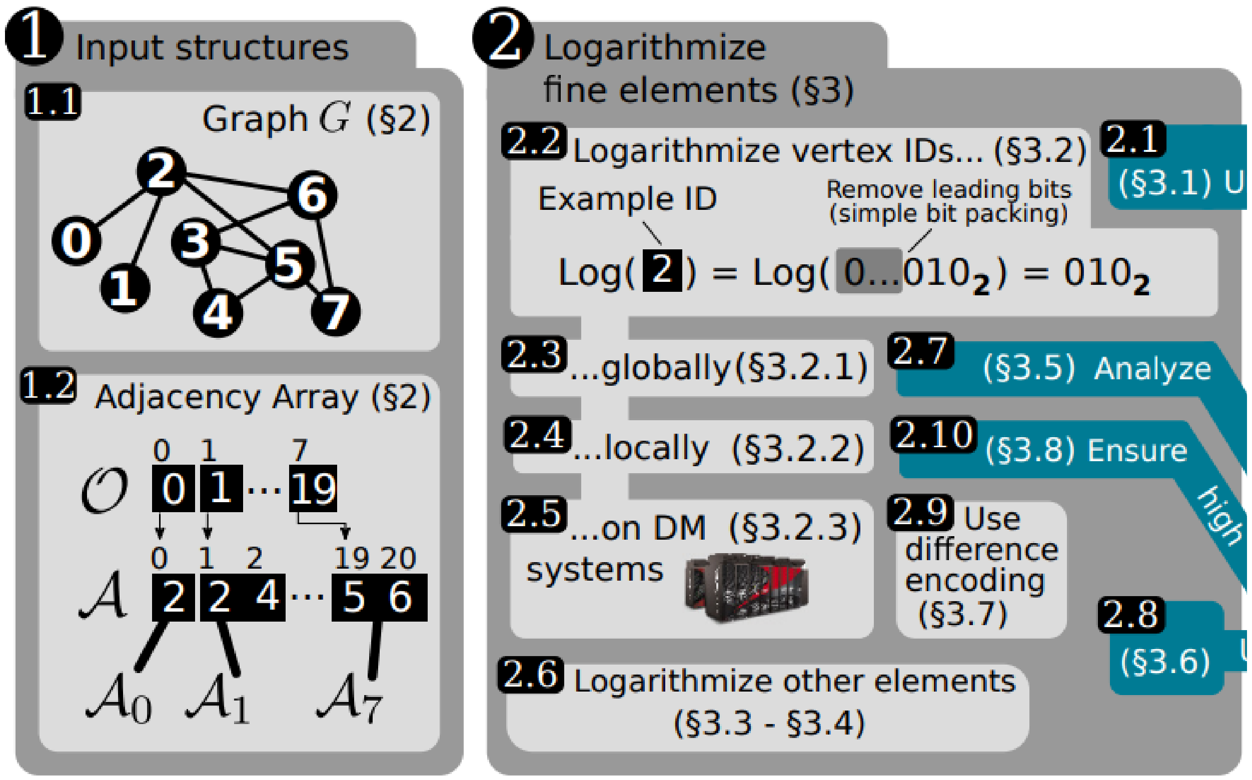

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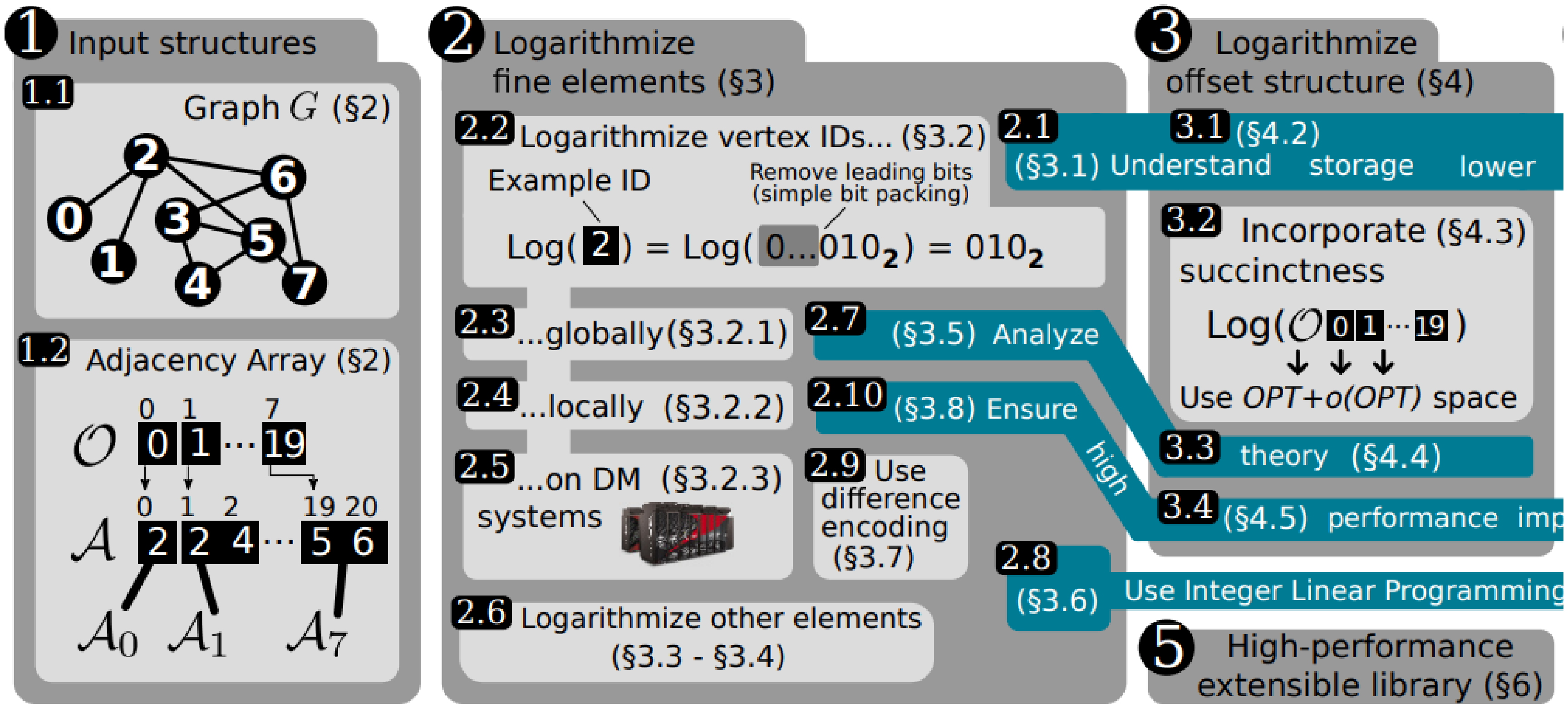

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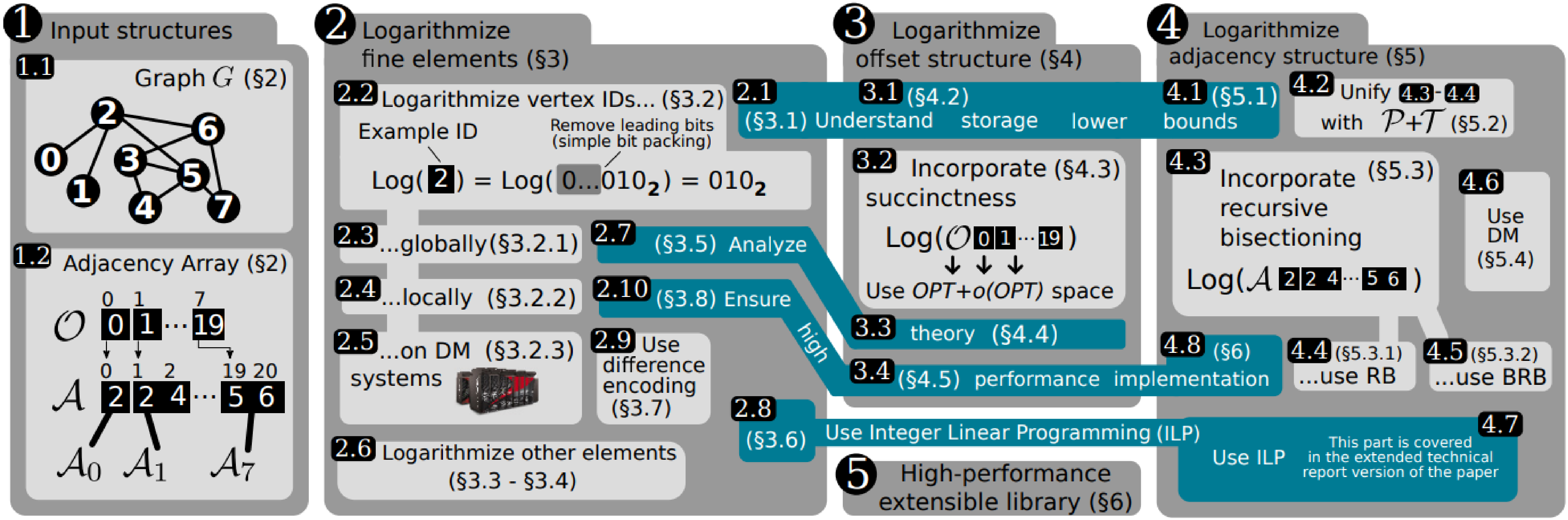

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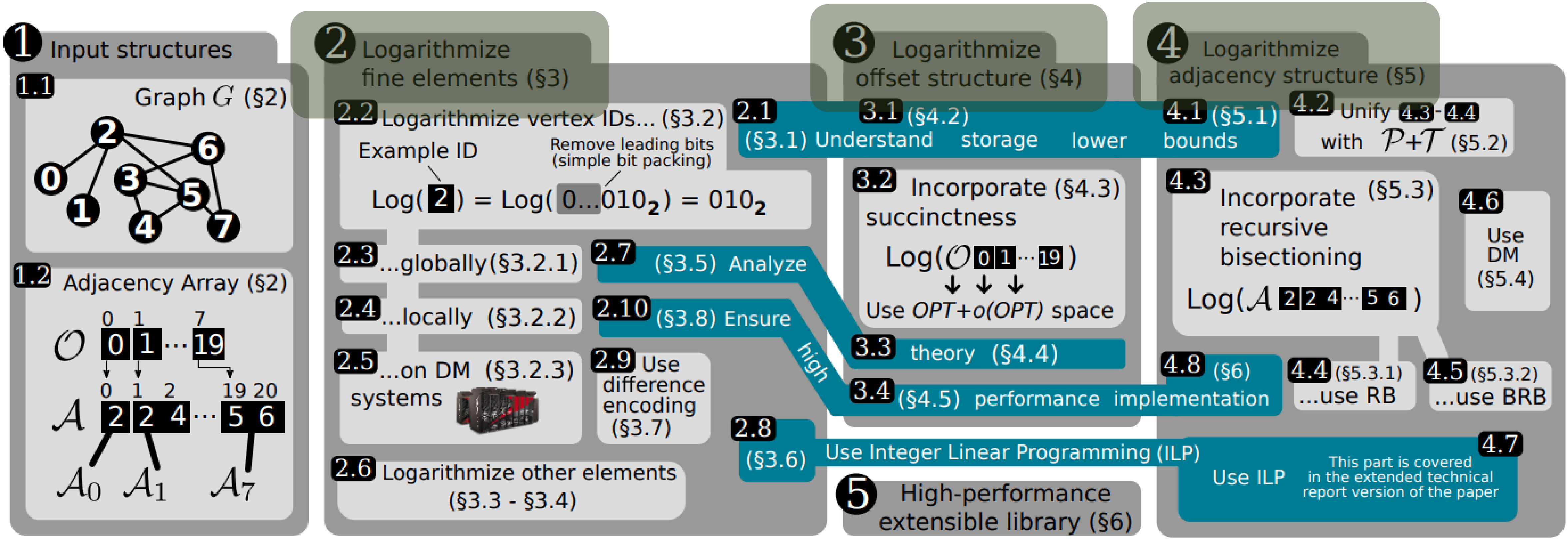

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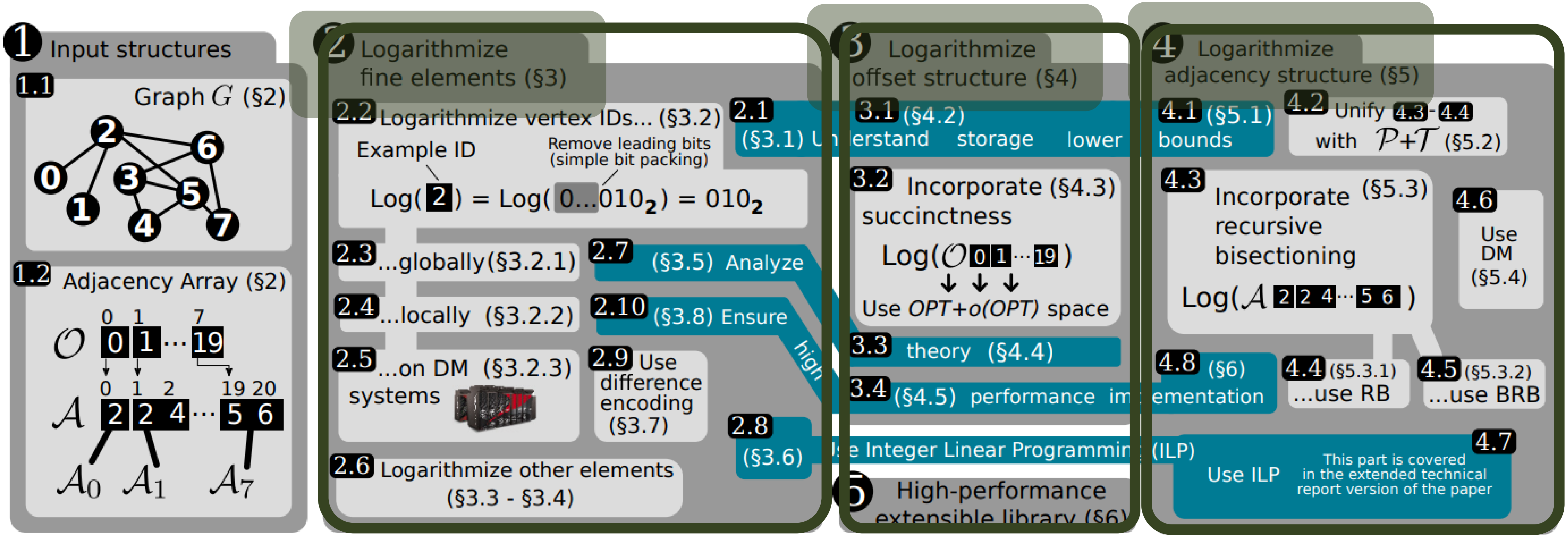
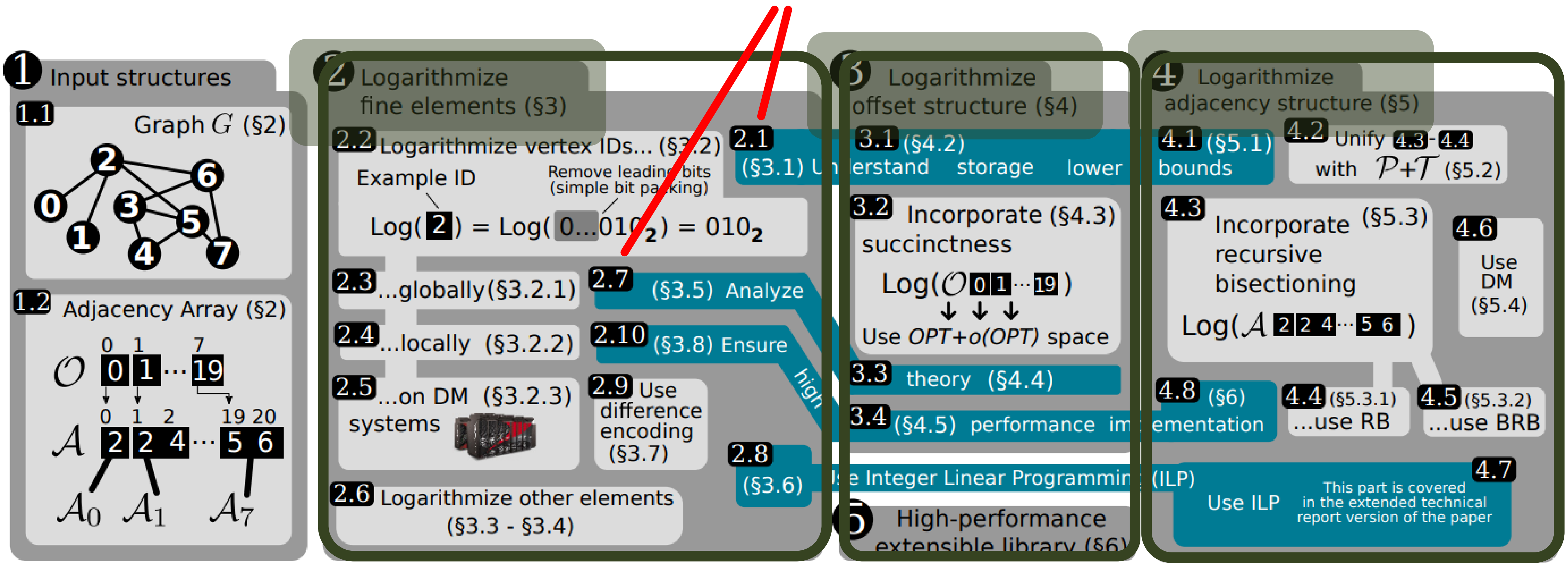


Log(Graph) full design…

Understand storage lower bounds and the theory



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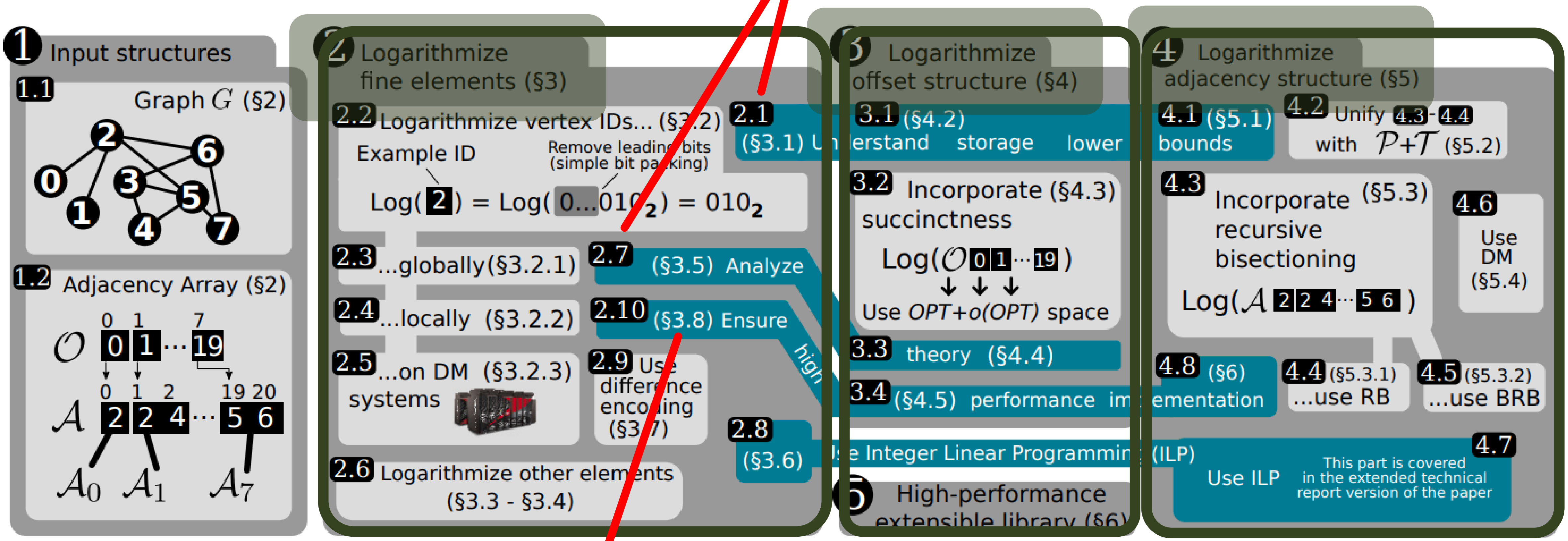
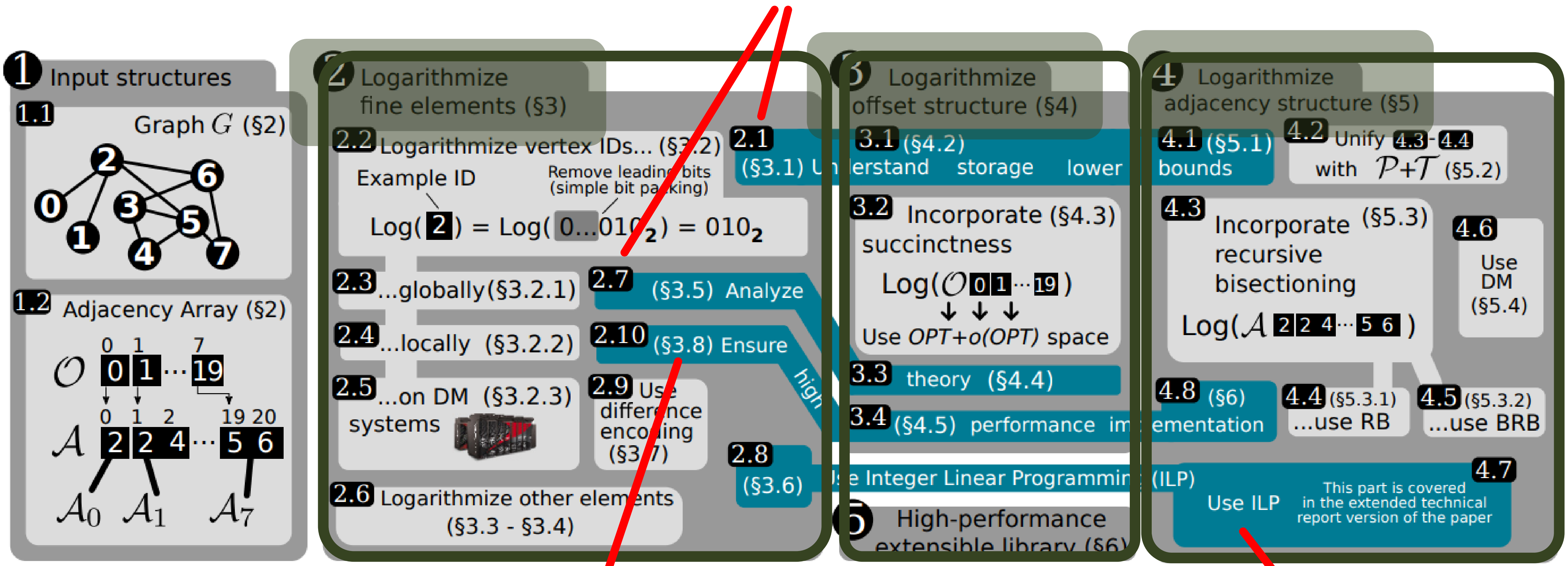


Ensure high-performance implementation

Log(Graph) full design...

Understand storage lower bounds and the theory



Ensure high-performance implementation

Use Integer Linear Programming (ILP) for more storage reductions

Key method (vertex labels)

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Bit packing: use $\lceil \log n \rceil$ bits for one vertex label

Key method (vertex labels)

Bit packing: use $\lceil \log n \rceil$ bits for one vertex label

Modern bitwise operations

Key method (vertex labels)

Bit packing: use $\lceil \log n \rceil$ bits for one vertex label

Modern bitwise operations

Key method (offsets)

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Succinct bit vectors:
understand state-of-the-art designs and use the best ones in a given context

$\mathcal{O}$	ID	**Asymptotic size** [bits]	**Exact size** [bits]	*select* or $\mathcal{O}[v]$
Pointer array	ptr$W$	$O(Wn)$	$W(n+1)$	$O(1)$
Plain [44]	bvPL	$O\left(\frac{Wm}{B}\right)$	$\frac{2Wm}{B}$	$O(1)$
Interleaved [44]	bvIL	$O\left(\frac{Wm}{B} + \frac{Wm}{L}\right)$	$2Wm\left(\frac{1}{B} + \frac{64}{L}\right)$	$O\left(\log \frac{Wm}{B}\right)$
Entropy based [31, 78]	bvEN	$O\left(\frac{Wm}{B} \log \frac{Wm}{B}\right)$	$\approx \log \binom{\frac{2Wm}{B}}{n}$	$O\left(\log \frac{Wm}{B}\right)$
Sparse [76]	bvSD	$O\left(n + n \log \frac{Wm}{Bn}\right)$	$\approx n\left(2 + \log \frac{2Wm}{Bn}\right)$	$O(1)$
B-tree based [1]	bvBT	$O\left(\frac{Wm}{B}\right)$	$\approx 1.1 \cdot \frac{2Wm}{B}$	$O(\log n)$
Gap-compressed [1]	bvGC	$O\left(\frac{Wm}{B} \log \frac{Wm}{Bn}\right)$	$\approx 1.3 \cdot \frac{2Wm}{B} \log \frac{2Wm}{Bn}$	$O(\log n)$

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Recursive partitioning: use representations that assume more about graph structure to enable better bounds

Key method (offsets)

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C++ templates to reduce overheads in performance-critical kernels

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